

Homework 7 (Posted 26th March, Due during or before class 2nd April)

Problem 1: 6 pts How many bits do you need to store the size and height of the trees in the Union Find data structure? Give your answer for different implementations (e.g., arbitrary unions, unions by size, union by height, path compression etc.).

Problem 2: 6 pts Let A be the adjacency matrix of a directed graph. Consider the matrix A^2 . Is there any relation between the entry $A^2[i, j]$ to the number of paths between vertices i, j in the original graphs? Justify your answer. Answer the same question about $A^n[i, j]$.

Problem 3: 6 pts Consider the graph representation of a complete binary tree. How many edges can a complete binary tree of n nodes have? How will you represent it (adjacency list or adjacency matrix)? Give the storage complexity of your representation.

Problem 4: 6 pts Consider an adjacency list representation of a digraph. Give algorithms to compute the in-degree and out-degree of a vertex using the adjacency list representation. Analyze their complexities.

Problem 5: 6 pts Let A be the adjacency matrix of a graph G . Let G^T be a graph with adjacency matrix A^T . (A^T is transpose of a matrix A . Transpose of a matrix A , A^T is related to A as follows: $A^T[i, j] = A[j, i]$.) Do you see any relation between G and G^T ? Answer the question for a digraph G .