Homework 1 (Posted 29th Jan, Due before class 5th February)

f(n)	$\log^k n$	n^k	$\sqrt{n}$	$\log(n!)$
g(n)	n^ϵ	c^n	n^{sinn}	$\log(n^n)$
Is $f(n)$ $O(g(n))$?				
Is $f(n)$ $\Omega(g(n))$?				
Is $f(n)$ $\Theta(g(n))$?				
Is $f(n)$ $o(g(n))$?				

Table 1:

Problem 1: (Grade 16 pts): In Table 1, $k \geq 1, \epsilon > 0, c > 1$. Please answer yes or no, and also justify your answer in each case.

Problem 2: (Grade 6 pts) State true or false. Justify your answer (Give reasons if your answer is true, give a counter-example if your answer is false). Assume throughout that $f(n) \geq 0$ and $g(n) \geq 0$ for all n.

1. If $f(n)$ is $O(g(n))$ then $\log f(n)$ is $O(\log g(n))$. Assume that $\log(g(n)) > 0$ and $f(n) \geq 1$ $\forall n$.
2. $f(n) + o(f(n))$ is $\Theta(f(n))$.
3. $f(n)$ is $O\left((f(n))^2\right)$.

Problem 3: (Grade 3 pts) Prove that $F_N \geq 2^{N/2}$ for all $N \geq 2$. Here, $F_0, F_1, F_2, \ldots$ are the Fibonacci numbers.

Problem 4: (Grade 8 pts) You have a list of n real numbers, and another number x. You need to find out whether the sum of any two consecutive numbers in the list equals x or not. Give an algorithm to solve the problem. Analyze its complexity. For full grade you need to give a $O(n \log n)$ algorithm.

Now give an algorithm which finds out whether there are p consecutive elements whose sum equals x, where p is an input. Analyze its complexity.

Problem 5: (Grade 7 pts) You have a real number x, and a sequence of real numbers $a_0, \ldots, a_{n-1}$. Give an algorithm to find out the value of the polynomial $\sum_{i=0}^{n-1} a_i x^i$. Analyze the complexity of your algorithm. For full grade you need to give a $O(n)$ algorithm. Please note that the basic operations are addition, multiplication, subtraction, division, memory access, read and write operations. In particular, i multiplications need i basic steps.