Fair Coalitions for Power-Aware Routing in Wireless Networks

Ratul K. Guha, Carl A. Gunter and Saswati Sarkar

Abstract

Several power aware routing schemes have been developed for wireless networks under the assumption that nodes are willing to sacrifice their power reserves in the interest of the network as a whole. But, in several applications of practical utility, nodes are organized in groups, and as a result a node is willing to sacrifice in the interest of other nodes in its group but not necessarily for nodes outside its group. Such groups arise naturally as sets of nodes associated with a single owner or task. We consider the premise that groups will share resources with other groups only if each group experiences a reduction in power consumption. Then, the groups may form a *coalition* in which they route each other's packets. We demonstrate that sharing between groups has different properties from sharing between individuals and investigate fair mutually-beneficial sharing between groups. In particular, we propose a pareto-efficient condition for group sharing based on max-min fairness called *fair coalition routing*. We propose distributed algorithms for computing the fair coalition routing. Using these algorithms we demonstrate that fair coalition routing allows different groups to mutually beneficially share their resources.

Index Terms

Wireless Communication, Algorithm design and analysis, Energy-aware systems and Routing.

I. Introduction

Wireless networks typically consist of nodes that must discharge increasingly complex computing and communication functionalities despite rigorous constraints on power, bandwidth, size and memory. Significant progress has been made to improve hardware to address these needs and much is being done to develop

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software that uses techniques like power-optimizing algorithms. Comparatively less has been done to exploit *sharing* amongst nodes as a way to address these challenges. This is unfortunate, since sharing can yield great benefits. A variety of challenges impede progress: (a) determining which resources can be shared, (b) deciding when to share resources, as sharing would evidently involve a cost, (c) deciding with whom to share resources, and (d) determining how to share resources.

Oftentimes, groups of nodes rather than individual nodes are basic entities in the sharing mechanism. The resource expenditure/utilization of the group as a whole is more important than that of a single node or the entire network. Groups are often formed on the basis of membership in an organization or a shared task. For example, employees of an organization A may carry wearable computers that belong to A. When these devices form an ad hoc network, they may share resources with other devices with the objective of minimizing the total resource consumed by the devices in A, rather than that of all devices in the network. Thus, the devices belonging to an organization form a natural group. Wearable computers involved in one distributed computation may form a group. In a sensor network, different groups would consist of sensors that monitor different attributes such as temperature, pressure, wildlife presence etc. Sensors can also be deployed in the same area by different organizations, e.g., seismic sensors can be deployed in the ocean by two different agencies. Then, sensors belonging to each agency will constitute a group. In the above cases, the resource consumed by groups is more important than that consumed by individual nodes as the distributed computation can be performed and the attributes can be measured even when some members fail. The research in this case must investigate issues pertinent to sharing of resources from the perspective of groups.

A *group* is an intermingled set of nodes having a purpose in common. We do not consider the motivation behind the group formation, but investigate the sharing of resources among different groups. The critical resource we focus on is power. Nodes in wireless networks are powered by battery, and size limitations compel the usage of low lifetime batteries. This calls for judicious consumption of battery power. Normally, communication consumes significantly higher power than other operations. Nodes share power by routing each others packets, and it is well-known that multihop routing substantially decreases the overall power consumption of the network [34]. We address the research challenges that arise when nodes decide to route each others packets with the sole objective of reducing the power consumption of their groups.

We now enumerate some of these challenges. The nodes in a group share power by routing each other's packets to common destinations. Groups are said to form *coalitions*[*] when they route each other's packets. The first challenge is to determine which groups would form coalitions. Presumably, a precondition for forming coalitions among groups is that *each* group communicates the same amount of information to the chosen destinations while consuming *less* power after the coalition is formed. Whether or not the precondition is satisfied depends on the routing in the coalition, and the number of possible routes can be an exponential function of the number of nodes in the groups. There need not even exist a routing that reduces the power consumption of each group. Fig.1(a) and (b) show that if each group consists of a single node,

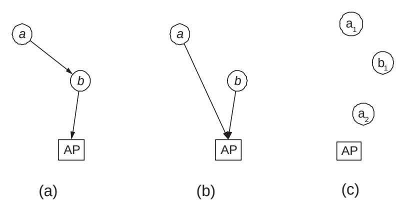


Fig. 1. In (a) and (b), we show two different routings where node a constitutes group A and node b constitutes group B. Both groups need to send traffic to the access point(AP). In (a) the farther node a routes its traffic to b and b sends to AP. So the routing in (a), reduces the power cost of a but increases that for b. In (b) each node routes directly to AP and there is no reduction in power costs for both groups.
In (c) nodes a_1 and a_2 constitute group A and b_1 constitutes group B. Here a_1 can send its traffic through b_1 and b_1 can in turn send through a_2. This could result in a decrease in the total power for group A and B as against the case when the groups route to AP independently.

then groups do not *mutually* benefit from the coalition; but this no longer holds if the groups consist of two or more nodes (Fig.1(c)). The challenge then is to answer whether there exists at least one joint routing that makes the coalition mutually beneficial. The next challenge is to compute such a joint routing. We will show in Section III-C that the routing that minimizes the total power consumption of all groups may not result in mutually beneficial coalitions as it may increase the power consumption of some groups. The benefit

[*]Even after forming a coalition, different groups maintain their separate identities, associations with their individual organizations and discharge their individual responsibilities. The coalition operation just allows joint routing.

incurred by a group due to the coalition operation is the decrease in its power consumption after it joins the coalition. We need to determine a routing that shares the benefit equitably. A simplistic approach is to insist that the groups each get the same benefit, but this can be wasteful if one group can gain benefit without harming the others. A max-min fair [1] routing uses the following strategy for a pair of groups: determine the greatest minimum benefit to be gained by either of the two groups when sharing and maximize the benefit of the other group so long as the changes do not reduce this minimum. This strategy can be generalized to multiple groups. The challenge now is to compute a max-min fair power aware coalition routing.

Finally, the network topology is dynamic since nodes move and the transmission condition in the links significantly change over time. Thus, the benefits obtained through coalition and hence the decisions to remain in coalition change with time. When topology changes, even if the coalition operation remains mutually beneficial, the max-min fair power aware coalition routing may change. We therefore need a distributed and dynamic algorithm that seamlessly adapts the computations in the event of topology change.

In Section II we survey the relevant literature. In Section III we provide a mathematical framework for a coalition of two groups. This section presents several distinctive properties of coalition routings. For example, a max-min fair power aware coalition routing exhibits important characteristics that do not hold for max-min fair allocation of other resources such as bandwidth. We show that the max-min fair coalition routing is guaranteed to attain the desired minimum benefits for each group should the coalition be feasible. In Section IV we present a polynomial complexity algorithm for computing the fair coalition routing. This algorithm needs to solve a linear program at a central processor, which requires the knowledge of the global topology. In Section V we present a distributed computing scheme which allows the routing to be computed via simple iterative computations and message exchanges at each participating node. In Section VI we generalize the framework and the computation algorithms for a coalition among multiple groups in more general networks, and also consider more general models for power consumption and signal propagation. These coalition routing algorithms provide foundations for developing operational protocols. Design of such protocols would require deployment of mechanisms to enforce group routings e.g., security checks. In

Section VI we briefly discuss some of these issues. Refer to appendix for all proofs.

II. Related Work

The existing research on efficient utilization of power in wireless networks can be classified into the following broad categories. The first maximizes the lifetime of any given node through optimum battery discharge strategy [6], [19]. The second varies the transmission power levels of nodes so as to control the network topology as desired [8], [14], [23], [25], [32]. The third reduces the power consumption by optimizing several parameters at the MAC layer [11], [21], [22], [31]. The last maximizes the lifetime of the network by balancing the power consumption of different nodes [3], [4], [17]. Another prevalent approach is to route in accordance with a power based metric rather than a distance metric [34]. However the common feature of the existing research is that the basic entity is a node. The performance of the network is either quantified in terms of the aggregate performance of the nodes or that of the bottleneck node. Hou *et al.* [10] propose a polynomial time algorithm to compute lexicographic max-min(LMM) fair rate allocation and show that this rate allocation attains the LMM node lifetimes. The distinctive feature of our work is that the basic entity is a group rather than a single node, and the operations are coalitions. The performance objective we consider is fairness and the issues significantly differ due to the choice of the basic entity. We are concerned about the performance of each group rather than the network as a whole. Relaying and caching strategies have been proposed for node cooperation when a node decides to relay the requests of other nodes based on its selfish interests [24], [30]. Our research is complementary since we assume that a group of nodes decide to route the packets of other groups based on the interest of the group as a whole. We present an algorithm that obtains a specific pareto optimal objective, the max-min fair operating point.

III. Mathematical Framework for Coalition of Groups

A. Power Model

We first present the mathematical model we use for power consumption [7], [33]. Let the transmitted energy per bit be E_t. The received energy depends on the distance between the transmitter and the receiver and on other phenomena like refraction (e.g., through walls), diffraction (e.g., around buildings), reflection (e.g., on ground and objects), scattering and absorption. The collective variation due to these phenomena is referred to as shadowing [26]. The received energy at a distance d is then $E_t\kappa^{-1}d^{-\alpha}$ where $2 \leq \alpha \leq 6$ and κ represents the link attenuation due to shadowing. For simplification, we assume that κ does not change with time and is the same for all links [7], [33] and we relax these assumptions in Section VI-C. We assume that the noise level is the same at all nodes. Let E_{rx} be the energy per bit required to maintain the SNR necessary for successful decoding at the receiver. Then for successful communication a node must transmit each bit at energy E_{tx}, where $E_{\text{tx}}\kappa^{-1}d^{-\alpha} \geq E_{\text{rx}}$. The power consumed by a transmitting node then is of the form $K_1 + K'rE_{\text{rx}}\kappa d^{\alpha}$ where K' is a constant, r is the node's data rate and K_1 is the node's idle power consumption. The node dissipates power K_1 even if it does not transmit or receive any traffic. Let constant $K = K'E_{\text{rx}}\kappa$.

The MAC and the physical layers determine K_1, K and α. For example, α is higher for obstructed paths within buildings. Unless otherwise stated we will use $\alpha = 4$ which corresponds to the path-loss in closed areas; however all analysis hold for any $\alpha \geq 0$. Nodes may exchange control packets for transmitting data packets; the control packet exchange depends on the MAC protocol e.g., IEEE 802.11 uses RTS, CTS packets. The energy consumed in exchanging control packets determine the constant K'. The linear relation between transmission power and data rate implicitly assumes that the expected number of control packets exchanged per data packet does not depend on the data rate. But, for example, in IEEE 802.11, the expected number of control packets exchanged per data packet increases with increase in data rates due to increase in collisions of RTS, CTS. Thus, strictly speaking the dependence is not linear. But, the inaccuracy due to the linear assumption is negligible except when the energy consumed in transmitting the control packets is comparable to that for transmitting data packets (Fig.2). Since the size of each control packet is significantly less than that of a data packet, this happens only when the expected number of control packets exchanged per data packet is very high which happens only at very high data rates. Usually, in order to avoid excessive energy consumption in retransmitting control packets, the system does not operate at these data rates. Thus

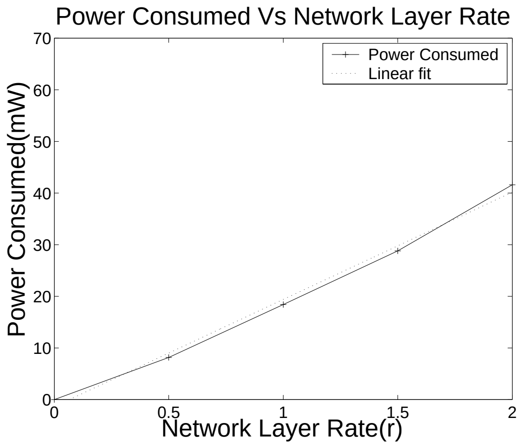


Fig. 2. We consider a network with 10 nodes such that all nodes are in each other's transmission range and share a single channel of capacity 11 Mbps. Node i transmits data to node $(i+1)\%10$ at network layer rate r. The MAC protocol is IEEE 802.11. We plot the power consumed by node 1 as a function of r. The power includes the power consumed in transmitting both control and data packets.

most power aware routing schemes assume this linear dependence e.g., [3], [4], [7], [17].

B. Formulation For a Single Group

We consider a network with M exit points. We denote the set of exit points (EP) as **e**$=$ $(\mathbf{e}_1, \ldots, \mathbf{e}_M)$.

We model the network nodes as a Weighted Directed Graph $\mathbf{G}\langle V, E, \mathbf{e}, W\rangle$ where V is the node set for the group, E is the edge set, **e** is the exit point set and W denotes the edge weights which are positive real numbers. Every node $v \in V$ has at least one path to an exit point and the outdegree of each exit point is 0. Hence the exit points act as a sink for data traffic. The node set V and the exit points are defined through their co-ordinates in the euclidean plane. The distance $d(v, v')$ is the distance between node $v \in V$ and node $v' \in V \cup \mathbf{e}$. If $(v, v') \in E$, weight $w(v, v') = d(v, v')^4$ and $w(v, v') \in W$. The edge set E is usually determined at the MAC and physical layers, and can be arbitrary except that the exit points only have incoming edges. We now describe an example edge set. When the node radios have limitations on maximum transmission power for each bit, then an acceptable SNR level can be maintained at the receiver only if the distance from the transmitter is below a certain maximum value which is referred to as the transmission range (D). In such networks, a directed edge exists from $v \in V$ to $v' \in V \cup \mathbf{e}$ if $d(v, v') < D$. Origin function $O_i : V \rightarrow \Re$ defines the traffic originating at a node $v \in V$ for each exit point ($\mathbf{e}_i$ in **e**. The graph

G and the origin functions are given.

Let the traffic on an edge (v, v') intended for exit point $\mathbf{e_i}$ be $r_i(v, v') \in \Re$. If $(v, v') \notin E$ then $r(v, v') = 0$. The total outgoing traffic from a node v for exit point $\mathbf{e_i}$ is then $\sum_{v' \in V \cup \{\mathbf{e_i}\}} r_i(v, v')$ which is the load on node v, $L_i(v)$. The sum of the incoming traffic and the originating traffic at a node must equal the exiting traffic. Thus, $\forall\, i$ and $\forall\, v \ \in V$

$$\sum_{v' \in V \cup \{\mathbf{e_i}\}} r_i(v, v') = O_i(v) + \sum_{v'' \in V} r_i(v'', v) = L_i(v). \tag{1}$$

Traffic routing is an $|E|M$ dimensional vector $\vec{r}$ whose components satisfy (1). The components of $\vec{r}$ are the traffics on the corresponding edges. Under routing $\vec{r}$, a node v spends power $N_{\vec{r}}(v)$ and $N_{\vec{r}}(v) = K_1 + K \sum_i \sum_{v' \in V \cup \{\mathbf{e_i}\}} r_i(v, v') d(v, v')^4$, where the constants K_1 and K are defined in Section III-A.

Different nodes may have different energy limitations. Thus, we assume that for each node v, the average power consumption is upper bounded by $B(v)$. Hence,

$$K_1 + K \sum_i \sum_{v' \in V \cup \{\mathbf{e_i}\}} r_i(v, v') d(v, v')^4 \leq B(v).$$

The power expenditure of a group $P_{\vec{r}}$ is then the total power consumed by all nodes in the group i.e., $P_{\vec{r}} = \sum_{v \in V} N_{\vec{r}}(v)$. The group optimal power expenditure P_{opt} is the minimum value of $P_{\vec{r}}$ over all possible $\vec{r}$, and can be obtained by routing the traffic over the minimum weight path from any node $v \in V$ to each exit point $\mathbf{e_i} \in \mathbf{e}$ for the weights $W^{\dagger}$. Such minimum weight paths can be computed by well-known algorithms like Dijkstra, Bellman ford, etc. Let v_i' be the next hop node to v in such a path. If $N_{opt}(v)$ is the power spent by a node v under optimal routing, then

$$N_{opt}(v) = K_1 + K \times \sum_i L_i(v) \times d(v, v_i')^4 \text{ and } P_{opt} = \sum_{v \in V} N_{opt}(v). \tag{2}$$

C. Coalition of Groups

We have described the terminology and the equations for a group of nodes. Now consider two groups of nodes A and B. Let their node sets be V^a and V^b respectively. Let their group optimal power expenditures before forming a coalition be P^a_{opt} and P^b_{opt}.

†Here, the weight of a path is the sum of the weights of the links in the path.

Next, we consider a combined network with groups A and B jointly routing to the exit points. Depending on the network scenario each group may route to one or more exit points. For example, when groups correspond to an organization, they could route to their own exit point. On the other hand, in sensor networks each group could route to multiple exit points. These scenarios constitute specific cases of our model.

The vertex set V for the combined network then is $V^a \cup V^b$. The edge set E^{joint} can be determined from V and the MAC and physical layer considerations. For example, E^{joint} can be obtained using the transmission range D, i.e., directed edge $(v, v') \in E^{\text{joint}}$ for any $v \in V^a \cup V^b$ and $v' \in V^a \cup V^b \cup \mathbf{e}$ if $d(v, v') < D$. Also, E^{joint} is a superset of the edge sets of each group. Again, for any $(v, v') \in E^{\text{joint}}$, weight $w(v, v') = d(v, v')^4$. The origin functions for all the nodes remain the same. Any vector in $R^{M|E^{\text{joint}}|}$ whose components are non-negative and satisfy (1) is a routing in the joint network, and will be referred to as a *coalition routing*. Note that $r(v, v') = 0$ if $(v, v') \notin E^{\text{joint}}$. For an arbitrary coalition routing $\vec{r}$, we now evaluate the power expenditure for each node. Let $J^a_{\vec{r}}$ and $J^b_{\vec{r}}$ be the total power expenditure for nodes in groups A and B respectively, under routing $\vec{r}$.

$$\text{Then, } J^a_{\vec{r}} = \sum_{v \in V^a} N_{\vec{r}}(v) \text{ and } J^b_{\vec{r}} = \sum_{v \in V^b} N_{\vec{r}}(v).$$

Definition 1: Group benefit under coalition routing $\vec{r}$ is the difference between the power spent by the group under individual optimal routing before merging, and the power spent by the group for coalition routing $\vec{r}$. The group benefits form the benefit vector $\vec{B_{\vec{r}}}$, where $\vec{B_{\vec{r}}} \equiv (B^a_{\vec{r}}, B^b_{\vec{r}})$, $B^a_{\vec{r}} = P^a_{opt} - J^a_{\vec{r}}$ and $B^b_{\vec{r}} = P^b_{opt} - J^b_{\vec{r}}$.

The idea behind combining two groups is to reduce the total power each group was spending initially. Depending on the system, group coalition may introduce some additional operational cost and groups would want to benefit over and above this cost. Let t be the benefit below which groups will not be willing to enter into a coalition. The value of t would depend on group policies and the overhead for the coalition.

Definition 2: A coalition is *useful with a routing* $\vec{r}$ if $\min(B^a_{\vec{r}}, B^b_{\vec{r}}) \geq t$.

Definition 3: A coalition is *useful* if it is useful with some routing $\vec{r}$.

We will present an algorithm to compute such a routing $\vec{r}$ if one exists.

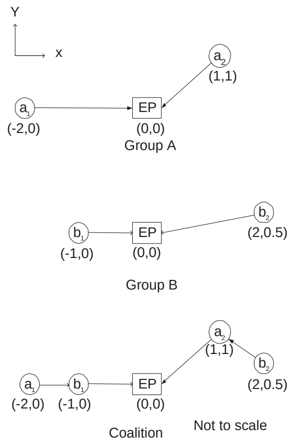


Fig. 3. Groups A(a_1, a_2) and B(b_1, b_2) route to the exit point EP. Each node sends 1 Mbps.

Definition 4: A *minimal coalition* routing is a coalition routing that results in the optimal or the minimal total power expenditure for groups A and B combined.

Next we illustrate the combination of two groups with an example. Consider Fig.3 in which groups A and B route to a single exit point. Each node generates traffic at the rate of 1 Mbps. Let $K = 1, K_1 = 0$. Optimal power expenditure for group A is $2^4 + \sqrt{2}^4 = 20$ and for group B is $1^4 + \sqrt{4.25}^4 \approx 19$. For the minimal power coalition routing shown, power expenditure for A is $1^4 + 2(\sqrt{2})^4 = 9$ and for B is $2(1)^4 + \sqrt{1.25}^4 \approx 3.6$. Benefit for group A is $20 - 9 = 11$ and for B is $19 - 3.6 = 15.4$ and both the components are positive. Consider now that node b_2 has a higher load to send, e.g., 5 Mbps. This will be relayed through a_2 in the coalition routing of Fig.3. Node a_2 will have a high power consumption (24) and the benefit of group A will be negative (-5). This demonstrates that the minimal coalition routing may not benefit each group.

Definition 5: A *feasible benefit vector* is one that results from a coalition routing $\vec{r}$. The set of all feasible benefit vectors is the *feasible benefit region*.

D. Properties of the Feasible Benefit Region

Theorem 1: The set of feasible benefit vectors is convex and closed.

We now demonstrate that different feasible benefit vectors can lead to disparate benefits for the groups.

For the minimal coalition routing, we can find the power expenditure for each node, i.e., $N_{opt}(v)$ for each $v \in V^a \cup V^b$. Further let J^a_{opt} and J^b_{opt} be the powers spent by nodes of groups A and B respectively under

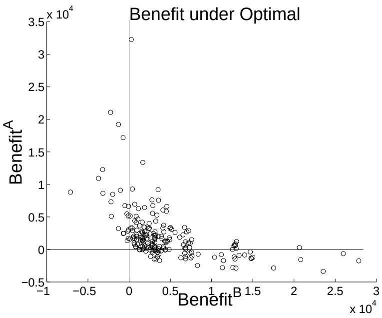


Fig. 4. 'Benefit vectors' under 'minimal coalition.'

the minimal coalition routing.

$$J^a_{opt} = \sum_{v \in V^a} N_{opt}(v) \text{ and } J^b_{opt} = \sum_{v \in V^b} N_{opt}(v).$$

Note again that the subscript 'opt' to J refers to the minimal coalition routing for nodes of groups A and B combined. The benefit vector $\vec{L}$ corresponding to the minimal coalition routing is then (L^a_{opt}, L^b_{opt}) where $L^a_{opt} = P^a_{opt} - J^a_{opt}$ and $L^b_{opt} = P^b_{opt} - J^b_{opt}$. Let $K = 1$ and let there be a single exit point. The vector $\vec{L}$ is plotted in Fig.4 for different random placements of nodes. Each group has 20 nodes uniformly distributed over a square of side $100m$, and the network is fully connected, i.e., each node can directly transmit to every other node. If the benefit vector is in the first quadrant (both coordinates are positive), then the groups mutually benefit from being merged, otherwise one of the groups is a loser. Most pairs of groups benefit from a minimal coalition, but there are many instances in which only one group benefits. Even when a pair of groups mutually benefits, there is often some disproportion in the extent of benefit, with one group getting somewhat more than the other. This motivates fair allocation of benefits.

E. Max-min Fair Benefit Vector

Definition 6: A feasible benefit vector $B_{\vec{r}}$ is *max-min fair* if for all i, $B^i_{\vec{r}}$ cannot be increased while maintaining feasibility without decreasing $B^j_{\vec{r}}$ for some group j, for which $B^j_{\vec{r}} \leq B^i_{\vec{r}}$.

Corollary 1: The max-min fair benefit vector exists and is unique.

The corollary follows as a consequence of Theorem 1 and results from [28].

Definition 7: A *fair coalition routing* is a coalition routing that results in a max-min fair benefit vector.

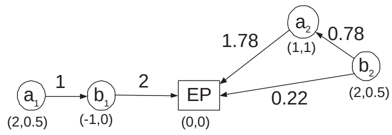


Fig. 5. Fair coalition routing when each node sends 1 Mbps. The numbers next to the links are the rates.

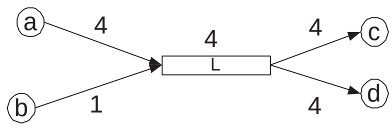


Fig. 6. Consider two sessions (a,c) and (b,d). The numbers next to the links are the link bandwidths. The max-min fair bandwidth for session (a,c) and (b,d) are 3 and 1 respectively.

Minimum component property: If $\vec{r}$ is a fair coalition routing, then $\min(B^a_{\vec{r}}, B^b_{\vec{r}}) \geq \min(B^a_{\vec{r_1}}, B^b_{\vec{r_1}})$ for any other coalition routing $\vec{r_1}$. This property follows from the definition of the max-min fair benefit vector.

In Fig.3 the max-min fair benefit vector when $K = 1$ and $M = 1$ is (11.9,11.9). This is achieved when node b_2 sends 0.78 Mbps to a_2 and 0.22 Mbps directly to EP like in Fig.5.

Proposition 1: Let $\vec{r}$ be a fair coalition routing. Then min$(B^a_{\vec{r}}, B^b_{\vec{r}}) \geq 0$.

Thus a coalition does not increase the power consumption of any group if fair coalition routing is used.

Theorem 2: A coalition will be useful if and only if it is useful with a fair coalition routing $\vec{r}$.

Theorem 2 presents a necessary and a sufficient condition for deciding whether the coalition would be useful.

Theorem 3: For two groups the max-min fair benefit vector has equal components.

Theorem 3 will be used in developing an efficient algorithm for computing a fair coalition routing for two groups.

Note that for other resource allocation problems, e.g., bandwidth allocation, the max-min fair vector need not have equal components even for two contenders (Fig.6) [5].

IV. Fair Coalition Algorithm(FC)

A. Description

We show that the fair coalition routing and the associated benefit vector can be computed by solving the following linear program.

FC: Maximize Z:

Subject to:

$$
\begin{aligned}
Z - B^a_{\vec{r}} &\leq 0, \\
Z - B^b_{\vec{r}} &\leq 0, \\
K_1 + K \sum_i \sum_{v' \in V^a \cup V^b \cup \{\mathbf{e_i}\}} r_i(v,v') d(v,v')^4 &\leq B(v) \; \forall \, v \in V^a \cup V^b, \quad (3) \\
\sum_{v' \in V^a \cup V^b \cup \{\mathbf{e_i}\}} r_i(v,v') - \sum_{v'' \in V^a \cup V^b} r_i(v'',v) &= O_i(v) \; \forall \, v, v' \in V^a \cup V^b \text{and } i. \quad (4)
\end{aligned}
$$

The power consumption of each node is constrained in (3) and the flows are balanced in (4). Let Z^* be the objective function value obtained from **FC**.

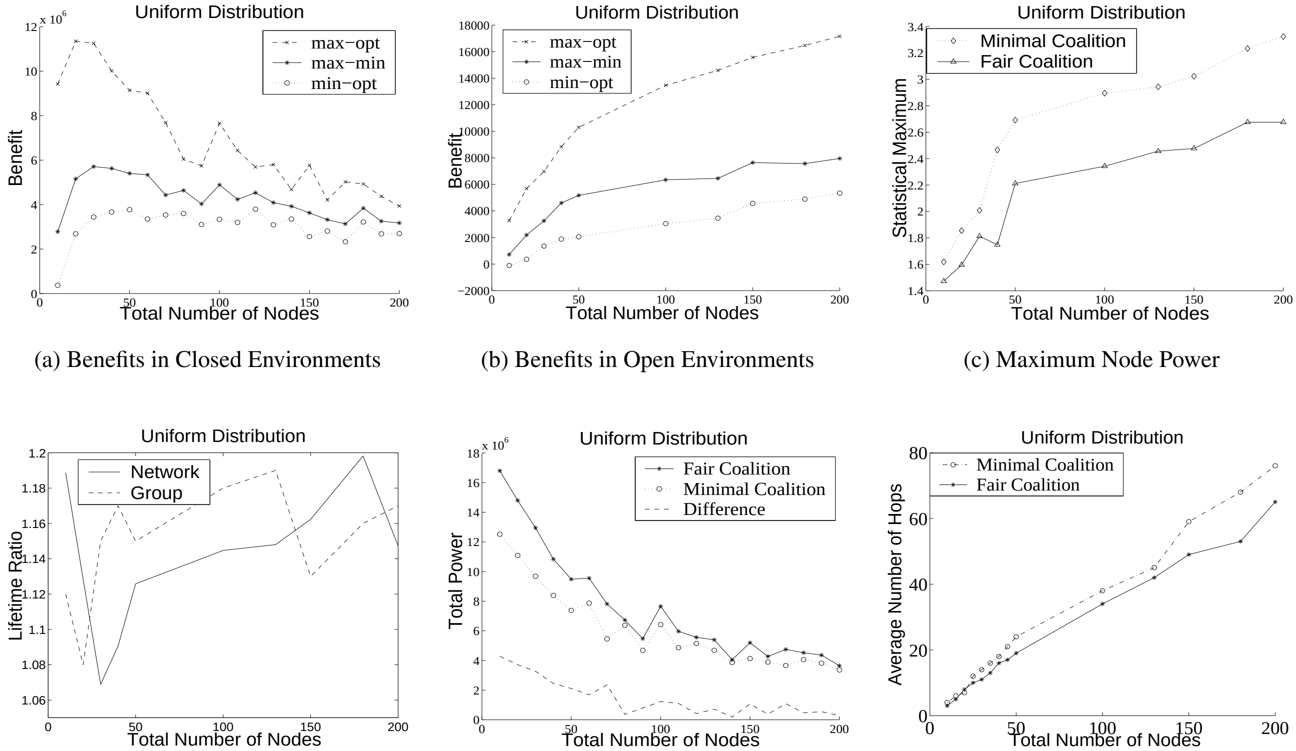
Theorem 4: The routing $\vec{r}$ obtained as a solution of **FC** is a fair coalition routing.

Proof: Let $\mathrm{minben}(\vec{r})$ =min$(B^a_{\vec{r}}, B^b_{\vec{r}})$. From Theorem 3 and the minimum component property, any feasible routing that attains the maximum value of $\mathrm{minben}(\vec{r})$ is a fair coalition routing $\vec{r}$. Thus **FC** computes the fair coalition routing.

The exit point can solve **FC** to compute the fair coalition routing and the max-min fair benefit. The linear program involves $(M+1)|V^a \cup V^b| + 2$ constraints and $M|E^{\text{joint}}| + 1$ variables. Hence the max-min fair benefit vector and the fair coalition routing are polynomial complexity computable [13].

For solving **FC**, an exit point needs to know the edge set E^{joint} and the distances between the nodes. Initially, the nodes inform the exit point their incident edges and the distances from their neighbors, and later they inform the exit point only when these change. The MAC and the physical layers of a node v determines its incident edges (v, v') and (v', v) in E^{joint}. Nodes can learn the distances from their neighbors by power measurements and positioning algorithms, some of which do not need GPS [2].

B. Simulation Results



(a) Benefits in Closed Environments (b) Benefits in Open Environments (c) Maximum Node Power

(d) Network Lifetime (e) Fairness Overhead (f) Hop Delay Performance

Fig. 7. Performance of coalition routings in networks consisting of two groups of equal sizes and nodes uniformly distributed in a square of size $100m$.

We investigate the efficacy of fair coalition routing through simulations using MATLAB. We evaluate the benefits attained by different coalition routing schemes. We also consider other performance attributes such as network lifetime, end-to-end path lengths, additional power consumption for providing fairness, etc. We consider a network with one exit point $(M = 1)$ and a coalition of two groups. Nodes of both groups are distributed in a square of side $100m$. Each node generates traffic at the rate of 1 Mbps. The value of K depends on the choice of the wireless interface, and its effect is to scale our measurements. Thus, without loss of generality, we consider $K = 1$. We will later mention details for a specific interface. Note that the benefit values do not depend on K_1. We consider different number of nodes, different distributions of nodes, different locations of the exit point, different sizes of the groups, different distances between groups and report averages over 100 random topologies in each case.

We first consider a fully connected network, i.e., each node can transmit directly to every other node. We assume that both groups have equal number of nodes, the exit point is at the center of the square, and all nodes are uniformly distributed in the square. In Fig. 7(a), we plot the benefit values as a function of the number of nodes. As proved before, the max-min fair benefit vector will have equal components. We plot the average values of the maximum component of the benefit vector of the minimal coalition routing (max-opt), the minimum component of the benefit vector of the minimal coalition routing (min-opt) and the max-min fair benefit (max-min). As expected the max-min group benefit is between the maximum and the minimum components of the benefit vector of the minimal coalition routing. Benefits initially increase and later decrease with increase in the number of nodes. This can be explained as follows. Power consumption in a routing scheme decreases if the distance between consecutive nodes in a path decreases. This holds even if such a decrease increases the number of hops. This is because the power consumed in any routing is proportional to (i) the expectation of the fourth power of the distance in each hop and (ii) the number of hops. When the number of nodes is small, each group has a small number of nodes and thus joint routings allow packet transmissions across hops that are significantly shorter than those in the individually optimal routings in each group. Thus, joint routings have substantially lower power consumption. This effect becomes more pronounced with increase in the number of nodes for moderate number of nodes. But, when the number of nodes becomes really large, each group has a large number of nodes, and the hop distances and hence the power consumptions in the individual optimal routings become small as well[‡]. Thus, the benefits of joint routing decrease. Nevertheless, the benefit values are still considerable even for networks with 200 nodes.

In Fig.7(b), we consider a different path loss exponent $\alpha = 2$ which arises in open environments. Here, the trends are similar to Fig. 7(a), but the benefits are somewhat smaller. This is because the reduction in power consumption due to the reduction in hop-distances $d(v, v')$ obtained by the joint routings are less for

[‡]Recently Zhao *et al.* [35] has proved that when nodes are uniformly distributed and their number n becomes large, the network transfers $\Omega(n/\log n)$ amount of data before any node dies. In other words, the data transferred by a network in its lifetime becomes arbitrarily large with increase in n. This happens because of reduction in the distance between consecutive nodes in the routes. Although Zhao *et al.* do not consider networks with groups, their result is consistent with our observation.

$\alpha = 2$ than for $\alpha = 4$, as the power consumed in a link (v, v') is proportional to $d(v, v')^{\alpha}$.

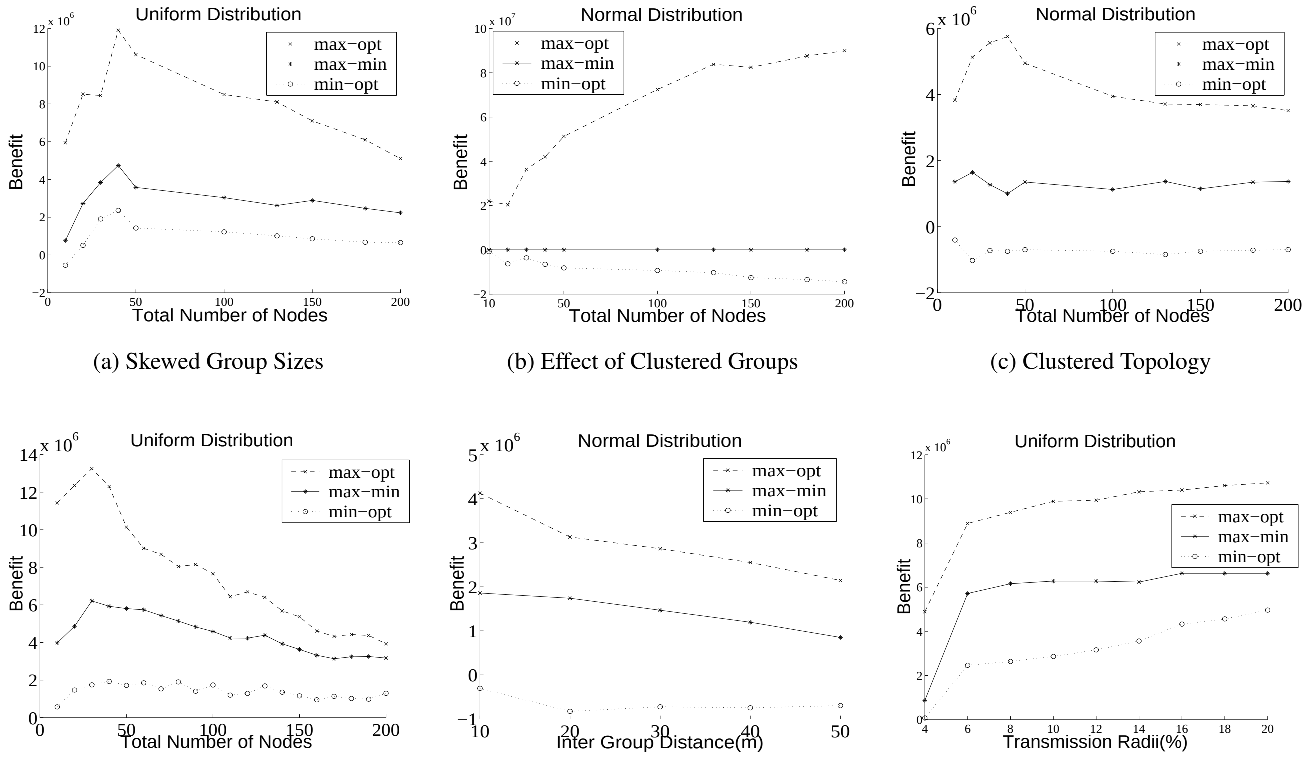
We now revert to the closed environment, $\alpha = 4$, and compare the lifetime of the network attained under different coalition routing schemes. The network lifetime can be defined in different ways, e.g., it can be considered as the time required for a certain fraction of nodes to die, or the first time instant at which the network is disconnected etc.[3], [4], [34]. The lifetime of a network for all these metrics is governed by the power consumption of the nodes that spend high power and die faster than others. Thus in Fig. 7(c) we plot the quantity $(\bar{X} + \sigma_x)/\bar{X}$ where $\bar{X}$ is the mean power over all nodes and σ_x is the standard deviation. Note that this quantity is a measure of the statistical maximum of the power spent by any node. Fair coalition routing has a lower value of this quantity as compared to the minimal. This happens because the minimal coalition routing derives its advantages by routing significant amount of traffic through a few nodes. We therefore expect that fair coalition routing will have higher lifetime under most metrics (i.e., all metrics that depend on the power consumption of the nodes that consume more power than others). To demonstrate that this is indeed the case, we choose a particular notion of lifetime namely the time required for a certain fraction (e.g., 5%) of nodes to die. We assume that all nodes have the same initial energy. In Fig. 7(d), we plot the ratio between the lifetimes of the network under the fair and the minimal coalition routings that are computed when all nodes are functional. We also plot the ratio of the lifetimes of the group with the minimum lifetime under fair coalition and the group with the minimum lifetime under minimal coalition routings. Consistent with our expectation, the ratio is always above 1.

Fig. 7(e) plots the total powers spent under the minimal and fair coalition routings and their difference. This difference can be looked upon as the cost for providing fairness. Here $K_1 = 0$. The average cost is modest (18%) considering the benefit (46%)[§] obtained and the fairness achieved.

In Fig. 7(f), we plot the average number of hops traversed by each packet before it reaches the exit point. We notice that on an average, the fair and minimal coalition routings use similar number of hops. The hop count affects the average end-to-end delay experienced by packets. But, the delay also depends on other

[§]The cost % is obtained from Fig. 7(e). The benefit % is with respect to the total power consumed prior to the coalition and is obtained from Fig. 7(a) and Fig. 7(e).

factors such as interference. The detailed investigation of the delay and interference issues in coalition routing is beyond the scope of this paper.



(a) Skewed Group Sizes

(b) Effect of Clustered Groups

(c) Clustered Topology

(d) Exit point at end

(e) Inter Group Distance

(f) Lower transmission range D

Fig. 8. Benefit values for coalition routings for different network scenarios.

We now evaluate the benefits for different distributions of nodes, different locations of the exit point, different sizes of the groups and different distances between groups. But, the trends and the conclusions remain the same as in the previous cases.

Fig. 8(a) shows the results for unequal group sizes. One group is four times as large as the other. The nodes are still uniformly distributed. The smaller group has a lesser benefit under the minimal coalition routing in this case. The remaining trends are the same as for groups with equal sizes.

We now investigate the effect of clustered topologies on the benefit values (Fig. 8(b), 8(c)). Both groups have equal number of nodes. In Fig. 8(b), nodes of each group are normally distributed with a variance of 25 around the respective group centroids that are uniformly distributed. The group with the centroid closer to the exit point has negative benefit under the minimal coalition routing, and zero benefit under the fair coalition routing. The group closer to the exit point loses after coalition when the minimal coalition routing is used, but not when the fair coalition routing is used. Here, the benefits of the fair coalition routing starts decreasing for much larger number of nodes than in the uniform distribution case (Fig. 7(a)), as the topology becomes pervasive only for much larger number of nodes. For example, when the number of nodes in the network is 400 the benefit reduces by 25% as compared to the benefit in a network with 200 nodes. In Fig. 8(c) we consider a network with two clusters of equal sizes, but now the clusters include equal number of nodes from both groups. The nodes in each cluster are normally distributed with a variance of 25 around the respective group centroids that are uniformly distributed. Here both groups obtain positive benefits under fair coalition.

We now investigate the case when the exit point is at the edge of the square. We consider two different distributions of nodes: (i) uniform (Fig. 8(d)) and (ii) normal (Fig. 8(e)). For uniform distribution, the trends are similar to the case with the exit point at the center (Fig. 7(a)). But, since all nodes are now in the same side of the exit point, the paths to the exit point contain larger number of nodes of both groups, and hence the benefits are higher. For normal distribution, the nodes of each group are normally distributed around the centroid of the group with a variance of 25. The centroids are equidistant from the exit point and at a distance d from each other where d is a measure of the separation between the groups. In Fig. 8(e), we plot the benefits as a function of d. The benefits decrease as d increases as then fewer nodes from one group can route the packets of the other group due to the larger separation between the groups.

We now relax the assumption that the network is fully connected, and assume that each node can transmit directly to only nodes within distance D. We investigate the effect of different transmission ranges D on the benefits in Fig. 8(f). The network has 20 nodes in each group, but the characteristics are otherwise similar to that considered in Fig. 7(a). Lower values of D will result in fewer edges in the network. The benefit increases significantly with increase in D for lower values of D as more and more nodes can be included in potential routes to the exit point. Note that the maximum possible distance between any two nodes in this network is $100\sqrt{2}$. A slight drop can be noticed when D is around $10\sqrt{2}$. This is because the power consumption of the group optimal decreases by a smaller amount than that of the fair coalition routing. When D exceeds $18\sqrt{2}$, the curves level off. The transmission range is now high enough to include those nodes which would have been a part of the coalition routing in the fully connected case.

For the Lucent 802.11b Orinoco card, a rate of 1 Mbps in closed environment corresponds to 15dBm of output power [18]. The constant K is then roughly $5.5 \times 10^{-6} W/\text{Mbit} * \text{m}^4$. For any value of K_1, this translates to a benefit of 30 Watts for a group with 10 nodes for the uniform case with equal group sizes. It is also worthwhile to note that the CPU time to compute **FC**, for any of the above topologies was not more than 0.5secs on a 700Mhz/256MB RAM laptop using a simplex algorithm implementation [9].

V. DISTRIBUTED IMPLEMENTATION

The algorithm in Section IV-A for computing the fair coalition routing requires a centralized computation at the exit point. Though the simplest solution, it will not be computationally tractable when the exit points have capability similar to the nodes themselves. Consider for example a sensor network where a group of sensors communicate their measurements to a common node which in turn transmits to say a satellite. Here we would not want to overwhelm the relay node with the linear programming computation. Furthermore, when nodes move, the edge set E changes. For example, when a node can directly transmit to only nodes within its transmission range D, then links between two nodes will be created (cease to exist) when one moves in to (out of) the transmission range of another. Finally, the power consumed for transmission of each bit in a link will change with change in the distance between the incident nodes. The traffic generation rate of each node will also change with time. Due to these changes, the coalition may no longer be useful or may start being useful or the fair coalition routing may change. Thus, **FC** must be solved every time such changes occur. Rather than having the exit point repeat the entire computation in every such instance, it is beneficial to have a distributed implementation where every node performs some simple iterative computations and the values seamlessly converge to the max-min fair solution. Based on the new max-min fair solution, the groups can determine whether the coalition is useful (Theorem 3), and use the fair coalition routing if they remain in or join the coalition.

Now we present an iterative approach to compute a fair coalition routing for two groups. This has been motivated by recently proposed solutions for optimization problems in other resource allocation settings [12], [29]. Let Z_n and $\vec{r_n}$ denote the corresponding quantities in iteration n, where Z_0 and $\vec{r_0}$ can be arbitrarily chosen. The initial choices need not satisfy any of the constraints. Thus each node can select the initial values of the loads for each of its outgoing edges without any co-ordination with the other nodes. Similarly Z_0 is selected at an exit point. Now we define some indicators. The benefit indicator of a group is 1 if Z_n is more than the group benefit.

$$\epsilon_n^a = \begin{cases} 0, & \text{if} \quad Z_n + J^a_{\vec{r_n}} \leq P^a_{opt}, \\ 1, & \text{if} \quad Z_n + J^a_{\vec{r_n}} > P^a_{opt}. \end{cases}$$

$$\epsilon_n^b = \begin{cases} 0, & \text{if} \quad Z_n + J^b_{\vec{r_n}} \leq P^b_{opt}, \\ 1, & \text{if} \quad Z_n + J^b_{\vec{r_n}} > P^b_{opt}. \end{cases}$$

We now outline the rate update mechanism for the traffic intended for each of the M exit points. Node congestion $c^v_{n,i}$ is the difference between the outgoing and the sum of the originating and incoming traffic at node v for exit point i. From (4),

$$c^v_{n,i} = \sum_{v' \in V^a \cup V^b \cup \{\mathbf{e_i}\}} r_{n,i}(v, v') - \left(O_i(v) + \sum_{v'' \in V^a \cup V^b} r_{n,i}(v'', v) \right).$$

Node congestion indicator for node v for traffic directed to exit point i is

$$s^v_{n,i} = \begin{cases} 0 & \text{if} \quad c^v_{n,i} = 0, \\ 1 & \text{if} \quad c^v_{n,i} > 0, \\ -1 & \text{if} \quad c^v_{n,i} < 0. \end{cases}$$

Traffic for exit point i at node v is considered balanced, lightly loaded or heavily loaded as $s^v_{n,i}$ is 0,1 and -1 respectively. For the exit point, $s^{\mathbf{e}}_n = 0$. The power level indicator at node v, t^v_n is set to 1 if the current power consumption exceeds the limit $B(v)$ and 0 otherwise. Hence,

$$t^v_n = \begin{cases} 0 & \text{if} \quad K_1 + K \sum_i \sum_{v' \in V^a \cup V^b \cup \{\mathbf{e_i}\}} r_i(v, v') d(v, v')^4 \leq B(v), \\ 1 & \text{if} \quad K_1 + K \sum_i \sum_{v' \in V^a \cup V^b \cup \{\mathbf{e_i}\}} r_i(v, v') d(v, v')^4 > B(v). \end{cases}$$

We present an iterative approach using the above indicators. Note that $s^v_{n,i}$ and t^v_n can be updated at node v using the incoming rates in the previous iteration. Now, update of ϵ^a_n and ϵ^b_n require a knowledge of the total power being spent by the nodes of a group. We will discuss how to acquire this information in a distributed manner.

Let $\{\delta_n\}$ be the step-sizes that satisfy $\lim_{n\to\infty}\delta_n = 0$ and $\sum_{n=1}^{\infty}\delta_n = \infty$. For example $\delta_n = 1/n$ satisfies the conditions. Each node v updates its outgoing traffic in edges $(v, v') \in E^{\text{joint}}$ as follows. $[\cdot]_+$ denotes the projection on $[0, \infty)$.

$r_{n+1,i}(v, v') = \left[r_{n,i}(v, v') - \gamma\delta_n\left(s^v_{n,i} - s^{v'}_{n,i} + +d(v, v')^4(t^v_n + \epsilon^a_n)\right)\right]_+$ if $v \in V^a$.

$r_{n+1,i}(v, v') = \left[r_{n,i}(v, v') - \gamma\delta_n\left(s^v_{n,i} - s^{v'}_{n,i} + +d(v, v')^4(t^v_n + \epsilon^b_n)\right)\right]_+$ if $v \in V^b$.

Trivially, $r_{n+1,i}(v, v') = 0$ if $(v, v') \notin E^{\text{joint}}$.

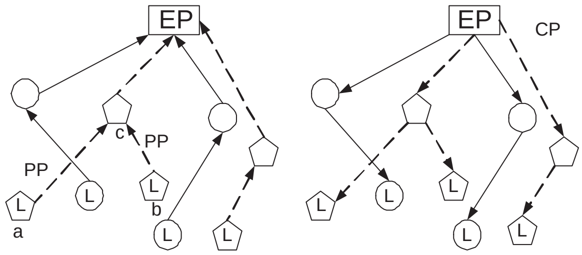
The exit point updates Z as follows: $\quad Z_{n+1} = [Z_n + \delta_n(1 - \gamma(\epsilon^a_n + \epsilon^b_n))]_+$.

Theorem 5: For all $\gamma > 1$ the iterative procedure stated above will converge to the max-min fair benefit vector and fair coalition routing, irrespective of the initial choice of the iterates.

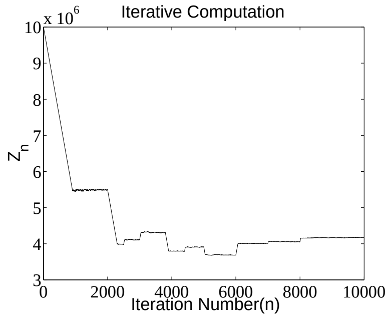
Since the convergence guarantees in Theorem 5 hold irrespective of the initial choice of the iterates, the procedure converges to the fair allocations even after changes in E^{joint} and the power consumed in the links.

Now we outline a distributed scheme to implement the iterations. Assume that we have a spanning tree connecting nodes of each group to any one of the exit points. Refer to Fig.9(a). Each leaf node L sends a power packet (PP) upstream that contains the power expended by L. Each node of a group adds all the power values in the PP arriving from its downstream branches, adds its own power expenditure to the sum, and sends a PP upstream with the resulting power value. Using these group powers the exit point determines ϵ^a_{n+1} and ϵ^b_{n+1} and updates Z_n. The exit point communicates ϵ^a_n and ϵ^b_n to each group through congestion indicator packet CP, and the nodes can use these to update their rates. The PP and CP can be separate packets, or they can be piggybacked on the data and acknowledgement packets.

We now evaluate the convergence time of the distributed implementation. We consider a fully connected network with 10 nodes in each group where the nodes are uniformly distributed in a square of side 100m,



(a) Exchange of PP and CP. Circles and pentagons denote the two groups. Let the power spent by nodes a, b and c be 1,2 and 3 respectively. The PPs sent by a, b and c have power values 1,2 and 6 respectively.



(b) Convergence for the distributed computation. Here $\gamma = 2500$ and $\delta_n = 1/n, \forall\, n, Z_0 = 10^7$ and $\vec{r_0} = \vec{0}$. During iteration 2000, nodes change their positions. After iteration 2400 nodes change their positions one by one (by $\pm 10\%$) till iteration 6000. After iteration 6100 the transmission rates change (by $\pm 5\%$).

Fig. 9. Distributed Implementation

and one exit point is at the center. Each node generates traffic at the rate of 1 Mbps. We assume that the size of each CP and PP packet is 15 bytes. The CP and the PP packets traverse a total of 12 hops per iteration. Now, if the transmission rate in each link is 11 Mbps, then each iteration consumes approximately 0.13 milliseconds. Here $K = 1, \gamma = 2500$ and $\delta_n = 1/n, \forall\, n, Z_0 = 10^7$ and $\vec{r_0} = \vec{0}$. The benefit Z_n converges to the max-min fair benefit value of 5.5×10^6 in 1000 iterations which consume 130 milliseconds (Fig.9(b)). In general the initial convergence time will depend on how far the initial guess is from the optimal.

We next demonstrate that the re-computations that result from incremental changes in topology and traffic generation rates converge much faster. We assume that during iteration number 2000 (i.e., after the initial convergence) all nodes select new locations - the new locations are also uniformly distributed. The power consumptions in the links now change due to the topology rearrangement, but Z_n converges to the new max-min fair value in 400 iterations which consumed 50 milliseconds. The convergence is faster as compared to the initial convergence because only the node positions were changed while their traffic generation rates remained same. Thereafter, between iterations 2400 and 6000, nodes change their positions one by one. If a node i's current x-coordinate (y-coordinate) is x_i, then it selects its new x-coordinate (y-coordinate) uniformly within $[0.9x_i, 1.1x_i]$ ($[0.9y_i, 1.1z_i]$). On an average, 60 iterations ($\approx$ 8ms) are required for convergence for each change. Finally, between iteration 6100 and 8100 the nodes change their traffic generation rates one by one. If a node i's current generation rate is $O(i)$, then its new rate is uniformly distributed within $[0.95O(i), 1.05O(i)]$. Now, on an average after each change, Z_n converges to the new max-min fair value in 20 iterations ($\approx$ 3ms).

Groups join or remain in the coalition if and only if the new max-min fair benefit Z_n exceeds the minimum required benefit t (Theorem 3), and use the corresponding fair coalition routing whenever they are in a coalition. To prevent routing instability and oscillations, the groups evaluate the coalition formation decision and alter the routing only when (i) the current value of Z_n substantially differs from that at the previous decision epoch and (ii) Z_n remains at its current value for some time which ensures convergence. Determination of these necessary deviations and time durations as also the security mechanisms required to enforce the coalition formation decisions and the fair coaliton routing constitute separate research topics and are beyond the scope of the current work. We however briefly discuss some of the security issues in Section VI-D.

VI. Discussion and Generalizations

We now describe how the framework we have proposed and the analytical results we have obtained can be generalized to include several additional features of practical relevance.

A. Multi-Group Fair Coalition Algorithm

We now investigate the max-min fair benefit vector and fair coalition routing when multiple (n) groups attempt to form a coalition. Definition 6 also defines the max-min fair benefit vector in this case. This case is significantly different from the two group case discussed earlier. Let P^i_{opt} be the minimum possible

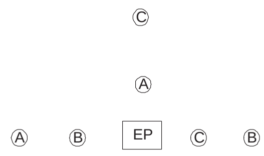


Fig. 10. All three groups A, B and C will benefit if routing together but no two taken at a time will mutually benefit.

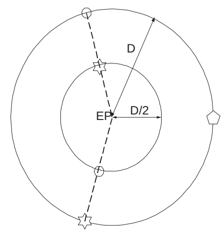


Fig. 11. D is the transmission range. The max-min fair benefit for the star and the circle group is $14D^4/16$, while that for the pentagon group is zero.

power spent by group i to route to the exit points before joining the coalition. Also let $J^i_{\vec{r}}$ be the power spent by nodes of group i under coalition routing $\vec{r}$. The benefit for group i is then $B^i_{\vec{r}}\ \forall\ i = 1 \dots n$ with $B^i_{\vec{r}} = P^i_{opt} - J^i_{\vec{r}}$. The benefit vector for the coalition routing is $\vec{B_{\vec{r}}} \equiv (B^1_{\vec{r}}, B^2_{\vec{r}} \dots B^n_{\vec{r}})$.

We mention some important properties of fair coalition routing for multiple group coalition.

Proposition 2: Consider three groups A,B and C. Consider three separate coalitions (A,B), (B,C) and (A,B,C). If the pairwise coalitions (A,B) and (B,C) are mutually beneficial for each group (i.e., the benefit for each group under some coalition routing is positive), then the coalition (A,B,C) is beneficial for each group.

A counterexample presented in Fig.10 shows that the converse is not true.

The components of the max-min fair benefit vector need not be equal when more than two groups combine. Refer to Fig.11 where each node generates 1 Mbps. Here, $M = 1$ and $K = 1$.

Now we present the *multi-group FC* algorithm. This algorithm solves a sequence of linear programs. Note that solving a single linear program is not sufficient since the components of the max-min fair benefit allocation need not be equal in this general case.

Let $I = \{1 \dots n\}$, INC refer to the individual node constraints (3) and LF refer to the load flow condition (4) generalized to multiple groups.

$\mathbf{Stage_1}$: **Maximize:** Z:

Subject to: $Z \leq B^i_{\vec{r}} \, \forall i \in I$

$\vec{r}$ satisfies INC and LF.

Let Z_1^* be the objective value and $\vec{r_1^*}$ be the routing obtained from above. Let $equal$ = $\{t : \ B^t_{\vec{r_1^*}} = Z_1^*\}$.

$\mathbf{Substage_1}$ For each $k \in equal$,

Maximize: $B^k_{\vec{r}}$:

Subject to: $B^i_{\vec{r}} \geq Z_1^* \, \forall i \in I \setminus \{k\}$

$\vec{r}$ satisfies INC and LF.

Let $\vec{r_k}$ be the routing corresponding to the k^{th} maximization $\forall \, k \in equal$. Let $e_1 = \{n : B^n_{\vec{r_n}} = Z_1^*\}$.

$\mathbf{Stage_2}$: **Maximize:** Z:

Subject to: $Z \leq B^i_{\vec{r}} \, \forall i \in I \setminus e_1$

$B^i_{\vec{r}} \geq Z_1^* \, \forall i \in e_1$

$\vec{r}$ satisfies INC and LF.

Let Z_2^* be the objective value and $\vec{r_2^*}$ be the routing obtained from above. Let $equal$ = $\{t : \ B^t_{\vec{r_2^*}} = Z_2^*\}$.

$\mathbf{Substage_2}$ For each $k \in equal$

Maximize: $B^k_{\vec{r}}$:

Subject to: $B^i_{\vec{r}} \geq Z_2^* \, \forall i \in I \setminus e_1 \setminus \{k\}$

$B^i_{\vec{r}} \geq Z_1^* \, \forall i \in e_1$

$\vec{r}$ satisfies INC and LF.

Let $\vec{r_j}$ be the routing corresponding to the j^{th} maximization $\forall \, j \in equal$. Let $e_2 = \{n : B^n_{\vec{r_n}} = Z_2^*\}$.

Similarly in the ith step.

$\mathbf{Stage_i}$: **Maximize:** Z:

Subject to: $Z \leq B^i_{\vec{r}} \, \forall i \in I \setminus e_1 \setminus e_2 \ldots \setminus e_{i-1}$

$B^i_{\vec{r}} \geq Z_t^* \, \forall i \in e_t \, \forall t = 1 \ldots (i-1)$

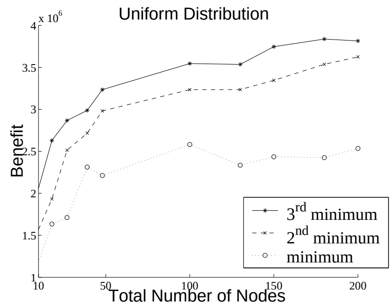


Fig. 12. We average the minimum, second minimum and the largest component over 100 topologies.

$\vec{r}$ satisfies INC and LF.

Theorem 6: The routing $\vec{r}$ obtained as a solution of multi-group FC is a fair coalition routing.

Fig.12 shows benefits for fair coalition routing for three equal sized groups spread over a square of side 100m. Here, $M = 1$ and $K = 1$.

B. Receiving Power

We have so far assumed that a node does not consume any power when it is receiving information. We now relax this assumption, and assume that the receiving power of a node is proportional to the incoming traffic rate. The total power expenditure of a node v is the sum of the power spent to transmit load $\sum_i L_i(v)$ and to receive load $\sum_i (L_i(v) - O_i(v))$. Thus

$$\begin{aligned} N_{\vec{r}}(v) &= K_1 + K \sum_i \sum_{v' \in V \cup \{\mathbf{e_i}\}} r_i(v, v') d(v, v')^4 + K' \sum_i (L_i(v) - O_i(v)) \\ &= K_1 + K \sum_i \sum_{v' \in V \cup \{\mathbf{e_i}\}} r_i(v, v') d(v, v')^4 + K' \sum_i \sum_{v'' \in V} r_i(v'', v) \text{ (from (1)).} \\ J^a_{\vec{r}} &= \sum_{v \in V^a} N_{\vec{r}}(v) \\ \text{Similarly, } J^b_{\vec{r}} &= \sum_{v \in V^b} N_{\vec{r}}(v). \end{aligned}$$

The max-min fair benefit vector and the fair coalition routing can be computed by substituting the expressions for $J^a_{\vec{r}}, J^b_{\vec{r}}$ in **FC** with the above¶.

¶Now, P^a_{opt} (P^b_{opt}) can still be obtained by routing the traffic using the minimum weight path in group a (b). But, the weight of a link (v, v') is now $Kd(v, v')^4 + K'$ instead of $d(v, v')^4$. This happens since we assume that the receiving power depends only on the received rate.

The distributed algorithm remains similar except for the rate update strategy which needs to be modified. We describe the update strategy for $r_{n+1}(v, v')$ when $v \in V^a$ and $(v, v') \in E^{\text{joint}}$. The update strategy for $r_{n+1,i}(v, v')$ when $v \in V^b$ and $(v, v') \in E^{\text{joint}}$ can be obtained by interchanging a with b in the following

$$\begin{aligned}
r_{n+1,i}(v, v') &= [r_{n,i}(v, v') - \gamma\delta_n(s^v_{n,i} - s^{v'}_{n,i} + d(v, v')^4\epsilon^a_n)]_+ \text{ if } v \in V^a, v' \in \mathbf{e}. \\
r_{n+1,i}(v, v') &= [r_{n,i}(v, v') - \gamma\delta_n \left(s^v_{n,i} - s^{v'}_{n,i} + \left(K'/K + d(v, v')^4\right) \epsilon^a_n\right)]_+ \text{ if } v, v' \in V^a. \\
r_{n+1,i}(v, v') &= [r_{n,i}(v, v') - \gamma\delta_n(s^v_{n,i} - s^{v'}_{n,i} + d(v, v')^4\epsilon^a_n + \epsilon^b_n)]_+ \text{ if } v \in V^a, v' \in V^b.
\end{aligned}$$

The convergence guarantees in Theorem 5 hold.

C. Generalized Propagation Model

We first consider a simple generalization where $\kappa(v, v')$s are different for different links, but do not change with time. This happens when the environment is static. Now, for successful communication to v' a node v must transmit each bit at energy E_{tx}, where $E_{\text{tx}}\kappa(v, v')^{-1}d(v, v')^{-\alpha} \geq E_{\text{rx}}$. The power consumed by node v under routing $\vec{r}$ is then $K_1 + K \sum_{v' \in V \cup \{\mathbf{e}\}} r(v, v')\kappa(v, v')d(v, v')^\alpha$. Thus, now $d(v, v')^\alpha$ must be replaced with $\kappa(v, v')d(v, v')^\alpha$ everywhere (note that $\kappa(v, v')d(v, v')^\alpha$ can be obtained by measuring the signal strength at receiver v'). The framework remains the same other than this change, and all analytical guarantees hold.

We next consider the case that the environment and hence $\kappa(v, v')$ changes with time for each link $(v, v')^{\|}$. The time duration during which $\kappa(v, v')$ does not change for a link (v, v') is referred to as the coherence time of the link. Coherence times are large when nodes move around slowly, e.g., when the maximum node velocity $v_{\max}$ is lower than 5 m/s, the coherence time is $c/(v_{\max} \times f) = (3 \times 10^8)/(5 \times 2.4 \times 10^9) = 25$ ms [26] (p. 165). Here f is the centre frequency of the signal and c is the speed of light. The fair coalition routing can now be recomputed every time $\kappa(v, v')$ changes. Since the distributed algorithm converges fast in presence of incremental changes, the rate allocation can seamlessly adapt to changes in $\kappa(v, v')$. If however

$^{\|}$When the environment is not static, $\kappa(v, v')$ is modeled as a random variable whose logarithm is normally distributed with mean zero and a variance 5-12dB depending on the environment [26].

Measurement	Plain	Authenticated	Encrypted
Time (mins)	95	93	86
Rate (Mbps)	4.016	4.034	4.061
Data (MB)	2861	2813	2622

Fig. 13. IPSec Costs

$\kappa(v, v')$ changes rapidly, statistical information must be used to determine the link rates and the transmission powers. Specifically, the transmission powers and the routing can be determined assuming that $\kappa(v, v') = E[\kappa(v, v')] + 2\sqrt{\mathrm{Var}[\kappa(v, v')]}$, as with a high probability, $\kappa(v, v') \leq E[\kappa(v, v')] + 2\sqrt{\mathrm{Var}[\kappa(v, v')]}$.

D. Trust Issues

We assume that members of a group trust one another and are willing to jointly route packets to save power in the interest of the group as a whole. We assume that when groups agree to form a coalition, they trust one another to use the fair coalition routing. There is related work [20] on how to detect cheating in which one or more parties do not support their agreed routing rules. Nodes in a group can use security schemes to ensure that they route for other nodes in the same group and in groups that are participating in the coalition. Within a group one can identify trusted members with public key certificates and thereafter establish a symmetric key for authenticating individual packets. Different groups can be authenticated via third party public key repository. This can prevent nodes from masquerading as nodes of some other group that is already a part of an active coalition. This leads to a natural question as to what is the cost incurred to enforce group routing.

We tested whether this incurs significant additional power if it is done with IPSec tunnels [16] between neighboring nodes. To get an idea of the processing overhead, we let a Dell L400 laptop running Windows 2000 generate constant bit rate UDP traffic over an 802.11b network. The payload rate was fixed at 4 Mbps. For various security parameters, we measured the time for the laptop to die down[**]. Fig. 13 shows the results

[**]Before each experiment, the laptop was charged fully from a completely dead battery to nullify battery memory and hysteresis, and was subsequently switched off for 2 hours to eliminate heating-related discrepancies.

for three cases averaged over five runs of the experiment. The first column shows that the laptop battery died in 95 minutes after sending 2861MB of data in plaintext. Header overhead accounts for the rate of 4.016 Mbps to send 4 Mbps of payload. Authentication used null-encrypted ESP [15] with SHA1 for message authentication codes; encryption used ESP with SHA1 and 3DES. Encryption has a significant effect on power, but it is not really needed to enforce group routing. We can assume that nodes encrypt end-to-end and do not need hop-by-hop encryption. Hence it is possible to enforce group routing efficiently with only modest power costs by using authentication with null encryption. Thus, it is clearly worthwhile to use group routing.

IPSec is a sufficiently efficient enforcement mechanism when the number of nodes is less than 50. This is because each node is likely to route to only a few others. Thus about 50-100 tunnels are required and these can all use null encryption. There are techniques that work efficiently for larger groups (see, for example, the IETF documents from the Multicast Security working group, msec) but these seem unnecessary if the nodes are laptops. For sensor networks, a more specialized security protocol may be necessary. A comprehensive design of security mechanisms is beyond the focus of this paper.

VII. Conclusions

We have studied the problem of forming coalitions between groups of nodes with the intent of saving power. We found that an application of max-min fair techniques to this problem yields an efficient and balanced approach which we call fair coalition routing. We developed theory and algorithms for fair coalition routing. We have carried out a range of simulations that demonstrate that fair coalition routing is practical and beneficial in common cases.

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APPENDIX

Proof of Theorem 1: Consider two possible coalition routings between A and B. Let $P^a_{\vec{r_1}}$ and $P^b_{\vec{r_1}}$ be the powers expended for routing $\vec{r_1}$ by groups A and B respectively, and $P^a_{\vec{r_2}}$ and $P^b_{\vec{r_2}}$ similarly for routing $\vec{r_2}$. The benefits vector for routing $\vec{r_1}$ is $(P^a_{opt} - P^a_{\vec{r_1}}, P^b_{opt} - P^b_{\vec{r_1}})$ and for routing $\vec{r_2}$ is $(P^a_{opt} - P^a_{\vec{r_2}}, P^b_{opt} - P^b_{\vec{r_2}})$. Consider a new routing that sends α fraction of traffic through routing $\vec{r_1}$ and $1-\alpha$ fraction through routing $\vec{r_2}$. Since $P_{\vec{r}}$ is a linear function of $\vec{r}$, we have the new power expenditure as $\alpha * P^a_{\vec{r_1}} + (1-\alpha) * P^a_{\vec{r_2}}$ for group A and $\alpha * P^b_{\vec{r_1}} + (1-\alpha) * P^b_{\vec{r_2}}$for group B. The benefit vector for the new routing is then,

$$(P^a_{opt} - (\alpha * P^a_{\vec{r_1}} + (1-\alpha) * P^a_{\vec{r_2}}), P^b_{opt} - (\alpha * P^b_{\vec{r_1}} + (1-\alpha) * P^b_{\vec{r_2}})),$$

which is, $$\alpha * (P^a_{opt} - P^a_{\vec{r_1}}, P^b_{opt} - P^b_{\vec{r_1}}) + (1-\alpha) * (P^a_{opt} - P^a_{\vec{r_2}}, P^b_{opt} - P^b_{\vec{r_2}}).$$

Hence the set of feasible benefit vectors is convex. $\diamond$

Proof for Proposition 1: Let $\vec{r}$ be a fair coalition routing, and $\min(B^a_{\vec{r}}, B^b_{\vec{r}}) < 0$. Consider the routing $\vec{r_1}$ in which each group uses its group optimal. Then $B^a_{\vec{r_1}} = 0$ $B^b_{\vec{r_1}} = 0$, and $\min(B^a_{\vec{r}}, B^b_{\vec{r}}) < \min(B^a_{\vec{r_1}}, B^b_{\vec{r_1}})$. Thus from the minimum component property $\vec{r}$ is not a fair coalition routing which is a contradiction. $\diamond$

Proof for Theorem 2: Let $(B^a_{\vec{r}}, B^b_{\vec{r}})$ be the benefit vector under fair coalition routing $\vec{r}$. If the minimum is greater that t, then all other components are also greater than t. Hence $\vec{r}$ will result in a useful coalition.

Now we prove the 'only if' condition using contradiction. Let the minimum component of max-min fair benefit vector be less that t. Also suppose a routing $\vec{r_1}$ exists, such that $\min(B^a_{\vec{r_1}}, B^b_{\vec{r_1}}) \geq t$. Thus $\vec{r}$ is not a fair coalition routing from the minimum component property. This is a contradiction. $\diamond$

Proof for Theorem 3: Consider two groups A and B. Let $\vec{r}$ be a fair coalition routing. Suppose that $B^a_{\vec{r}} > B^b_{\vec{r}}$. From Proposition 1 $B^a_{\vec{r}} \geq 0$ and $B^b_{\vec{r}} \geq 0$. Thus $B^a_{\vec{r}} > 0$. Since A benefits from the coalition it sends traffic to at least one node in B. Now consider a coalition routing $\vec{r^*}$ in which group A sends α fraction of traffic through the joint routing $\vec{r}$ and $1 - \alpha$ fraction of traffic through its group optimal, $0 < \alpha < 1$. B routes as in $\vec{r}$. Clearly $\vec{r^*}$ is feasible. Now consider the links (v, v') from group A nodes ($v \in V^a$) to group B nodes ($v \in V^b$) in the joint routing. Since in the optimal routing nodes in A do not route their traffic through the nodes in B, for each such (v, v'), $\sum_i r^*(v, v') \leq \sum_i r(v, v')$ and for some $v \in V^a$ and $v' \in V^b$, $\sum_i r^*(v, v') < \sum_i r(v, v')$. Hence $J^b_{\vec{r^*}} < J^b_{\vec{r}}$. Now, $B^b_{\vec{r}} = P^b_{opt} - J^b_{\vec{r}}$ and $B^b_{\vec{r^*}} = P^b_{opt} - J^b_{\vec{r^*}}$. Since $J^b_{\vec{r^*}} < J^b_{\vec{r}}$, $B^b_{\vec{r^*}} > B^b_{\vec{r}}$ for any $\alpha \in (0, 1)$. Since $B^a_{\vec{r}} > B^b_{\vec{r}}$, when α is sufficiently close to 1, $B^a_{\vec{r*}} > B^b_{\vec{r*}}$, but then $\vec{r}$ does not satisfy the minimum component property. This is a contradiction. $\diamond$

We will use the following concepts in proving Theorem 5.

Consider a convex and continuous function f defined on a convex set $F \subseteq \mathbf{R}^k$. Then a vector $w_0 \in \mathbf{R}^k$ is called a *subgradient* of f at a point $y_0 \in F$ if it satisfies $f(y) - f(y_0) \geq (w_0, y - y_0)\ \forall y \in F$. An interior point y_0 of F is the minimum point of f in F if and only if $\vec{0}$ belongs to the set of subgradients at y_0.

Proof for Theorem 5: Let $g(v) = \sum_i (\sum_{v'} r_i(v, v') - O_i(v) - \sum_{v''} r_i(v'', v))$ and $z(v) = K_1 + K \sum_i \sum_{v' \in V^a \cup V^b \cup \{\mathbf{e_i}\}} r_i(v, v') d(v, v')^4 - B(v)$.

P: Maximize : $F(\vec{r}, Z) = Z - \gamma s(\vec{r}, Z)$ where $s(\vec{r}, Z) = \sum_{v \in V^a \cup V^b} (|g(v)| + \max(0, z(v))) + \max(0, Z -$

$B^a_{\vec{r}}) + \max(0, Z - B^b_{\vec{r}})$. Let $\vec{Q} \equiv (\vec{r}, Z)$. Let $\vec{Q}^* \equiv (\vec{r}^*, Z^*)$ be the optimal solution and U^* be the optimal value of $F(\vec{r}, Z)$. We prove in two steps. In the first step, we prove that **P** has the same solution as **FC** for $\gamma > 1$. In the second step, we prove that the routing obtained by the iterative approach converges to the optimal solution of **P**, i.e., $\lim_{n\to\infty} ||\vec{r_n} - \vec{r^*}|| = 0$ where $\vec{r_n}$ is the routing obtained in the nth iteration, and $||\vec{X}||$ denotes the norm of $\vec{X}$, i.e., if $X \equiv (x_1, x_2 \ldots)$ then $||\vec{X}|| = \sqrt{x_1^2 + x_2^2 \ldots}$. The result follows.

Step 1: Select $\vec{Q}$ such that $s(\vec{Q}) > 0$. For such $\vec{Q}$, there always exists a component of the subgradient that is less than or equal to $1-\gamma$ and $1-\gamma$ is less than 0. Therefore $\vec{0}$ does not belong to the set of subgradients. Hence $\vec{Q}$ cannot be an optimal solution for **P** . Therefore all solutions of **P** involve $\vec{Q}$ for which $s(\vec{Q}) = 0$. Also for $s(\vec{Q}) = 0$ the value of the objective function of **FC** and **P** are equal. Therefore for $\gamma > 1$, any optimal solution of **P** is an optimal solution of **FC**.

Step 2: Choose an arbitrary $\kappa > 0$. Let $\kappa' = \kappa/2$. For any $\epsilon' > 0$ define $D_{\epsilon'}$ as $D_{\epsilon'} = \{\vec{Q} : F(\vec{Q}) \geq U^* - \epsilon'\}$. From Theorem 27.2 [27] it follows that there exists an $\epsilon = \epsilon(\kappa') > 0$ such that

$$D_\epsilon \subset \{\vec{Q} : ||\vec{Q} - \vec{Q}^*|| \leq \kappa'\}. \tag{5}$$

Consider n for which $\vec{Q}_n \notin D_\epsilon$. Therefore $F(\vec{Q}_n) < U^* - \epsilon$.

The update equations at the nodes of group A and B can be compactly stated as $\vec{Q}_{n+1}(v, v') = [\vec{Q}_n(v, v') + \delta_n \vec{\nu_n}]_+$, where $\vec{\nu_n}$ is the subgradient of $F(\vec{r}, Z)$. It follows from the definition of subgradients that $(\vec{\nu_n}, \vec{Q}_n - \vec{Q}^*) \leq F(\vec{Q}_n) - U^* < -\epsilon$. Now, $||\vec{\nu_n}|| \leq T$, where $T = \sqrt{2\gamma^2(1+L^4)^2N^2 + (2\gamma+1)^2}$, L is the maximum distance between any two nodes and N is the total number of nodes in the network.

$$\begin{aligned}
||\vec{Q}_{n+1} - \vec{Q}^*||^2 &= ||[\vec{Q}_n + \delta_n \vec{\nu_n}]_+ - \vec{Q}^*||^2 \\
&\leq ||\vec{Q}_n + \delta_n \vec{\nu_n} - \vec{Q}^*||^2 \\
&= ||\vec{Q}_n - \vec{Q}^*||^2 + \delta_n^2 ||\vec{\nu_n}||^2 + 2\delta_n(\vec{\nu_n}, \vec{Q}_n - \vec{Q}^*) \\
&< ||\vec{Q}_n - \vec{Q}^*||^2 + T^2\delta_n^2 - 2\epsilon\delta_n.
\end{aligned}$$

Since $\delta_n \to 0$, $\delta_n \leq \epsilon/T^2$ when n is sufficiently large. For all such n,

$$||\vec{Q}_{n+1} - \vec{Q}^*||^2 < ||\vec{Q}_n - \vec{Q}^*||^2 - \epsilon\delta_n. \tag{6}$$

Suppose there exists a $N'_\epsilon < \infty$ such that $\vec{Q}_n \notin D_\epsilon$ for all $n \geq N'_\epsilon$. Therefore, there exists $N_\epsilon \geq N'_\epsilon$ such that (6) holds for all $n \geq N_\epsilon$. Adding the inequalities obtained from (6) for $n = N_\epsilon$ to $N_\epsilon + m$ we obtain

$$||\vec{Q}_{N_\epsilon+m+1} - \vec{Q}^*||^2 < ||\vec{Q}_{N_\epsilon} - \vec{Q}^*||^2 - \epsilon \sum_{n=N_\epsilon}^{N_\epsilon+m} \delta_n,$$

which implies that $||\vec{Q}_{N_\epsilon+m+1} - \vec{Q}^*|| \to -\infty$ as $m \to \infty$ since $\sum_1^\infty \delta_n = \infty$. This is not possible since $||\vec{Q}_{N_\epsilon+m+1} - \vec{Q}^*|| \geq 0$. Hence the supposition was incorrect. Hence there exists a sequence $n_{1,\epsilon} < n_{2,\epsilon} < \ldots$ such that $\vec{Q}_{n_{i,\epsilon}} \in D_\epsilon$ for all $i = 1, 2, \ldots$. Let $i_1 = n_{1,\epsilon}$. Since $\delta_n \to 0$, there exists i_2 s.t. $\delta_n \leq \min(\kappa'/T, \epsilon/T^2)$, $\forall\, n \geq n_{i_2,\epsilon}$. Let $i' = \max(i_1, i_2)$. Consider the following cases.

Case 1: $n = n_{j,\epsilon}$ for some $j \geq i'$. Here $\vec{Q}_n \in D_\epsilon$ and from (5) it follows that $||\vec{Q}_n - \vec{Q}^*|| \leq \kappa' < \kappa$.

Case 2: $n = n_{j,\epsilon} + 1$ for some $j \geq i'$. Then $\vec{Q}_n = \vec{Q}_{n_{j,\epsilon}+1} = [\vec{Q}_{n_{j,\epsilon}} + \delta_{n_{j,\epsilon}} \vec{\nu}_{n_{j,\epsilon}}]_+$. Thus $||\vec{Q}_n - \vec{Q}_{n_{j,\epsilon}}|| = ||[\vec{Q}_{n_{j,\epsilon}} + \delta_{n_{j,\epsilon}} \vec{\nu}_{n_{j,\epsilon}}]_+ - \vec{Q}_{n_{j,\epsilon}}|| \leq ||\vec{Q}_{n_{j,\epsilon}} + \delta_{n_{j,\epsilon}} \vec{\nu}_{n_{j,\epsilon}} - \vec{Q}_{n_{j,\epsilon}}|| = \delta_{n_{j,\epsilon}} ||\vec{\nu}_{n_{j,\epsilon}}|| \leq U \delta_{n_{j,\epsilon}} \leq \kappa'$. From the above and since $||\vec{Q}_{n_{j,\epsilon}} - \vec{Q}^*|| \leq \kappa'$ (Case 1) we get $||\vec{Q}_n - \vec{Q}^*|| \leq ||\vec{Q}_{n_{j,\epsilon}} - \vec{Q}^*|| + ||\vec{Q}_n - \vec{Q}_{n_{j,\epsilon}}|| \leq \kappa' + \kappa' = 2\kappa' = \kappa$.

Case 3: $n_{j,\epsilon} + 1 < n < n_{j+1,\epsilon}$ for some $j \geq i'$. Also $\vec{Q}_{n'} \notin D_\epsilon \forall n_{j,\epsilon} < n' < n_{j+1,\epsilon}$. From (6), it follows that $||\vec{Q}_{n'+1} - \vec{Q}^*|| < ||\vec{Q}_{n'} - \vec{Q}^*||$. Thus, $||\vec{Q}_n - \vec{Q}^*|| < ||\vec{Q}_{n_{j,\epsilon}+1} - \vec{Q}^*||$. Since $||\vec{Q}_{n_{j,\epsilon}+1} - \vec{Q}^*|| \leq \kappa$ (Case 2), $||\vec{Q}_n - \vec{Q}^*|| \leq \kappa$.

From cases 1,2 and 3, it follows that $||\vec{Q}_n - \vec{Q}^*|| \leq \kappa\, \forall n \geq n_{i',\epsilon}$. Since κ is arbitrary, $\lim_{n\to\infty} ||\vec{Q}_n - \vec{Q}^*|| = 0$ and since $\vec{Q} \equiv (\vec{r}, Z)$ we have $\lim_{n\to\infty} ||\vec{r}_n - \vec{r^*}|| = 0$. $\diamond$

Proof for Proposition 2: Consider the joint routing $\vec{r_1}$ under which (a) A and B jointly route to the exit points without using any node in C, and both groups have positive benefits and (b) C routes optimally to the exit point without using nodes of groups A and B. Under $\vec{r_1}$, group C has zero benefit, and groups A and B have positive benefits. Such $\vec{r_1}$ exists because the coalition between A and B is mutually beneficial. Now, using $\vec{r_1}$ we construct a coalition routing $\vec{r}$ that will make benefits of all three groups positive. Since the coalition between B and C is mutually beneficial, at least one node in C can send traffic through at least one node in B. Let b1 and c1 be such a node pair. Let c1 send α fraction of its traffic to b1 where $\alpha > 0$ and $1 - \alpha$ fraction of its traffic using its group optimal. Now for any $\alpha > 0$ under $\vec{r}$ the benefit of group C will be greater than that under $\vec{r_1}$ (as nodes in C route less traffic under $\vec{r}$ than under $\vec{r_1}$), and hence positive.

Also, the benefits of groups A and B under $\vec{r}$ is less than that under $\vec{r_1}$, as nodes in groups A and B route more traffic under $\vec{r}$ than $\vec{r_1}$. But, α can be suitably reduced to keep the benefits of groups A and B positive. Hence a routing $\vec{r}$ exists under which all three groups have a positive benefit.

Proof for Theorem 6: Consider a feasible benefit vector $\vec{B_{\vec{r}}}$ such that there exists subsets $y_1, y_2 \dots y_k$ such that for $k \leq n$, $y_1 \cup \dots y_k = \{1 \dots n\}$ and the following conditions hold.

1) $B^i_{\vec{r}} = B^j_{\vec{r}}$ if $i, j \in y_m$ for each $m \in \{1 \dots k\}$.
2) $B^i_{\vec{r}} > B^j_{\vec{r}}$ if $i \in y_m$ and $j \in y_{m-1}$ for each $m \in \{2 \dots k\}$.
3) For any $i \in y_m$ while maintaining feasibility $B^i_{\vec{r}}$ cannot be increased without reducing $B^j_{\vec{r}}$ for some $j \in y_1 \cup \dots y_m$.

Then $\vec{B_{\vec{r}}}$ is a max-min fair benefit vector.

Each stage of the linear program has a feasible solution. Let the program yield a routing $\vec{r^*}$ and terminate at stage k. Clearly $\vec{B_{\vec{r^*}}}$ is feasible. Note that $e_1 \cup \dots \cup e_k = \{1 \dots n\}$. We will show that $\vec{B_{\vec{r^*}}}$ satisfies the above properties with $y_1 = e_1, \dots, y_k = e_k$. Note that $B^i_{\vec{r^*}} = Z^*_m \; \forall \; i \in e_m$ and $1 \leq m \leq k$. Also $Z^*_1 < Z^*_2 \dots < Z^*_k$. Thus properties 1 and 2 hold. Let property3 not hold. Then there exists a routing $\vec{r_1}$ such that $B^i_{\vec{r^*}} > Z^*_m$ for some $i \in e_m$ and $B^j_{\vec{r_1}} \geq B^j_{\vec{r^*}}$ for each $j \in \{e_1 \cup \dots \cup e_m\}$.

Case A: Let $B^j_{\vec{r_1}} \geq Z^*_m$ for each $j \in \{e_{m+1} \cup \dots \cup e_k\}$. But then $\vec{r_1}$ is a feasible solution of a substage of stage m and therefore $i \notin e_m$.

Case B: Let $B^j_{\vec{r_1}} < Z^*_m$ for some $j \in \{e_{m+1} \cup \dots \cup e_k\}$. Then we have two feasible benefit vectors $\vec{B_{\vec{r_1}}}$ and $\vec{B_{\vec{r^*}}}$ such that $B^j_{\vec{r_1}} \geq B^j_{\vec{r^*}}$ for each $j \in \{e_1 \cup \dots e_m\}$, $B^i_{\vec{r_1}} > Z^*_m$ and $B^j_{\vec{r^*}} > Z^*_m$ for each $j \in \{e_{m+1} \cup \dots e_k\}$.

Let $\vec{A(\alpha)} = \alpha \vec{B_{\vec{r^*}}} + (1-\alpha)\vec{B_{\vec{r_1}}}$ for $0 < \alpha < 1$. Now, from Theorem 1, $\vec{A(\alpha)}$ is a feasible benefit vector. For each $\alpha > 0$, $A^j(\alpha) \geq B^j_{\vec{r^*}}$ for each $j \in \{e_1 \cup \dots e_m\}$ and $A^i(\alpha) > Z^*_m$. For α close to 1, $A^j(\alpha) > Z^*_m$ for each $j \in \{e_{m+1} \cup \dots e_k\}$. Let α_0 be one such α. Then like in Case A, $\vec{A(\alpha_0)}$ is a feasible solution of a substage of stage m and $i \notin e_m$. This is a contradiction and thus property (c) also holds. $\diamond$