# Achieving Proportional Fairness using Local Information in Multi-hop Aloha Networks

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**Abstract**

We address the problem of fair medium access control in multi-hop Aloha networks, with the objective of achieving proportionally fair rates. We consider a general multi-hop wireless network, and show how the attempt probabilities/attempt rates in Aloha protocols should be set so that the achieved rates are globally proportionally fair. For both slotted and unslotted Aloha, we argue that it is possible to achieve globally fair rates with local topology information.

## I. Introduction and System Model

We address the issue of fair medium access control (MAC) in multi-hop aloha networks. Our notion of fairness is that of *proportional fairness* [2], defined as a fairness measure that maximizes the aggregate log-utility of the system. We consider both the cases of slotted and unslotted Aloha, we argue that it is possible to achieve globally fair rates with local topology information. More specifically, we show that a node can set its attempt probability/attempt rate optimally only by knowing some minimal information about its two-hop neighborhood.

Next we describe the system model that we use. A general wireless ad-hoc network can be modeled as an undirected graph $G = (N, L)$, where $N$ and $L$ respectively denote the set of nodes and the set of (undirected) links in the network, and a link exists between two nodes if and only if they can hear each other (we assume a symmetric hearing matrix). Note that there are $2|L|$ possible communication pairs in graph $G$, out of which only a few pairs may be actively communicating. The set of active communication pairs is represented by the set of (directed) edges $E$. We assume that every edge $e \in E$ is always backlogged. We assume, without loss of generality, that all the nodes share a single wireless channel of unit capacity.

For any node $i$, $K_i = \{j : (i, j) \in L\}$ represents the set of nodes that can hear node $i$ (excluding node $i$ itself), and is referred as the set of *neighbors* of node $i$. For any node $i$, the set $O_i = \{j : (i, j) \in E\} \subseteq K_i$ represents the set of neighbors to which $i$ is sending traffic, and is referred as the set of *out-neighbors* of node $i$. Also, for any node $i$, the set $I_i = \{j : (j, i) \in E\} \subseteq K_i$ represents the set of neighbors from which $i$ is receiving traffic, and is referred as the set of *in-neighbors* of node $i$. We assume omnidirectional transmission, i.e., a transmission by node $i$ to any of its neighbors reaches all of its neighbors. We assume that each node is associated with a single transceiver i.e., a node can not be transmitting and receiving at the same time. We also assume *no capture*, i.e., node $j$ is unable to listen to any of the transmissions if more than one of its neighbors are transmitting simultaneously.

## II. Fair Medium Access Control in Slotted Aloha

In slotted Aloha, each node $i$ makes an attempt to transmit a packet in a slot with a certain probability, to a randomly chosen destination node $j \in O_i$. Let the attempt probability of any node $i \in N$ be denoted by $P_i$. Once node $i$ decides to make an attempt in a slot, a destination $j \in O_i$ is chosen randomly with probability $\frac{p_{(i,j)}}{P_i}$, where $\sum_{j \in O_i} p_{(i,j)} = P_i$. Therefore, the unconditional probability that a transmission attempt is made on edge $(i, j)$ in a slot is $p_{(i,j)}$. Let $\underline{p} = (p_e, e \in E)$ denote the vector of attempt probabilities on all edges. Then, $x_e = x_{(i,j)}$, the effective data rate (or throughput) on edge $e = (i, j)$, is obtained as [1] (Section 4.6.2):

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$$x_{(i,j)}(\underline{p}) \;=\; p_{(i,j)}\,(1-P_j)\prod_{k\in K_j\setminus\{i\}}(1-P_k)\,. \tag{1}$$

Our objective is to set the attempt probabilities optimally so that proportional fairness is attained. Assuming that the edge rates $x_e(\underline{p})$ are as given in (1), the optimal attempt probability vector, $\underline{p}^* = (p_e^*, e \in E)$, is defined as [2]:

$$\underline{p}^* \;=\; \arg\max_{0\leq\underline{p}\leq 1}\sum_{e\in E}\log(x_e(\underline{p}))\,. \tag{2}$$

The following result shows that the optimal attempt probabilities can be calculated based only on local topology information.

*Theorem 1:* $p^*_{(i,j)}$, the optimal attempt probability (as defined by (2)) on any edge $(i,j)\in E$, is given by

$$p^*_{(i,j)} \;=\; \frac{1}{|I_i| + \sum_{k\in K_i}|I_k|}\,. \tag{3}$$

## III. Fair Medium Access Control in Unslotted Aloha

In unslotted Aloha, each node makes an attempt to transmit at a certain rate, which is assumed to be generated according to a poisson process. If the node is already transmitting when a transmission attempt is generated, then this newly generated transmission attempt is aborted (i.e., ignored). Otherwise, the node starts transmitting, even if it was receiving a packet at that instant. This “transmission takes precedence over reception” assumption is necessary to keep the analysis tractable, and has been used by previous researchers as well [3]. Packets are assumed to have a fixed length $T$. Let the poisson attempt rate of any node $i \in N$ be denoted by $\lambda_i$. Once node $i$ decides to transmit a packet, a destination $j \in O_i$ is chosen randomly with probability $\frac{\lambda_{(i,j)}}{\lambda_i}$, where $\sum_{j\in O_i}\lambda_{(i,j)} = \lambda_i$. Thus $\lambda_{(i,j)}$ can be viewed as the attempt rate on edge $(i,j)$. Let $\underline{\lambda} = (\lambda_e, e \in E)$ denote the vector of attempt rates on all edges.

*Lemma 2:* $x_{(i,j)}$, the throughput on edge $(i,j)$, can be expressed as

$$x_{(i,j)}(\underline{\lambda}) \;=\; T\lambda_{(i,j)}\,e^{-T\sum_{k\in K_j\cup\{j\}\setminus\{i\}}\lambda_k}\prod_{k\in K_j\cup\{j\}}\frac{1}{1+T\lambda_k}\,.$$

The attempt rates being defined as in Lemma 2, the optimal attempt rate vector, $\underline{\lambda}^* = (\lambda_e^*, e \in E)$, is given as

$$\underline{\lambda}^* \;=\; \arg\sup_{\underline{\lambda}\geq 0}\sum_{e\in E}\log(x_e(\underline{\lambda}))\,. \tag{4}$$

*Theorem 3:* $\lambda^*_{(i,j)}$, the optimal attempt rate (as defined by (4)) on any edge $(i,j)\in E$, is obtained as $\lambda^*_{(i,j)} = \lambda^*_i/|O_i|$, where $\lambda^*_i$ is given by

$$\lambda_i^* \;=\; \begin{cases} \text{approaches } \infty & \text{if } \sum_{k\in K_i\cup\{i\}}|I_k| = |O_i| \\[2ex] \dfrac{\sqrt{1+\frac{|O_i|}{\sum_{k\in K_i\cup\{i\}}|I_k|-|O_i|}}-1}{T} & \text{otherwise}\,. \end{cases} \tag{5}$$

## References


[1] D. Bertsekas and R. Gallagher, *Data Networks*, Prentice Hall, 1992.

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[3] F. A. Tobagi, “Modeling and Performance Analysis of Multihop Packet Radio Networks,” *Proceedings of the IEEE*, Vol. 75, No. 1, pp. 135-155, Jan. 1987.