Cooperative Profit Sharing in Coalition Based Resource Allocation in Wireless Networks

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Abstract*—We consider a network in which several service providers offer wireless access service to their respective subscribed customers through potentially multi-hop routes. If providers cooperate, i.e., pool their resources, such as spectrum and service units, and agree to serve each others' customers, their aggregate payoffs, and individual shares, can potentially substantially increase through efficient utilization of resources and statistical multiplexing. The potential of such cooperation can however be realized only if each provider intelligently determines who it would cooperate with, when it would cooperate, and how it would share its resources during such cooperation. Also, when the providers share their aggregate payoffs, developing a rational basis for such sharing is imperative for the stability of the coalitions. We model such cooperation using the theory of *transferable payoff coalitional games.* We first consider the scenario where the locations of providers' service units and the set of channels they have access to have been decided a priori. We show that the optimum cooperation strategy, which involves the allocations of the channels and service units to mobile customers, can be obtained as the solution of a convex optimization. We next show that the grand coalition is stable in this case, i.e., if all providers cooperate, there is always an operating point that maximizes the providers' aggregate payoff, while offering each a share that removes any incentive to split from the coalition. Next, we consider scenarios where the providers decide where to open their service units and which channels to lease. We show how the optimal cooperation strategy can be obtained by solving integer/convex optimizations, and that the previous results hold in some important special cases. Next we investigate the stability of two other sharing rules, the nucleolus and the Shapley value. Finally we study the problem of optimal selection of service level agreements (SLA)s by providers.

I. Introduction

A. Motivation

We have witnessed a significant growth in commercial wireless services in the past few years, and the trend is likely to continue in the foreseeable future. Satisfaction of this increasing demand is contingent upon efficient utilization of the transmission resources, which are either under-utilized (e.g., spectrum - utilization of licensed spectrum is at times only 15% [1]), or costly (e.g. infrastructure). Cooperation among different wireless providers has the potential for substantially improving the utilization of the available resources, and should therefore enhance the proliferation of wireless services. In particular, different providers may form a coalition and pool their resources, such as spectrum and infrastructure like base stations or access points (which we refer to as service units) and relay nodes, and serve each others' customers. Such coalitions may lead to substantially higher throughput through statistical multiplexing and lower overall energy consumptions of the customers through multi-hop relaying. Both of these in turn lead to higher customer satisfaction, and higher payoff for the providers. Cooperation may also be instrumental in reducing the costs incurred by the providers in acquiring spectrum and deploying infrastructure like service units. This would again lead to higher net payoff for the providers. We now elucidate the above benefits using a sequence of examples.

We first demonstrate how cooperation may substantially enhance throughput and decrease energy consumption of customers. Transmission qualities of available channels randomly fluctuate with time and space, owing to customer mobility and propagation conditions. Also, in secondary access networks, the providers may be secondary users who do not license channels but communicate when the license holders (primary users) do not use the channels. Such access opportunities may only arise sporadically. Since all customers of all providers do not need to be served simultaneously, and the channels of different providers may not be unavailable or have poor qualities simultaneously, spectrum pooling can enhance throughput by mitigating service fluctuations resulting from occasional variations in channel qualities and availabilities, and instantaneous traffic overloads. In multi-hop wireless networks (e.g., mesh networks), cooperation increases the number of available relays (mesh points). This in turn increases the number of multi-hop routes to each customer, thereby decreasing the total power usage and the total throughput of the customers. Also, the customers may be induced to serve as relays, perhaps, in lieu of service discounts. Then the enhancement in throughput and energy consumption owing to cooperation magnifies as the coalitions have a larger set of customers, and therefore a larger number of multi-hop routes.

We now demonstrate how cooperation may substantially reduce the costs incurred by the providers. A provider can acquire a channel by paying a fixed licensing cost or usage based charges, or a combination of the two. The first case arises when the providers are primary users who license the channels from government agencies, and the other options arise when they are secondary users who use the channels licensed by the primaries. When the providers do not cooperate, they may need to operate as secondary users and opt primarily for usage based charges, as the volume of their individual traffic may not justify other options. Since cooperation allows the providers to pool the customers, the resulting higher aggregate traffic may allow them to license channels, share the licensing fees

*C. Singh & A. Kumar are in the Dep. of Electrical Communications Eng., Indian Institute of Science, Bangalore, and their research has been supported by an INRIA Associates program DAWN, and also by the Indo-French Centre for the Promotion of Advanced Research (IFCPAR), Project No. 4000-IT-A.
S. Sarkar & A. Aram are in the Dep. of Electrical and Systems Eng., University of Pennsylvania. Their contributions have been supported by NSF grants NCR- 0238340, CNS-0721308, ECS-0622176.

and thereby reduce the individual costs. Next, deploying new service units (e.g., access points) and subsequently maintaining them, is one of the major costs in expanding the networks. Cooperation may reduce the expansion costs by eliminating the need for deploying additional service units, and also allow the providers to deliver desired coverage and throughput guarantees while deploying fewer service units. For example, consider a provider whose customer base is concentrated in a particular region. Traffic demand is therefore low but non-zero (owing to customer mobility) in other regions. The provider must however deploy service units even in the regions of low traffic intensity so as to provide universal coverage (otherwise the customers would desert). If instead, the provider cooperates with another whose traffic demand is concentrated in a different region, both may satisfy coverage requirements by deploying service units only in the regions where their individual demands are concentrated, and thereby reduce individual operational expenses.

B. Research challenges and Contributions

Several research challenges must, however, be addressed before large scale cooperation can be realized. First, commercial service providers are selfish entities who seek to maximize their individual payoffs. Therefore, they will cooperate with others only when cooperation increases their individual incomes. Even so, a provider is likely to refuse to join a coalition if it perceives that its share of the aggregate payoff is not commensurate to the amount it invested and the wealth it generated. The former depends on the transmission rates in the channels it has acquired and the locations and the number of service units it has deployed, while the latter depends on its customer base. So, developing a rational basis for determining the individual shares of the aggregate payoff is imperative. Note that the aggregate payoff and the individual shares depend on the providers' cooperation strategies. Specifically, each provider needs to decide which providers it would cooperate with, which channels it is going to use, the locations of its service units, and when it should serve the customers of another provider. The sharing mechanism and the optimal cooperation strategies for each provider depend on each other and must be obtained jointly.

We present a framework to determine the optimal decisions of the providers using tools from *transferable payoff coalitional game theory*. The framework also provides a rational basis for sharing the aggregate payoff. The first network setup we consider is an access network where providers pool their spectrum, service units and customers (Section IV). We assume that the locations of service units and the set of channels they have access to, are determined a priori, but the providers decide how they would allocate the service units and the channels of the coalition, to the customers. We then obtain optimal decision rules for the providers and a strategy for sharing the resulting aggregate payoff as solutions of convex optimization problems. This sharing strategy can be computed in polynomial time and ensures that it is optimal for all providers to cooperate. Specifically, if any subset of providers split from the grand coalition (the coalition of all providers), irrespective of how they cooperate and the way they share their aggregate payoff, at least one provider in this subset receives less net payoff than what it received in the grand coalition. In coalitional game terminology, such a sharing scheme exists only when the *core* of the game is nonempty. This result is of interest in itself as many cooperative games have empty cores, and the specific games we consider do not satisfy some standard sufficiency conditions for non-emptiness of the core (e.g., convexity of the game).

In the following sections, we extend the formulation and results. We first consider the cases where in addition, the providers need to determine the locations of their service units or the set of channels each service unit has access to (Section V). The optimal cooperation strategy can now be obtained by solving an integer optimization where the duality gap is nonzero unlike that in convex optimizations used before. We obtain the optimal decision rules and the payoff sharing mechanism in some important special cases of this general problem. Subsequently, we extend the results in Section IV to networks with multi-hop transmissions (Section VI).

As the core of a coalitional game (including the games we study) need not be a singleton, it is not obvious which element in the core of the game should determine the shares of the providers. Thus, we discuss two other sharing mechanisms with the property that both existent uniquely, namely, the nucleolus and the Shapley value (Section VII). We numerically evaluate and compare the providers' payoff increases resulting from cooperation under different sharing mechanisms and different payoff functions as a function of the number of customers and service units (Section IX). Finally, we consider the problem of optimal selection of service level agreements (SLAs) by the service providers. This problem is of interest since a) providers are limited in resources and can not accept all SLAs and b) accepting or rejecting an SLA could increase or decrease a providers payoff, which depends on the set of providers in the coalition as well as the actions they take. We then propose, and subsequently qualitatively compare the relative strengths and weaknesses of, two SLA selection strategies (Section VIII).

II. Related Work

Interactions between different entities in wireless networks have primarily been investigated from the following extreme perspectives. In the first, each entity is assumed to select its actions so as to maximize its individual incentive without coordinating with others, e.g., [2]–[8]. This scenario, which has been investigated using noncooperative game theory, in general suffers from inefficient utilization of resources [9]. The other perspective has been to assume that entities selflessly choose their actions so as to optimize a global utility function even when such actions may deteriorate individual incentives of some entities (e.g., [2], [6]). We investigate interactions among providers assuming that each provider would be willing to cooperate and coordinate its actions with others when such cooperation enhances its individual incentives.

We obtained optimal cooperation schemes using the framework of cooperative game theory. This choice of tools allowed us to combine the desirable features of the extreme approaches studied in the existing literature, that of allowing entities to choose their actions guided by selfish objectives, and of maximizing global utility functions. Surprisingly, cooperative game theory has seen only limited use in wireless context so far. Nash bargaining solutions have been proposed for power control and spectrum sharing among multiple users [10]. Coalitional games have been used recently for modeling cooperation among nodes in the physical layer [11], [12], collaborative sensing by secondary users in cognitive radio networks [13], rate allocation in multiple access channels (MAC) [14], rate allocation among mobiles and admission control in heterogeneous wireless access environments [15], and studying cooperation between single antenna receivers and transmitters in an interference channel [16]. Our problem formulation, solution techniques, and results significantly differ from the above owing to the difference in contexts - our focus is on cooperative resource allocation and resulting payoff sharing among providers at the network and MAC layers. To our knowledge, our work is the first to investigate cooperation among wireless providers.

Coalitional game theory has been used for studying cooperation in other communication networks (see [17] for a survey). For instance, Shapley value based profit sharing has been proposed and investigated for incentivizing cooperation among peers [18] and among internet service providers [19]. Our formulations and solution techniques may be used for establishing the non-emptiness of the core and computing an allocation in the core in polynomial time for coalitional games among internet service providers (Section VI).

III. System Model

Consider a network with a set of providers $\mathcal{N}$. Each provider i deploys a set of service units $\mathcal{B}_i$ in order to serve its set of customers $\mathcal{M}_i$. Let $\mathcal{B}_i \cap \mathcal{B}_j = \emptyset$ and $\mathcal{M}_i \cap \mathcal{M}_j = \emptyset$ for $i \neq j$. For a $\mathcal{S} \subseteq \mathcal{N}$, let $\mathcal{B}_\mathcal{S}$ and $\mathcal{M}_\mathcal{S}$ denote the set of service units and customers associated with providers in $\mathcal{S}$. Thus $\mathcal{B}_\mathcal{N}$ and $\mathcal{M}_\mathcal{N}$ are the sets of all service units and all customers in the network, respectively. We assume, unless mentioned otherwise, that a) the locations of service units and the channels they have access to are predetermined and b) the service units and the customers communicate through single-hop links. We show how these assumptions can be relaxed in Sections V and VI, respectively. Each customer j negotiates a minimum rate guarantee with its provider; we refer to these negotiations as *service level agreements* (SLAs). We present the communication model and the coalitional game among providers in Sections III-A and III-B and describe in Section III-C how the formulations capture the essence of existing wireless technologies.

A. Communication Model

We assume that each service unit has access to a single channel or a frequency band[1]. We assume that the achievable rates of a customer-service unit pair do not depend on communications of other customers and service units. This is equivalent to different service units having access to different channels. Thus, we use the same set of indices for the set of service units and the channels they have access to. At a given time, each service unit can serve at most one customer, and each customer can be served by at most one service unit (*time sharing*).

For ease of exposition, we consider only downlink communications in our model (the results easily extend to the case where communications involve both uplinks and downlinks). The instantaneous rates the customers receive depend on their current locations and the current quality of the channels accessed by the associated service units (which in case of secondary users also includes the current actions of the channels' primary users), both of which can be random. We therefore assume that when customer j is served by service unit k, j receives at a rate r_{jk}, a random variable which is a function of the location of customer j and the state of channel k (we therefore allow frequency selective fading). Let Ω_{jk} be the joint state space of customer j's location and channel k's state. We assume that $|\Omega_{jk}|$ is finite, since (i) feasible service rates in any practical communication system belong to a finite set, and (ii) we can partition the service region in such a way that the service rates received by the customers inside a member of the partition do not depend on the locations of the customers. Let $\Omega = \prod_{\substack{j \in \mathcal{M}_\mathcal{N} \\ k \in \mathcal{B}_\mathcal{N}}} \Omega_{jk}$. An $\omega \in \Omega$ denotes a network realization. Let $\mathbb{P}(\omega)$ be the probability of the outcome ω.

B. Utility and Game Model

We now propose a coalitional game theory framework that models the interactions of the providers who would cooperate only when such cooperation enhances their individual profits (payoffs in the terminology of economics).

Definition III.1. *A coalition $\mathcal{S} \subseteq \mathcal{N}$ is a subset of providers who cooperate. We refer to $\mathcal{N}$ as the grand coalition.*

Definition III.2. *A coalitional game with transferable payoff $< \mathcal{N}, v >$ consists of a finite set $\mathcal{N}$ (set of providers) and a characteristic function $v(\cdot)$ that associates with every nonempty subset $\mathcal{S}$ of $\mathcal{N}$, a real number $v(\mathcal{S})$. For each coalition $\mathcal{S}$, $v(\mathcal{S})$ is the maximum aggregate payoff available for division* in any arbitrary way *among the members of $\mathcal{S}$.*

A service unit can serve a customer only when either both are associated with the same provider, or the providers associated with them are in a coalition. Let $\alpha_{jk}(\omega) \in [0,1]$ be the fraction of time service unit k serves customer j, when the realization of rates is ω. When the provider associated

[1] This assumption causes no loss of generality. In the case where service units have access to multiple channels with a radio available for every channel, each service unit channel combination can be considered as one unit in $\mathcal{B}_i$. We will see later how the case where service units have a limited number of radios can also be captured.

with customer j is in coalition $\mathcal{S}$ and the network realization is ω, the rate received by j is a random variable $y_j(\omega) = \sum_{k\in\mathcal{B}_\mathcal{S}} \alpha_{jk}(\omega) r_{jk}(\omega)$.

When customers associated with provider i receive rates $\mathbf{y}_i(\omega) = \{y_j(\omega), j \in \mathcal{M}_i\}$, i gains a benefit (e.g., revenue from the customers) of $U_i(\mathbf{y}_i(\omega))$, where $U_i(\cdot)$ is an increasing concave function and equal to 0 at the origin[2]. Next, owing to the tariffs imposed by spectrum regulators or by the license holders of the channels who allow the providers to use the channels, provider i incurs a cost of $V_i(\mathbf{z}_i(\omega))$, where $\mathbf{z}_i(\omega) = \{z_k(\omega), k \in \mathcal{B}_i\}$ and $z_k(\omega) = \sum_{j\in\mathcal{M}_\mathcal{S}} \alpha_{jk}(\omega)$ is the total fraction of time channel (or, service unit) k is used. Functions $V_i(\cdot)$ are increasing, convex and equal to 0 at the origin. Then the profit (or payoff) of a coalition $\mathcal{S}$ is the sum of the U_is for $i \in S$ minus the sum of the V_is for $i \in S$. We assume that the benefit and cost functions $U_i(.), V_i(.)$ are decided apriori (based on spectrum regulation, customer charging policies etc.), and do not investigate the optimal selections of these functions.

Providers in a coalition $\mathcal{S}$ have to decide how to schedule service units to customers, i.e., select the variables $\alpha_{jk}(\omega)$s, for each $\omega \in \Omega$, based on the benefit and cost functions $U(.), V(.)$ so as to maximize their total profit, subject to possible service level agreements. Let $v(\mathcal{S})$ denote the maximum aggregate profit available to a coalition $\mathcal{S}$. In Sections, IV, V and VI, we show how to obtain $v(\mathcal{S})$ by solving convex/integer optimizations.

C. How the formulations relate to existing wireless communication systems

We now illustrate via examples how our framework can be used to model specific communication systems. Consider elastic data transfers in the downlink of a CDMA cellular system (e.g., used for internet access of cellular subscribers) [20, Chapter 5] with provider set $\mathcal{N}$. Owing to simplicity of physical layer implementations, a base station (service unit) always transmits at a pre-determined fixed power (which may be different for different cells). This happens even when a base station's downlink queue is empty (i.e., no mobiles associated with it require downlink transmission). Similar communication model has extensively been used in related contexts [21]. Customers in a cell are served on *time-sharing* basis, i.e., a base station transmits to at most one customer at a given time. Also, at any given time, a customer is associated with only one base-station and thus receives transmissions from at most one base station. Then, $\{\alpha_{jk}(\omega)\}$ represent the fraction of times customers are served by different base stations. Now, let P_k be the fixed transmission power of base station k. The channel gains between customer-base station pairs, h_{jk}s, are random. When base station k transmits to customer j and the channel gain realization is ω, the downlink SINR to j is [20, Chapter 5]

$$\mathrm{SINR}_{jk}(\omega) = \frac{h_{jk}(\omega) P_k}{\sum_{i'\in\mathcal{B}_\mathcal{N}\setminus\{k\}} h_{ji'}(\omega) P_{i'} + N_0 W},$$

where N_0 is the power spectral density of the additive noise and W is the spectrum bandwidth[3]. Thus, $\mathrm{SINR}_{jk}(\omega)$ is independent of which customers are being served by other base stations. Further, the rate achievable between pair j-k, $r_{jk}(\omega)$, is a function of $\mathrm{SINR}_{jk}(\omega)$, hence $r_{jk}(\omega)$ is also independent of transmissions to other customers.

In a variant of the above service discipline (*power sharing*), a base station distributes its total power among the downlink transmissions in its cell. Orthogonal codes and chip synchronous transmissions can ensure that the intra-cell interference for a customer is negligible. As in the earlier case, the inter-cell interference remains fixed. The quantity r_{jk} is the fixed peak rate between customer j and base station k, and is achieved when k uses its entire power to transmit to j. The variables $\{\alpha_{jk}(\omega)\}$ account for the fractional power allocation[4]. We formulate the characteristic functions considering time-sharing, and point out the modifications required for incorporating power sharing.

Next, consider downlink communications in a multi-cell OFDMA system [20, Chapter 6]. The system bandwidth is divided into several, say C, channels (sub-carriers in OFDM terminology). In order to manage interference, fractional frequency reuse is employed; i.e., the set of sub-carriers is partitioned into reuse groups, with one such group of sub-carriers being assigned to each base-station. Customers are permanent, as would be the case if the system is being used to provide internet access service to apartments and offices. The base-station allocates sub-carriers from its assigned reuse group to customers in the periphery of its coverage area. It assigns any of the remaining channels to customers in the remaining part of its coverage area (proximate to the base station). With such reuse partitioning and spatial allocation of subcarriers we can assume that the interference is zero. Also assume that each base-station, in each state ω, assigns a fixed transmit power to each of its carriers. With these assumptions the downlink rate that a user j gets over service unit k (which denotes a base-station and sub-carrier pair) depends only on the channel gain from the corresponding base-station to itself, and not on which user is served by each service unit. At any given time, a sub-channel can

[2] For example, $U_i(\mathbf{y}_i)$ may equal $\nu_i \sum_{j\in\mathcal{M}_i} y_j$, where ν_i is the cost per unit throughput imposed by provider i on its customers, or more generally, $U_i(\mathbf{y}_i(\omega))$ may equal $\sum_{j\in\mathcal{M}_i} g_i(y_j)$, where $g_i(.)$ is an increasing concave revenue function chosen by provider i. Customer satisfactions turn out to be concave functions of rates and as such revenue functions are usually chosen as concave (and increase sub-linearly in practice). Also, note that we allow the revenue functions to be different for different customers of the same provider.

[3] The mobiles at cell boundaries experience poor SNR owing to high interference from neighboring base stations. Thus, in some implementations, neighboring base stations are allocated different bands; but again sometimes all base stations are allocated the same band so as to facilitate smooth hand-overs and since CDMA technology can provide acceptable rates even in presence of low SINRs. Note that our framework allows any general band allocation across base stations (i.e., different bands can be assigned to disjoint sets of base stations in an arbitrary manner). Also note that the SINR expression above, in particular, assumes that all base stations use same band; in general we sum over all co-channel base stations to obtain the aggregate interference in the denominator.

[4] In the low SNR regime, the rates are proportional to the SNR, and thus the peak rates are shared among the mobiles in the same proportion as the overall power.

be assigned to only one customer, but more than one sub-channel can be assigned to a customer (*multiple allocation*). The communication model presented in Section III-A captures all these attributes except the multiple allocation condition. We will point out the modifications required for allowing for multiple allocation while formulating the characteristic functions in the next section.

IV. Spectrum Pooling Game

We start with deriving the characteristic function $v(\cdot)$ for the resource pooling game. For a coalition $\mathcal{S} \subseteq \mathcal{N}$, the maximum aggregate payoff $v(\mathcal{S})$ is given by the following convex optimization problem.

$\mathrm{P}(\mathcal{S}) : \max \sum_{\substack{i \in \mathcal{S} \\ \omega \in \Omega}} \mathbb{P}(\omega) \Big(U_i(\mathbf{y}_i(\omega)) - V_i(\mathbf{z}_i(\omega)) \Big)$

subject to:

1) $y_j(\omega) = \sum_{k \in \mathcal{B}_\mathcal{S}} \alpha_{jk}(\omega) r_{jk}(\omega), \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega$
2) $z_k(\omega) = \sum_{j \in \mathcal{M}_\mathcal{S}} \alpha_{jk}(\omega), \quad k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$
3) $\sum_{k \in \mathcal{B}_\mathcal{S}} \alpha_{jk}(\omega) \leq 1, \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega$
4) $\sum_{j \in \mathcal{M}_\mathcal{S}} \alpha_{jk}(\omega) \leq 1, \quad k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$
5) $\sum_{\omega \in \Omega} \mathbb{P}(\omega) y_j(\omega) \geq m_j, \quad j \in \mathcal{M}_\mathcal{S}$
6) $\alpha_{jk}(\omega) \geq 0, \quad j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$

Constraints (3) ensure that for all $j \in \mathcal{M}_\mathcal{S}$, the fraction of time customer j is served is at most 1. Constraints (4) ensure that the fraction of time each service unit $k \in \mathcal{B}_\mathcal{S}$ serves is at most 1[5]. Constraints (5) provide the minimum service guarantees. Incidentally, constraints (3), (4) arise from the time-sharing model [6], but for power-sharing or for multiple allocation models, only constraints (4) suffice - all results presented below extend even in absence of constraint (3).

Assumption IV.1. *$P(\{i\})$ is feasible for each $i \in \mathcal{N}$, i.e., each provider can support the minimum rates of its customers even when it does not cooperate with other providers.*

Then $\mathrm{P}(\mathcal{S})$ is feasible for each $\mathcal{S} \subseteq \mathcal{N}$.

Thus, the optimization problem $\mathrm{P}(\mathcal{S})$ provides the maximum aggregate payoff of the providers in a coalition $\mathcal{S}$ and also the optimal service unit-customer allocations that attain this maximum. Clearly, $v(\mathcal{N}) \geq v(\mathcal{S})$ for any $\mathcal{S} \subseteq \mathcal{N}$, i.e., the grand coalition of all providers attains the maximum possible aggregate payoff among all coalitions.

Finally, we examine whether the above resource allocation framework captures the intricacies of existing wireless traffic. Until a few years back, wireless traffic predominantly consisted of voice calls. Voice calls are not overly sensitive to transmission rates, and may proceed even when the rates are low. Thus, minimum rate requirements (SLAs) were redundant, and customers were charged based only on their usage times (*air-times*), and not on the service rates they received. A typical pricing scheme would entail charging the customers linearly per unit air-time: then, $U_i(\mathbf{y}_i) = 0$, $V_i(\mathbf{z}_i) = -\alpha_i \sum_{k \in \mathcal{B}_i} z_k - \beta_i |\mathcal{M}_i|$ for each provider i, where $\beta_i > 0$ is the fixed fee provider i imposes on each of its customers for availing of its service and $\alpha_i > 0$ is the revenue per unit air-time provider i collects from its customers. Our framework can optimally allocate the customers to the service-units under the above pricing scheme since $U_i(\cdot), V_i(\cdot)$ are respectively concave and convex functions for each i. Incidentally[7], a customer must only be charged for the amount of airtime he needs and not for additional airtime the provider can provide. This can be incorporated in our framework by allowing the network state ω to also represent the activity states of the customers. Let $\mathcal{A}_\mathcal{S}(\omega)$ be the set of customers of $\mathcal{M}_\mathcal{S}$ that are active (i.e., involved in voice-calls) in ω. Constraint 2 in $\mathrm{P}(\mathcal{S})$ may be modified as $z_k(\omega) = \sum_{j \in \mathcal{A}_\mathcal{S}(\omega)} \alpha_{jk}(\omega)$ for each $k \in \mathcal{B}_\mathcal{S}$ and $\omega \in \Omega$. Thus, $z_k(\omega)$ considers the airtimes of only the "on-call" customers. The modified optimization $\mathrm{P}(\mathcal{S})$ continues to be a concave maximization with linear constraints, and all subsequent results apply.

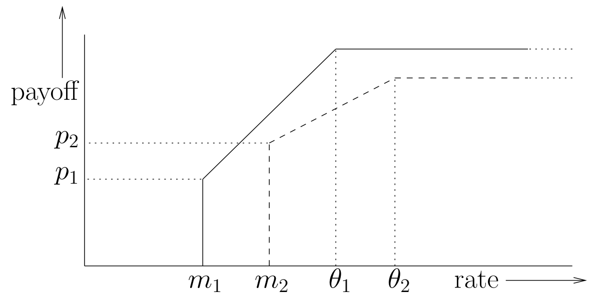

Fig. 1: Examples of revenue functions. The customers pay fixed costs p_js for being guaranteed minimum average rates m_js, but do not pay additional costs for rates beyond θ_js.

Data is fast emerging as the predominant component of wireless traffic. Many emerging applications, such as streaming video, require certain minimum rate, and the quality of service is critically sensitive to the service rate. Thus, minimum rate constraints are likely to be integral components of service agreements in near future, and providers are likely to charge (i) fixed fees that are increasing functions of the minimum rates agreed upon, and (ii) additionally for service rates they can provide over and above the required minimum value. A customer may however be willing to pay additionally for rates only up to a certain maximum rate value determined by his QoS requirements. The following simple pricing strategy captures the above features. If the

[5]This condition can be modified to capture the scenario when a service unit has access to multiple channels with only 1 radio, as follows. The modified Constraint (4) for a service unit, bounds the sum of $\alpha_{jk}(\omega)$ over customers $j \in \mathcal{M}_\mathcal{S}$, and channels k accessed by that service unit, by 1. It can be shown that all the subsequent results extend to this scenario.

[6]The system can be represented by a complete bipartite graph where the customers and the service units represent the nodes and there exists a link between every customer-service unit node pair. Under the time-sharing model, any customer-service unit assignment corresponds to a matching in the above graph. Note that for each ω, $\{\alpha_{jk}(\omega)\}$ comprise a feasible allocation of service units to customers if and only if there exists a corresponding collection of matchings $L_1, L_2, \ldots$ and a collection of non-negative real numbers $\gamma_1, \gamma_2, ..$ such that (i) $\sum_i \gamma_i = 1$, $\gamma_i \geq 0$ and (ii) if the service unit - customer allocation follows matching L_i for γ_i fraction of time for each i, then service unit k transmits to customer j for $\alpha_{jk}(\omega)$ fraction of time for all j, k. Constraints (3), (4) provide the necessary and sufficient condition for feasibility of $\{\alpha_{jk}(\omega)\}$ for each ω [22].

[7]More involved revenue schemes such as those that charge customers additionally only beyond a certain amount of airtime usage do not however constitute convex functions and can not therefore be incorporated in our framework.

average rate a customer of provider i receives is r, and he has negotiated a minimum rate guarantee of m, then he pays $\alpha_i \max\left(\min(r,\theta) - m, 0\right) + \beta_i' m$, where θ is the maximum rate the customer needs (Fig. 1 with $p_j = \beta_j' m_j$). Owing to the minimum rate constraints (5) in P($\mathcal{S}$), each customer's average rate is at least m_j. Thus, $V_i(\mathbf{z}_i) = 0$,

$$U_i(\mathbf{y}_i) = \alpha_i \sum_{j \in \mathcal{M}_i} \max(y_j, \gamma_j) + \beta_i m_j \text{ with } \beta_i = \beta_i' + \alpha_i$$

capture the above pricing strategy. Our framework can incorporate the above pricing scheme since $U_i(\cdot), V_i(\cdot)$ are respectively concave and convex functions for each i. Finally, constraints (5) in P($\mathcal{S}$) apply to the average service rates; more stringent QoS demands may require constraints on service rates in each ω, i.e., given certain desired minimum rates $m_j(\omega)$ for different $\omega \in \Omega$, $y_j(\omega) \geq m_j(\omega)$ for each $\omega \in \Omega$. The modified optimization P($\mathcal{S}$) continues to be a concave maximization with linear constraints, and all subsequent results apply. Alternatively, "soft" minimum rate guarantees may be ensured in each ω by choosing strict concave revenue functions. Specifically, higher the degree of concavity of the revenue functions (that is lower the second derivatives), a provider incurs higher additional revenue in any ω by enhancing the service rate of a customer who is receiving a low rate at that ω as opposed to enhancing that of a customer who is receiving a high rate at that ω. Thus, providers are more likely to equalize the service rates of all customers at each ω, and in the process ensure certain minimum rates to each customer at every ω.

A rational basis for splitting the maximum aggregate payoff is however imperative for motivating the providers to join the grand (or any other) coalition. We introduce a solution concept in coalitional games known as the *core* for providing such a rational basis. The idea behind the core in a cooperative game is analogous to that behind a Nash equilibrium of a noncooperative game: an outcome is stable if no deviation is profitable.

Definition IV.1. *For any real valued vector $\mathbf{x} = (x_i, i \in \mathcal{N})$ and any coalition $\mathcal{S}$, we let $x(\mathcal{S}) = \sum_{i \in S} x_i$. Such a vector is said to be an imputation if $x(\mathcal{N}) = v(\mathcal{N})$ and $x_i \geq v(\{i\})$ for all $i \in \mathcal{N}$. The core of the coalitional game with transferable payoff $\langle N, v\rangle$ is the set of all imputations $\mathbf{x}$ for which $x(\mathcal{S}) \geq v(\mathcal{S})$ for all $\mathcal{S} \subset \mathcal{N}$. In other words,*

$$\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^{\mathcal{N}} : x(\mathcal{N}) = v(\mathcal{N}), x(\mathcal{S}) \geq v(\mathcal{S}), \forall \mathcal{S} \subset \mathcal{N}\} \quad (1)$$

Note that an imputation provides the payoff shares of providers in a grand coalition such that no provider's payoff is below that it earns alone (i.e., in absence of any cooperation). The core consists of a collection of imputations that provide stronger guarantees: no coalition has any incentive to split from the grand coalition if the providers share the aggregate grand coalition payoff $v(\mathcal{N})$ as per an imputation $\mathbf{x}$ in the core. To see this, suppose a set of providers $\mathcal{S} \subset \mathcal{N}$ split from the grand coalition and form a separate coalition to share their aggregate payoff $v(\mathcal{S})$ as per $\mathbf{w}$. A provider $i \in \mathcal{S}$, however, would agree to split from the grand coalition only if $w_i > x_i$. This implies that $v(\mathcal{S}) = \sum_{i \in \mathcal{S}} w_i > \sum_{i \in \mathcal{S}} x_i$, and thus contradicts the fact that $\mathbf{x} \in \mathcal{C}$. Therefore, every imputation in the core renders the grand coalition stable. *This is a globally desirable outcome, since the grand coalition maximizes the aggregate payoff.*

We now elucidate $v(\cdot)$ and $\mathcal{C}$ using a simple example.

Example IV.1. *Let $\mathcal{N} = \{1, 2\}$, $\mathcal{B}_i = \{i\}, i = 1, 2$, and $\mathcal{M}_i = \{2i-1, 2i\}, i = 1, 2$. Let $r_{jk} = P$ for $j \in \mathcal{M}_1$, and $r_{jk} = Q$ for $j \in \mathcal{M}_2$, for all $k \in \mathcal{B}_{\mathcal{N}}$. Suppose $P < Q$ and $m_j = 0, \forall j \in \mathcal{M}_{\mathcal{N}}$. Let the benefit functions be the sum of the customers' service rates and costs be zero. Then $v(\{1\}) = P$, $v(\{2\}) = Q$, and $v(\{1, 2\}) = 2Q$ (when the providers cooperate, the aggregate benefit is maximized when only provider 2's customers are served and this maximum is $2Q$). Then, $\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^2 : x_1 + x_2 = 2Q, x_1 \geq P, x_2 \geq Q\}$. For instance, $(\frac{Q+P}{2}, \frac{3Q-P}{2})$ is an imputation in a core. Note that when $1, 2$ cooperate, the benefit (revenue) earned from provider 1's (2's, resp.) customers is 0 ($2Q$, resp.), and therefore less (more, resp.) than its payoff under the above imputation. Nevertheless, this imputation increases each provider's payoff by $\frac{Q-P}{2}$ as compared to that in absence of cooperation.*

In several coalitional games the core is empty, i.e., the grand coalition can not be stabilized [23, Example 260.3], and in general it is NP-hard to determine whether the core of a coalitional game is nonempty [24]. A sufficient condition for the core to be nonempty is the convexity of the coalitional game, i.e., $v(\mathcal{S}) + v(\mathcal{T}) \leq v(\mathcal{S} \cup \mathcal{T}) + v(\mathcal{S} \cap \mathcal{T})$ for all $\mathcal{S}, \mathcal{T} \subseteq \mathcal{N}$ [23, pp. 260]. But, as the following example illustrates, the game we are considering need not be convex.

Example IV.2. *Let $\mathcal{N} = \{1, 2, 3\}$, $\mathcal{B}_i = \{i\}, i = 1, 2, 3$, $\mathcal{M}_i = \{i\}, i = 1, 2, 3$. Let $r_{1k} = R, k \in \mathcal{B}_{\mathcal{N}}$, $r_{j1} = P, j \in \{2, 3\}$ and $r_{jk} = Q, j \in \{2, 3\}, k \in \{2, 3\}$ and $P > Q$. Let $m_j = 0$ for all $j \in \mathcal{M}_{\mathcal{N}}$. Let the benefit functions of the providers be the sum of the service rates and costs be zero. Thus $v(\{1\}) = R, v(\{1, 2\}) = v(\{1, 3\}) = R + P$, and $v(\{1, 2, 3\}) = R + P + Q$. Let $\mathcal{S} = \{1, 2\}$ and $\mathcal{T} = \{1, 3\}$. Then $v(\mathcal{S}) + v(\mathcal{T}) = 2R + 2P$ and $v(\mathcal{S} \cup \mathcal{T}) + v(\mathcal{S} \cap \mathcal{T}) = 2R + P + Q$. Thus $v(\mathcal{S}) + v(\mathcal{T}) > v(\mathcal{S} \cup \mathcal{T}) + v(\mathcal{S} \cap \mathcal{T})$. Hence, this game is not convex.*

Nevertheless, we next show that the game $< \mathcal{N}, v >$ always has a nonempty core (note that convexity is not a necessary condition for nonemptiness of the core). We use a proof technique similar to ones presented in [25]–[28]. The proof is constructive in that it provides an imputation in $\mathcal{C}$ as well.

Let[8] $\lambda, \beta \in \mathbb{R}^{\mathcal{M}_{\mathcal{S}} \times \Omega}$, $\nu, \gamma \in \mathbb{R}^{\mathcal{B}_{\mathcal{S}} \times \Omega}$, $\rho \in \mathbb{R}^{\mathcal{M}_{\mathcal{S}}}$, and $\varphi \in \mathbb{R}^{\mathcal{M}_{\mathcal{S}} \times \mathcal{B}_{\mathcal{S}} \times \Omega}$. Let $g_{i\omega}(\lambda, \rho) = \max_{y_j(\omega) \geq 0} \big(\mathbb{P}(\omega) U_i(\mathbf{y}_i(\omega)) + \sum_{j \in \mathcal{M}_i} y_j(\omega)(\lambda_j(\omega) + \rho_j \mathbb{P}(\omega))\big)$ and $h_{i\omega}(\nu) = \max_{z_k(\omega) \geq 0} \big(- \mathbb{P}(\omega) V_i(\mathbf{z}_i(\omega)) + \sum_{k \in \mathcal{B}_i} z_k(\omega) \nu_k(\omega)\big)$.

[8] The notations can be explained considering $|\Omega| = 1$, $\mathcal{M}_1 = \{4, 5, 6\}$ and $\mathcal{M}_2 = \{7, 8, 9\}$. A vector $x \in \mathbb{R}^{\mathcal{M}_1 \times \Omega}$ will have components x_4, x_5 and x_6 corresponding to customers $4, 5$ and 6 respectively. Similarly, a vector $x \in \mathbb{R}^{\mathcal{M}_2 \times \Omega}$ will have components x_7, x_8 and x_9 corresponding to customers $7, 8$ and 9 respectively.

Then we have the following as the dual of P(S):

D($\mathcal{S}$) : $\min \sum_{i\in\mathcal{S}} \Big(\sum_{\omega\in\Omega} \big(g_{i\omega} + h_{i\omega} + \sum_{k\in\mathcal{B}_i} \gamma_k(\omega) + \sum_{j\in\mathcal{M}_i} \beta_j(\omega)\big) - \sum_{j\in M_i} m_j\rho_j \Big)$
subject to:

I) $\lambda_j(\omega) r_{jk}(\omega) + \nu_k(\omega) + \beta_j(\omega) + \gamma_k(\omega) \geq 0, \quad j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$

II) $\beta_j(\omega), \gamma_k(\omega), \rho_j \geq 0, \quad j \in \mathcal{M}_\mathcal{S} k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$

Clearly, D($\mathcal{S}$) is feasible for each $\mathcal{S} \subseteq \mathcal{N}$. Formulate D($\mathcal{N}$) by appropriately defining vectors $\lambda, \beta, \gamma, \nu, \rho, \varphi$ and let $\mathcal{D}$ be the set of optimal solutions of D($\mathcal{N}$). Then, $\mathcal{D} \neq \emptyset$. Let

$$\begin{aligned}\mathcal{I} =\{\mathbf{x}^* \in \mathbb{R}^{\mathcal{N}} : x_i^* = \sum_{\omega\in\Omega} \Big(& g_{i\omega}(\lambda^*, \rho^*) + h_{i\omega}(\nu^*) \\ & + \sum_{k\in\mathcal{B}_i} \gamma_k^*(\omega) + \sum_{j\in\mathcal{M}_i} \beta_j^*(\omega) \Big) \\ & - \sum_{j\in\mathcal{M}_i} m_j\rho_j^* \text{ for some } \quad (\lambda^*, \nu^*, \beta^*, \gamma^*, \rho^*, \varphi^*) \in \mathcal{D}\}\end{aligned}$$

Here is the main result:

Theorem IV.1. *$\mathcal{I} \neq \emptyset$ and $\mathcal{I} \subseteq \mathcal{C}$.*

Thus, any imputation in $\mathcal{I}$ stabilizes the grand coalition - it also ensures that the payoffs of the providers are commensurate with the resource they invest and the wealth they generate. To see this, note that β_j^*, γ_k^* are Lagrange multipliers associated with the constraints (3), (4), respectively. For ease of exposition, let there be no minimum rate requirements and let the benefit and cost functions be linear. Then, $g_{i\omega}(\lambda^*, \rho^*) = h_{i\omega}(\nu^*) = 0$, and provider i's payoff x_i^* equals the sum of the Lagrange multipliers corresponding to the constraints (3), (4) for its customers and service units. Lagrange multiplier $\gamma_k^*(\omega)$ ($\beta_j^*(\omega)$, resp.) is high only when service unit k (customer j, resp.) is fully utilized, i.e., serves customers (is served, resp.) all the time, and provide high transmission rates, $r_{jk}(\omega)$ and cost less (pay more, resp.) per unit bandwidth. Thus, i's Lagrange multipliers and hence i's payoff is high when it invests more resource and/or generates more wealth.

Proof: Since $\mathcal{D} \neq \emptyset$, $\mathcal{I} \neq \emptyset$. We show that for an arbitrary $\mathbf{x}^* \in \mathcal{I}$, $\mathbf{x}^* \in \mathcal{C}$. Note that since $U_i(\cdot)$s and $V_i(\cdot)$s are (increasing) concave and convex functions respectively, the objective function of P($\mathcal{S}$) is concave. Also, the constraints of P($\mathcal{S}$) are all linear. Therefore, P($\mathcal{S}$) is maximizing a concave function over a convex set. Thus, strong duality holds.

Now, consider an arbitrary $\mathbf{x}^* \in \mathcal{I}$, corresponding to one $(\lambda^*, \nu^*, \beta^*, \gamma^*, \rho^*, \varphi^*) \in \mathcal{D}$. Clearly $x^*(\mathcal{N}) = \sum_{i\in\mathcal{N}} x_i^*$ is the optimal value of D($\mathcal{N}$). Since D($\mathcal{S}$) is the dual of P($\mathcal{S}$) for each $\mathcal{S} \subseteq \mathcal{N}$, by strong duality $x^*(\mathcal{N}) = v(\mathcal{N})$. Now we only need to show that $x^*(\mathcal{S}) = \sum_{i\in\mathcal{S}} x_i^* \geq v(\mathcal{S})$ for any $\mathcal{S} \subset \mathcal{N}$. By strong duality, $v(\mathcal{S})$ equals the optimum value of D($\mathcal{S}$). Consider the sub-vectors $\lambda_\mathcal{S}^*, \nu_\mathcal{S}^*, \beta_\mathcal{S}^*, \gamma_\mathcal{S}^*, \rho_\mathcal{S}^*, \varphi_\mathcal{S}^*$ consisting of the components of $\lambda^*, \nu^*, \beta^*, \gamma^*, \rho^*, \varphi^*$ in $\mathcal{S}$. Clearly these sub-vectors constitute a feasible solution of D($\mathcal{S}$) and $x^*(\mathcal{S})$ is the value of the objective function of D($\mathcal{S}$) for the above feasible solution. Therefore, the optimal value of D($\mathcal{S}$) is a lower bound for $x^*(\mathcal{S})$, i.e., $x^*(\mathcal{S}) \geq v(\mathcal{S})$. ∎

A. Computation Complexity and Distributed Computation

Note that P($\mathcal{S}$), D($\mathcal{S}$) are convex optimizations with linear constraints, and the number of the variables and constraints are polynomial in $|\Omega|, |\mathcal{M}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|$. Therefore, the computation times for the maximum aggregate payoff and optimal allocation for any given coalition, and an imputation in the core grows polynomially with the above [29]. The computation times can however be large since $|\Omega|$, typically, is large. This may not however pose a major challenge as the computations are done off-line using large work-stations and at a slower time-scale (only when the network state is updated or the coalitions are assessed). Also, whenever customers do not have minimum average rate constraints (see Constraints (5)), we can solve both P($\mathcal{S}$), D($\mathcal{S}$) by solving separate convex optimizations for each $\omega \in \Omega$ - the number of variables and constraints for each such optimization depends only on $|\mathcal{M}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|$ [9]. This separability allowed us to solve the above optimizations for reasonably large systems using Monte Carlo simulations (Section IX).[10]

Concave optimizations with linear constraints can be solved in distributed manner using the theory of subgradients, as described in [30], [31] for example. For brevity we describe the distributed computations only for P($\mathcal{S}$) - the same approach applies for computation of an imputation in the core via solving D($\mathcal{S}$). For simplicity, we consider the case that the customers do not have minimum rate requirements, and therefore owing to the separability described above focus on the optimization for only one ω. The advantage of this distributed computation is that each provider i needs to know only its benefit and cost functions $U_i(\cdot), V_i(\cdot)$ (and not those of the others), the link rates r_{jk} only when either j is its customer or k is its service unit. The need for limited access to global information ensures confidentiality of operations.

Based on message exchanges with other providers, each provider iteratively updates (i) the downlink allocations $\alpha_{jk}^{(n)}$ from its service-units to all customers, (ii) the rates of its customers $y_j^{(n)}$ and (iii) the total time allocation for its service units $z_k^{(n)}$ and the iterations provably converge to the optimum (the superscript n indicates the iteration index). At the end of each iteration, each provider i communicates (i) the $\{\alpha_{jk}^{(n)}\}$ iterates for all its service units (i.e., $k \in \mathcal{B}_i$), and (ii) indicators indicating the status of the satisfaction of the constraints (1), (3) for its customers (i.e., $j \in \mathcal{M}_i$), to the providers whose service units can serve its customers (i.e., those with positive r_{jk} to its customers). These indicators are used by other providers in the updates for the next iterations.

We describe the indicators and the update process next. Let

[9]This separability significantly speeds up the computations as the computation times for the optimizations are super-linear in the number of variables and constraints.

[10]In each run of the Monte Carlo simulation, we generated a network state ω, using the distribution on the service unit-customer rates, and solved the optimizations $P(\mathcal{S})$ for the coalitions $\mathcal{S}$ for the given ω. Subsequently, we computed the average of the payoffs of the providers over a large number of runs, and observed that the averages converged quite fast (Note that using ergodicity it can be analytically shown that as the number of runs tend to infinity, the averages converge to the optimum solution); we plotted the above empirical averages in Section IX.

(i) $a_{1j}^{(l)}$ be 1 if for customer j at the end of the lth iteration the L.H.S exceeds the R.H.S of constraint (1), -1 if R.H.S exceeds the L.H.S, and 0 otherwise.
(ii) $a_{2k}^{(l)}$ be defined similarly for constraint (2) (for service unit k).
(iii) $a_{3j}^{(l)}$ be 1 if for customer j the L.H.S exceeds the R.H.S of constraint (3) and 0 otherwise.
(iv) $a_{4k}^{(l)}$ be defined similarly for constraint (4) (for service unit k).

We now describe the update for each provider i, using constants $\delta^{(l)}, K$ that would be described later. In the $l+1$th iteration, provider i

1) for each of its customers j, (additively) increments $y_j^{(l)}$ by $\delta^{(l)}\left(\frac{\partial}{\partial y_j}U_i(\mathbf{y}_i) - Ka_{1j}^{(l)}\right)$,
2) for each of its service units k, decrements $z_k^{(l)}$ by $\delta^{(l)}\left(\frac{\partial}{\partial z_k}V_i(\mathbf{z}_i) + Ka_{2k}^{(l)}\right)$, and
3) for each customer j (not necessarily its customer though) and its service unit k such that $r_{jk} > 0$, increments $\alpha_{jk}^{(l)}$ by $\delta^{(l)}K\left(r_{jk}a_{1j}^{(l)} + a_{2k}^{(l)} - a_{3j}^{(l)} - a_{4k}^{(l)}\right)$ (note that the increments and decrements may be negative).

Now, we turn to the convergence analysis of the aforementioned update process. Recall that the optimization problems $\mathrm{P}(\mathcal{S})$ are assumed to be feasible ($\mathrm{P}(\mathcal{S})$ are trivially feasible in the absence of minimum rate requirements). We make a few additional assumptions in the following.

(i) There exists a $\Delta < \infty$ such that $\frac{\partial}{\partial y_j}U_i(\mathbf{y}_i) \leq \Delta$, $\frac{\partial}{\partial z_k}V_i(\mathbf{z}_i) \leq \Delta$ for all $j, k, \mathbf{y}_i, \mathbf{z}_i$ and i.
(ii) The step sizes $\{\delta^{(l)}\}$ satisfy $\lim_{l\to\infty}\delta^{(l)} = 0$ and $\sum_l \delta^{(l)} = \infty$. For example, the sequence $\{\delta^{(l)} = 1/l\}$ satisfies these assumptions.

The following analysis is similar to [30, Theorem 1] and [31, Theorem 5]. Let

1) $A_{1j}(\alpha, \mathbf{y}_i) = y_j - \sum_{k\in\mathcal{B}_\mathcal{S}} \alpha_{jk}r_{jk}, \quad j \in \mathcal{M}_i$,
2) $A_{2k}(\alpha, \mathbf{z}_i) = z_k - \sum_{j\in\mathcal{M}_\mathcal{S}} \alpha_{jk}, \quad k \in \mathcal{B}_i$,
3) $A_{3j}(\alpha) = \sum_{k\in\mathcal{B}_\mathcal{S}} \alpha_{jk} - 1, \quad j \in \mathcal{M}_\mathcal{S}$,
4) $A_{4k}(\alpha) = \sum_{j\in\mathcal{M}_\mathcal{S}} \alpha_{jk} - 1, \quad k \in \mathcal{B}_\mathcal{S}$.

Also define $\mathbf{Q} = (\alpha, \mathbf{y}, \mathbf{z})$,
$E_i(\mathbf{Q}) = \sum_{j\in\mathcal{M}_i}\Big(|A_{1j}(\mathbf{y}_i, \alpha)| + \max\{0, A_{3j}(\alpha)\}\Big) + \sum_{k\in\mathcal{B}_i}\Big(|A_{2k}(\mathbf{z}_i, \alpha)| + \max\{0, A_{4k}(\alpha)\}\Big)$,
and $W(\mathbf{Q}) = \sum_{i\in\mathcal{S}}\Big(U_i(\mathbf{y}_i) - V_i(\mathbf{z}_i) - KE_i(\mathbf{Q})\Big)$.
Let $\mathbf{Q}^* \equiv (\alpha^*, \mathbf{y}^*, \mathbf{z}^*)$ be an optimal solution of $\mathrm{P}(\mathcal{S})$. Now consider the following optimization problem.
$\tilde{\mathrm{P}}(\mathcal{S}) : \max \ W(\mathbf{Q})$
subject to: $\alpha, \mathbf{y}, \mathbf{z} \geq 0$
Let $\tilde{\mathbf{Q}}^* \equiv (\tilde{\alpha}^*, \tilde{\mathbf{y}}^*, \tilde{\mathbf{z}}^*)$ be an optimal solution and $\tilde{W}^*$ be the optimal values of $\tilde{\mathrm{P}}(\mathcal{S})$. The proof consists of two steps.

Step (i): In the first step we prove that for sufficiently large K, $\tilde{\mathbf{Q}}^*$ is also an optimal solution of $\mathrm{P}(\mathcal{S})$. This result is fairly intuitive. See [30] for a discussion.

Theorem IV.2. *If $K > \Delta$, then $\tilde{\mathbf{Q}}^*$ is also an optimal solution of P($\mathcal{S}$).*

Proof: The subgradient of $W(\cdot)$ at $\mathbf{Q}$ can be written as $\mathbf{s}(\mathbf{Q}) = \Big(K\big(r_{jk}a_{1j}^{(l)} + a_{2k}^{(l)} - a_{3j}^{(l)} - a_{4k}^{(l)}\big), \big(\frac{\partial}{\partial y_j}U_i(\mathbf{y}_i) - Ka_{1j}^{(l)}\big), -\big(\frac{\partial}{\partial z_k}V_i(\mathbf{z}_i) + Ka_{2k}^{(l)}\big), j \in \mathcal{M}_i, k \in \mathcal{B}_i, i \in \mathcal{S}\Big)$
Consider $\mathbf{Q}$ such that $E_i(\mathbf{Q}) > 0$ for some i. For such $\mathbf{Q}$, there always exists a component of the subgradient, $\mathbf{s}(\mathbf{Q})$, that has absolute value greater than or equal to $K - \Delta$. Therefore $\mathbf{0}$ does not belong to the set of subgradients. Hence, $\mathbf{Q}$ can not be an optimal solution of $\tilde{\mathrm{P}}(\mathcal{S})$. Thus, an optimal solution $\tilde{\mathbf{Q}}^*$ of $\tilde{\mathrm{P}}(\mathcal{S})$ satisfies $E_i(\tilde{\mathbf{Q}}^*) = 0$ for all i. Recall that $E_i(\mathbf{Q}^*) = 0$ for all i. Hence,

$$\begin{aligned}
&\sum_{i\in\mathcal{S}}\Big(U_i(\tilde{\mathbf{y}}_i^*) - V_i(\tilde{\mathbf{z}}_i^*)\Big) - \sum_{i\in\mathcal{S}}\Big(U_i(\mathbf{y}_i^*) - V_i(\mathbf{z}_i^*)\Big) \\
&= W(\tilde{\mathbf{Q}}^*) - W(\mathbf{Q}^*) + K\sum_{i\in\mathcal{S}}\Big(E_i(\tilde{\mathbf{Q}}^*) - E_i(\mathbf{Q}^*)\Big) \\
&= W(\tilde{\mathbf{Q}}^*) - W(\mathbf{Q}^*) \geq 0.
\end{aligned}$$

Thus, $\tilde{\mathbf{Q}}^*$ also is an optimal solution of $\mathrm{P}(\mathcal{S})$. ■

Step (ii): In the second step we show that the update process converges to an optimal solution of $\tilde{\mathrm{P}}(\mathcal{S})$.

Theorem IV.3. *The sequence of updates, $\{\mathbf{Q}^{(l)} \equiv (\alpha^{(l)}, \mathbf{y}^{(l)}), \mathbf{z}^{(l)})\}, l \geq 1$, converges to an optimal solution of $\tilde{\mathrm{P}}(\mathcal{S})$.*

Proof: Choose an arbitrary $e > 0$. Let $e' = e/2$. For any $\epsilon' > 0$ define $C_{\epsilon'}$ as $C_{\epsilon'} = \{\mathbf{Q} : L(\mathbf{Q}) \geq \tilde{W}^* - \epsilon'\}$. From [32, Theorem 27.2] it follows that there exists an $\epsilon = \epsilon(e') > 0$ such that

$$C_\epsilon \subset \{\mathbf{Q} : ||\mathbf{Q} - \tilde{\mathbf{Q}}^*|| \leq e'\}.$$

Where $\tilde{\mathbf{Q}}^*$ is an optimal solution of $\tilde{\mathrm{P}}(\mathcal{S})$. Consider l for which $\mathbf{Q}^{(l)} \notin C_\epsilon$. Therefore, $W(\mathbf{Q}^{(\mathbf{l})}) < \tilde{W}^* - \epsilon$. The update equations at the providers can be compactly stated as $\mathbf{Q}^{(l+1)} = [\mathbf{Q}^{(l)} + \delta^{(l)}\mathbf{s}^{(l)}]_+$, where $\mathbf{s}^{(l)}$ is the subgradient of $W(\cdot)$ at $\mathbf{Q}^{(l)}$. It follows from the definition of subgradients that

$$(\mathbf{s}^{(l)}, \mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*) \leq L(\mathbf{Q}^{(\mathbf{l})}) - \tilde{W}^* < -\epsilon.$$

Now, $||\mathbf{s}^{(l)}|| \leq T$, where

$$T = \sqrt{\big(|\mathcal{M}_\mathcal{S}| + |\mathcal{B}_\mathcal{S}|\big)(\Delta + K)^2 + |\mathcal{M}_\mathcal{S}||\mathcal{B}_\mathcal{S}|K^2(3+R)^2};$$

R is the maximum achievable rate for any customer-service unit pair.

$$\begin{aligned}
&||\mathbf{Q}^{(l+1)} - \tilde{\mathbf{Q}}^*||^2 \\
&= ||[\mathbf{Q}^{(l)} + \delta^{(l)}\mathbf{s}^{(l)}]_+ - \tilde{\mathbf{Q}}^*||^2 \\
&\leq ||\mathbf{Q}^{(l)} + \delta^{(l)}\mathbf{s}^{(l)} - \tilde{\mathbf{Q}}^*||^2 \\
&= ||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*||^2 + {\delta^{(l)}}^2||\mathbf{s}^{(l)}||^2 + 2\delta^{(l)}(\mathbf{s}^{(l)}, \mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*) \\
&< ||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*||^2 + T^2{\delta^{(l)}}^2 - 2\epsilon\delta^{(l)}.
\end{aligned}$$

Since $\delta^{(l)} \to 0$, $\delta^{(l)} \leq \epsilon/T^2$ when l is sufficiently large. For all such l,

$$||\mathbf{Q}^{(l+1)} - \tilde{\mathbf{Q}}^*||^2 < ||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*||^2 - \epsilon\delta^{(l)}.$$

Suppose there exists a $L'_\epsilon < \infty$ such that $\mathbf{Q}^{(l)} \notin C_\epsilon$ for all $l \geq L'_\epsilon$. Therefore, there exists $L_\epsilon \geq L'_\epsilon$ such that the above inequality holds for all $l \geq L_\epsilon$. Adding the inequalities corresponding to $l = L_\epsilon$ to $L_\epsilon + m$, we obtain

$$||\mathbf{Q}_{L_\epsilon+m+1} - \tilde{\mathbf{Q}}^*||^2 < ||\mathbf{Q}_{L_\epsilon} - \tilde{\mathbf{Q}}^*||^2 - \epsilon \sum_{l=L_\epsilon}^{L_\epsilon+m} \delta^{(l)},$$

which implies that $||\mathbf{Q}_{L_\epsilon+m+1} - \tilde{\mathbf{Q}}^*|| \to -\infty$ as $m \to \infty$ since $\sum_1^\infty \delta^{(l)} = \infty$. This is not possible since $||\mathbf{Q}_{L_\epsilon+m+1} - \tilde{\mathbf{Q}}^*|| \geq 0$. Hence the supposition was incorrect. Hence there exists a sequence $l_{1,\epsilon} < l_{2,\epsilon} < \ldots$ such that $\mathbf{Q}_{l_{i,\epsilon}} \in C_\epsilon$ for all $i = 1, 2, \ldots$. Since $\delta^{(l)} \to 0$, there exists i s.t. $\delta^{(l)} \leq \min(e'/T, \epsilon/T^2), \ \forall \ l \geq l_{i,\epsilon}$. Consider the following cases.

Case 1: $l = l_{j,\epsilon}$ for some $j \geq i$. Hence $\mathbf{Q}^{(l)} \in C_\epsilon$ and $||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*|| \leq e' < e$.

Case 2: : $l = l_{j,\epsilon} + 1$ for some $j \geq i$. Then

$$\mathbf{Q}^{(l)} = \mathbf{Q}^{(l_{j,\epsilon}+1)} = [\mathbf{Q}^{(l_{j,\epsilon})} + \delta^{(l_{j,\epsilon})}\mathbf{s}^{(l_{j,\epsilon})}]_+.$$

Thus

$$\begin{aligned} ||\mathbf{Q}^{(l)} - \mathbf{Q}^{(l_{j,\epsilon})}|| &= ||[\mathbf{Q}^{(l_{j,\epsilon})} + \delta^{(l_{j,\epsilon})}\mathbf{s}^{(l_{j,\epsilon})}]_+ - \mathbf{Q}^{(l_{j,\epsilon})}|| \\ &\leq ||\mathbf{Q}^{(l_{j,\epsilon})} + \delta^{(l_{j,\epsilon})}\mathbf{s}^{(l_{j,\epsilon})} - \mathbf{Q}^{(l_{j,\epsilon})}|| \\ &= \delta^{(l_{j,\epsilon})}||\mathbf{s}^{(l_{j,\epsilon})}|| \leq T\delta^{(l_{j,\epsilon})} \leq e'. \end{aligned}$$

From the above, and since $||\mathbf{Q}^{l_{j,\epsilon}} - \tilde{\mathbf{Q}}^*|| \leq e'$ (Case 1), we get

$$\begin{aligned} ||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*|| &\leq ||\mathbf{Q}^{(l_{j,\epsilon})} - \tilde{\mathbf{Q}}^*|| + ||\mathbf{Q}^{(l)} - \mathbf{Q}^{(l_{j,\epsilon})}|| \\ &\leq e' + e' = 2e' = e. \end{aligned}$$

Case 3: : $l_{j,\epsilon}+1 < l < l_{j+1,\epsilon}$ for some $j \geq i$. Also $\mathbf{Q}_{l'} \notin C_\epsilon \forall \ l_{j,\epsilon} < l' < l_{j+1,\epsilon}$. Recall that $||\mathbf{Q}^{(l'+1)} - \tilde{\mathbf{Q}}^*||^2 < ||\mathbf{Q}^{(l')} - \tilde{\mathbf{Q}}^*||^2 - \epsilon\delta^{(l')}$, implying $||\mathbf{Q}^{(l'+1)} - \tilde{\mathbf{Q}}^*|| < ||\mathbf{Q}^{(l')} - \tilde{\mathbf{Q}}^*||$. Thus, $||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*|| < ||\mathbf{Q}^{(l_{j,\epsilon}+1)} - \tilde{\mathbf{Q}}^*||$. Since $||\mathbf{Q}^{(l_{j,\epsilon}+1)} - \tilde{\mathbf{Q}}^*|| \leq e$ (Case 2), $||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*|| < e$.

From Cases 1,2 and 3, it follows that $||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*|| \leq e \forall \ l \geq l_{i,\epsilon}$. Since e is arbitrary, $\lim_{l\to\infty} ||\mathbf{Q}^{(l)} - \tilde{\mathbf{Q}}^*|| = 0$. ∎

Here is the main result.

Theorem IV.4. *The sequence of allocations,* $\{\alpha_{jk}^{(l)}, j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}\}, l \geq 1$, *converges to an optimum allocation.*

Proof: Combining Theorems IV.2 and IV.3, we obtain that the sequence of updates, $\{\mathbf{Q}^{(l)}\}, l \geq 1$, converges to an optimal solution, $\mathbf{Q}^* \equiv (\alpha^*, \mathbf{y}^*, \mathbf{z}^*)$, of P($\mathcal{S}$). Since $\mathbf{Q}^{(l)} \equiv (\alpha^{(l)}, \mathbf{y}^{(l)}, \mathbf{z}^{(l)})$, we have $\lim_{l\to\infty} ||\alpha^{(l)} - \alpha^*|| = 0$. ∎

Now we discuss how this framework can provide useful insights about the relation between a provider's payoff share, the resources it contributes, and the wealth it generates. Among the demands and assets in possession of a provider, one could be more constrained than the others. For instance, a provider might have a lot of customers, but few service units. Then, increasing the number of service units could boost the payoff generated by the provider, while adding to the number of customers might not change it. Using the rule of thumb that more demand adds to the value of an asset, an intuitive observation then is that in a coalition, the provider that offers more of the demand or asset that is sought most by the majority of the members of the coalition, is likely to receive a larger share of the aggregate payoff. The following example will further elucidate this.

Example IV.3. *Let* $\mathcal{N} = \{1, 2, 3\}$, $|\mathcal{M}_1| = 5$, *and* $|\mathcal{M}_2| = |\mathcal{M}_3| = 2$. *Also, let* $|\mathcal{B}_1| = 2$, $|\mathcal{B}_2| = 3$, *and* $|\mathcal{B}_3| = 4$. *Suppose* $r_{jk} = P$ *for all* $j \in \mathcal{M}_\mathcal{N}$ *and* $k \in \mathcal{B}_\mathcal{N}$. *Let* $m_j = 0$ *for all* $j \in \mathcal{M}_\mathcal{N}$. *Also, let the payoffs be equal to sum of the customers' service rates. Then,* $v(\{i\}) = 2P$ *for* $i \in \mathcal{N}$, $v(\{1, 2\}) = 5P$, $v(\{1, 3\}) = 6P$, $v(\{2, 3\}) = 4P$, *and* $v(\{1, 2, 3\}) = 9P$. *An example allocation in the core* $(\frac{7P}{2}, \frac{5P}{2}, 3P)$ *fetches payoff gains of* $\frac{3P}{2}$, $\frac{P}{2}$ *and* P *to the three providers as compared to the case when they do not cooperate. Also, somewhat contrary to intuition, provider* 1, *who has the least number of service units, attains the highest payoff. This is because the other providers, i.e.,* $2, 3$ *have fewer customers than service units, and these excess service units are utilized only when* 1 *joins the coalition along with its customers. Thus,* 1 *is adding the most value to the coalition by bringing in the demand that is sought out by others: note that* $v(\{2, 3\}) = v(\{2\}) + v(\{3\})$ *but* $v(\{1, 2, 3\}) > v(\{i\}) + v(\{2\}) + v(\{3\})$. *Also, the providers' shares of the aggregate payoff are usually largely determined by parameters other than their decision variables. For instance, the number of customers here is not a decision variable and yet it is critical in determining the payoff shares.*

Remark IV.1. *A provider can decide how to upgrade its resources, based on the above observation. For instance, in Example IV.3, if provider* 2 *can somehow expand its customer base, e.g., by extensive advertising, its share increases, although the aggregate payoff remains the same.*

V. Spectrum Acquisition And Service Unit Location Games

In the previous section, we investigated cooperation, assuming that the locations of service units and the set of channels they have access to are decided a priori. We showed that the core is nonempty and an imputation in the core can be obtained through solving a convex optimization problem. We now relax these assumptions. In particular, we examine the cooperation in a setup where providers can decide which channels to rent and where to open base stations. We generalize the model presented in Section III-A so as to incorporate these decision variables. Redefine $\mathcal{B}_i$ to be the set of candidate locations available to provider i for opening base stations. Let f_k be the cost of opening base station k. Let $b_k = 1$ if base station k is open and 0 otherwise. Also, i should determine which channels base station $k \in \mathcal{B}_i$ will have access to. Define C_k to be the set of channels available at base station k; $C_k, k \in \mathcal{B}_\mathcal{N}$ are assumed to be disjoint. Note that provider i needs to pay the spectrum regulator (a government agency or a license holder) a fixed fee (membership charge), g_l, if it intends to use a channel l in $\mathcal{C}_k$, $k \in \mathcal{B}_i$; this fee is in addition to any usage based charge the provider needs to pay for using the channel (the $V(\cdot)$ functions in the previous sections) which depends only on the amount of usage and is 0 if the channel is not used. Let $c_l = 1$ if base station k is allowed to use channel $l \in \mathcal{B}_k$ (we say that the channel is open) and 0 otherwise.

A channel $l \in C_k$, for some $k \in B_i$, can serve user j if a) Base station k is open, b) Channel l is open, and c) customer j and base station k are associated with same provider or the providers associated with them are in a coalition. We assume $m_j = 0$ for all $j \in \mathcal{M}_{\mathcal{N}}$, that is, there are no service level agreements. Note that, b_ks and c_ls are deterministic variables and cannot depend on ω, in contrast to α_{jl}s that are decided to best suit each network realization $\omega \in \Omega$.

We assume that all utility and cost functions are linear, and $u_{jl}(\omega) \in \mathbb{R}$ is the difference in the amount paid by customer j for using channel l and the usage-based charges incurred by channel l in serving customer j.

For a coalition $\mathcal{S} \subseteq \mathcal{N}$, the payoff $v(\mathcal{S})$ is then obtained by solving the following optimization problem.

$\mathrm{P}_{\mathrm{G}}(\mathcal{S})$: $\max \sum_{\substack{j \in \mathcal{M}_{\mathcal{S}} \\ l \in \mathcal{C}_{\mathcal{S}} \\ \omega \in \Omega}} \mathbb{P}(\omega) \alpha_{jl}(\omega) u_{jl}(\omega) - \sum_{k \in \mathcal{B}_{\mathcal{S}}} f_k b_k - \sum_{l \in \mathcal{C}_{\mathcal{S}}} g_l c_l$

subject to:

1) $\sum_{l \in \mathcal{C}_{\mathcal{S}}} \alpha_{jl}(\omega) \leq 1, \quad j \in \mathcal{M}_{\mathcal{S}}, \omega \in \Omega$
2) $\sum_{j \in \mathcal{M}_{\mathcal{S}}} \alpha_{jl}(\omega) \leq c_l, \quad l \in \mathcal{C}_{\mathcal{S}}, \omega \in \Omega$
3) $c_l \leq b_k, l \in \mathcal{C}_k, k \in \mathcal{B}_{\mathcal{S}}$
4) $\alpha_{jl}(\omega) \geq 0, \quad j \in \mathcal{M}_{\mathcal{S}}, l \in \mathcal{C}_{\mathcal{S}}, \omega \in \Omega$
5) $c_l, b_k \in \{0,1\}, \quad l \in \mathcal{C}_{\mathcal{S}}, k \in \mathcal{B}_{\mathcal{S}}$

Constraints (1) ensure that the total fraction of time customer j is being served, is upper bounded by 1. A channel l can serve at most the whole fraction of time if it is open and can not serve otherwise, by constraints (2). Finally, constraints (3) guarantee that only opened base stations can have open channels.

The following example illustrates how cooperation may change providers' decisions regarding the opening of channels.

Example V.1. *Consider a network with $\mathcal{N} = \{1,2\}$, $\mathcal{B}_1 = \mathcal{M}_1 = \{1\}$ and $\mathcal{B}_2 = \mathcal{M}_1 = \{2,3\}$, where $f_1 = 0$, and $f_2 = f_3 = f$. Let $r_{11} = r_{12} = r_{32} = Q$, $r_{21} = r_{22} = r_{33} = P$, and $r_{jk} = 0$ otherwise. Suppose and $m_j = 0$ for all $j \in \mathcal{M}_{\mathcal{N}}$, $f < P$, and $Q < P$. Let payoffs consist only of the sum of the customers' service rates. Now $v(\{1\}) = Q$. Also $v(\{2\}) = 2P - 2f$ and $v(\{1,2\}) = \max[2P - f, 2P + Q - 2f]$, where the former payoffs are the result of opening just channel 3, while the latter ones are in the event of opening both. Intuitively, if provider 2 cooperates with 1, opening channel 2 may not be necessary. In fact if $Q < f < P$, opening both channels is optimal when not in coalition, while opening just channel 3 is optimal under cooperation. This is in agreement with the intuition that deploying a service unit in the area that is covered by other service units might be redundant. However, if there is a relatively large traffic demand in the area, e.g., if $r_{21} = P$ (then $v(\{1,2\}) = \max[2P - f, 3P - 2f]$), opening both channels is optimal even when the providers cooperate.*

Note that the aggregate payoff of a coalition now is given by an integer (rather than convex) optimization problem. As a result, the strong duality does not hold in general. Thus the approach taken in Section IV to show that the core is nonempty, is inadequate here. However, we obtain the nonemptiness of the core in some special cases using different proof techniques.

A. Case 1

Consider the special case where customers do not move and the quality of channels do not vary with time, i.e. $|\Omega| = 1$. We also assume that each base-station is allowed to use only one channel, i.e., $|\mathcal{C}_k| = 1$ for all $k \in \mathcal{B}_{\mathcal{N}}$. Thus each base station corresponds to only one service unit. In particular, $\mathcal{B}_i$, as in Sections III,IV, is the set of service units available to provider i. Moreover, for each service unit $k \in \mathcal{B}_i$, provider i needs to pay the cost of opening the base station as well as the corresponding channel's membership cost. We redefine b_k to represent this total fixed cost whereas $b_k = 1$ if the service unit k is open and 0 otherwise. Then $\mathrm{P}_{\mathrm{G}}(\mathcal{S})$ will reduce to the following IP:

$\mathrm{P}_{\mathrm{c}}(\mathcal{S})$: $\max \sum_{\substack{j \in \mathcal{M}_{\mathcal{S}} \\ k \in \mathcal{B}_{\mathcal{S}}}} \alpha_{jk} u_{jk} - \sum_{k \in \mathcal{B}_{\mathcal{S}}} f_k b_k$

subject to:

1) $\sum_{k \in \mathcal{B}_{\mathcal{S}}} \alpha_{jk} \leq 1, \quad j \in \mathcal{M}_{\mathcal{S}}$
2) $\sum_{j \in \mathcal{M}_{\mathcal{S}}} \alpha_{jk} \leq b_k, \quad k \in \mathcal{B}_{\mathcal{S}}$
3) $\alpha_{jk} \geq 0, \quad j \in \mathcal{M}_{\mathcal{S}}, k \in \mathcal{B}_{\mathcal{S}}$
4) $b_k \in \{0,1\}, \quad k \in \mathcal{B}_{\mathcal{S}}$

We proceed to prove that the core of the coalitional game $< \mathcal{N}, v >$, with characteristic function $v(\cdot)$ given by $\mathrm{P}_{\mathrm{c}}(\mathcal{S})$, is nonempty. The proof consists of two steps.

Step (i): Consider the coalitional game $< \mathcal{N}, \hat{v} >$, where $\mathcal{N}$ is the same set of providers and the characteristic function $\hat{v}(\cdot)$ is given by the LP, $\mathrm{P}_{\mathrm{relaxed}}(\mathcal{S})$. $\mathrm{P}_{\mathrm{relaxed}}(\mathcal{S})$ is the linear relaxation of $\mathrm{P}_{\mathrm{c}}(S)$, where the constraints $b_k \in \{0,1\}$ are now replaced by $b_k \in [0,1]$. We show that the core of the coalitional game $< \mathcal{N}, \hat{v} >$, $\hat{\mathcal{C}}$, is nonempty.

Using $\lambda \in \mathbb{R}^{\mathcal{M}_{\mathcal{S}}}$, and $\nu, \gamma \in \mathbb{R}^{\mathcal{B}_{\mathcal{S}}}$, we construct the following LP as the dual of $\mathrm{P}_{\mathrm{relaxed}}(\mathcal{S})$

$\mathrm{D}_{\mathrm{relaxed}}(\mathcal{S})$: $\min \sum_{j \in \mathcal{M}_{\mathcal{S}}} \lambda_j + \sum_{k \in \mathcal{B}_{\mathcal{S}}} \gamma_k$

subject to:

1) $\lambda_j + \nu_k \geq u_{jk}, \quad j \in \mathcal{M}_{\mathcal{S}}, k \in \mathcal{B}_{\mathcal{S}}$
2) $\nu_k - \gamma_k \leq f_k, \quad k \in \mathcal{B}_{\mathcal{S}}$
3) $\lambda_j, \nu_k, \gamma_k \geq 0, \quad j \in \mathcal{M}_{\mathcal{S}}, k \in \mathcal{B}_{\mathcal{S}}$

Let $\mathcal{D}_{\mathrm{relaxed}}$ constitute the set of optimal solutions of $\mathrm{D}_{\mathrm{relaxed}}(\mathcal{N})$. Define: $\mathcal{I}_{\mathrm{c}} := \{\mathbf{x}^* \in \mathbb{R}^{\mathcal{N}} : x_i^* = \sum_{j \in \mathcal{M}_i} \lambda_j + \sum_{k \in \mathcal{B}_i} \gamma_k \quad \text{for some} \quad (\lambda^*, \nu^*, \beta^*, \gamma^*) \in \mathcal{D}_{\mathrm{relaxed}}\}$.

Theorem V.1. *$\mathcal{I}_c \neq \emptyset$, and $\mathcal{I}_c \subseteq \hat{\mathcal{C}}$*

Proof: The proof is identical to that of Theorem IV.1. ■

Step (ii): Next, we prove that, for any coalition $\mathcal{S} \subseteq \mathcal{N}$, $\mathrm{P}_{\mathrm{c}}(\mathcal{N})$ has zero integrality gap. In other words, $\mathrm{P}_{\mathrm{relaxed}}(\mathcal{S})$ has an integral optimum solution. In the proof we use the fact that, if the set of constraints of $\mathrm{P}_{\mathrm{relaxed}}(\mathcal{S})$ is written in matrix form, the corresponding matrix is totally unimodular.

Definition V.1. *A matrix A is totally unimodular if every square submatrix of A has determinant either* 0, 1 *or* -1.

We have the following sufficient conditions for the matrix A to be totally unimodular [33].

Theorem V.2. *Suppose A can be partitioned into two disjoint sets B and C, with the following properties:*

1) *Every column of A contains at most two non-zero entries;*
2) *Every entry in A is 0, $+1$, or -1;*
3) *If two non-zero entries in a column of A have the same sign, then the row of one is in B, and the other in C;*
4) *If two non-zero entries in a column of A have opposite signs, then the rows of both are in B or both in C.*

Then A is totally unimodular.

Now, consider the following linear program
P: $\max c^T x$
subject to: $Ax \leq b, \quad x \geq 0$
We have the following theorem [34].

Theorem V.3. *If $(1) A$ is totally unimodular, and $(2) b$ contains only integers, then linear program P has an optimal integral solution.*

Using above, we get the following result.

Theorem V.4. *For any coalition $\mathcal{S} \subseteq \mathcal{N}$, the integer program $\mathrm{P_c}(\mathcal{S})$ has zero integrality gap. In other words, $v(\mathcal{S}) = \hat{v}(\mathcal{S})$ for all $\mathcal{S} \subseteq \mathcal{N}$.*

Proof: Once we write $\mathrm{P_c}(\mathcal{S})$ in the form of P, it is trivial to verify that A, thus obtained, satisfies the sufficiency conditions of Theorem V.2. Hence A is totally unimodular. Also, b contains only 0 and 1. Thus, from Theorem V.3, $\mathrm{P_c}(\mathcal{S})$ will have an integral optimum solution. ■

Here is the main result.

Theorem V.5. *$\mathcal{I}_c \neq \emptyset$, and $\mathcal{I}_c \subseteq \mathcal{C}$*

Proof: Theorem V.4 implies that $\mathcal{C} = \hat{\mathcal{C}}$. Combining this with Theorem V.1, the claim immediately follows. ■

Remark V.1. *It follows directly from this theorem that an imputation in the core can be obtained by solving the linear optimization $D_{relaxed}(\mathcal{N})$, which can be done in polynomial time. This imputation again distributes the aggregate grand-coalition payoff among providers in accordance with the Lagrange-multipliers of $P_{relaxed}(\mathcal{N})$, which as explained in Section IV, are commensurate with the resource investments and wealth generated by the providers. Also, since both the primal $P_{relaxed}(\mathcal{N})$ and dual $P_{relaxed}(\mathcal{N})$ are linear programs, they can be solved by the providers in a distributed manner and without revealing their confidential information such as the revenue and costs, (i.e., u_{jk}s) to each other, using the subgradient technique as described in Section IV.*

Computation complexity: Now, we discuss the complexity of computing the optimal deployment and allocation of service units for the grand coalition via solving the primal linear program $\mathrm{P_{relaxed}}(\mathcal{N})$, and an element of the core via solving the dual program $\mathrm{D_{relaxed}}(\mathcal{N})$. The primal linear program has $V = (|\mathcal{M}_{\mathcal{N}}| + 1)|\mathcal{B}_{\mathcal{N}}|$ variables and $C = |\mathcal{M}_{\mathcal{N}}| + (|\mathcal{M}_{\mathcal{N}}| + 3)|\mathcal{B}_{\mathcal{N}}|$ constraints. The dual linear program $\mathrm{D_{relaxed}}(\mathcal{N})$ has $V = |\mathcal{M}_{\mathcal{N}}| + 2|\mathcal{B}_{\mathcal{N}}|$ variables and $C = |\mathcal{M}_{\mathcal{N}}| + (|\mathcal{M}_{\mathcal{N}}| + 3)|\mathcal{B}_{\mathcal{N}}|$ constraints[11]. Thus each can be solved using Karmarkar's interior point algorithm [35] in $O(C^{\frac{3}{2}} V^2 L)$ time where the obtained solution and the optimal solution match in L most significant digits [12].

[11]Note that we have fewer dual variables as compared to primal constraints as the dual variables corresponding to some primal non-negativity constraints can be omitted without any imprecision.

B. Case 2

Now, we relax the simplifying assumptions made in Section V-A. We allow the customers' locations and channels' qualities to be random, i.e., $|\Omega| > 1$. We also let each base-station access multiple channels, i.e., $|\mathcal{C}_k| \geq 1$ for all $k \in \mathcal{B}_{\mathcal{N}}$. However, we impose an upper bound on the scheduling random variables. More precisely we assume that $\alpha_{jl}(\omega) \in [0, \alpha], j \in \mathcal{M}_{\mathcal{N}}, l \in \mathcal{C}_{\mathcal{N}}, \omega \in \Omega$, for an α such that $\alpha|\mathcal{C}_{\mathcal{N}}| \leq 1$.

Remark V.2. *The assumption does not cause any loss of generality when for each ω there are several customers with identical transmission rates from the service-units (in practice this case arises when the overall number of customers is large). In such cases, the maximum aggregate payoff may be attained if the service times are equally split among the customers that have identical transmission conditions from the service units - thus, even the optimizations that do not impose this condition will choose small $\alpha_{jk}(\omega)$s.*

Let $v_{\mathrm{f}}(\mathcal{S})$ be the aggregate payoff of coalition $\mathcal{S}$ in this case. Then $v_{\mathrm{f}}(\cdot)$ is given by an optimization problem $\mathrm{P_f}(\mathcal{S})$ derived by omitting constraints (1) and replacing constraints (4) with the above stronger ones in $\mathrm{P_G}(\mathcal{S})$. In the following, we proceed to show that the core of the coalitional game $< \mathcal{N}, v_{\mathrm{f}} >$, $\mathcal{C}_{\mathrm{f}}$, is nonempty.

We use the following result [36].

Theorem V.6. *Consider the optimization problem:*
$\min f(z)$
$\text{subject to}: g(z) \leq 0$
where $z = (x, y)$, $x \in X$ is the continuous part, where X is a compact set of $\mathbb{R}^n$, and $y \in Y$ is the discrete part, where Y is a finite discrete set of K-element integer vectors. f is lower bounded and continuous and differentiable with respect to x, whereas the constraints $g = (g_1, \ldots, g_r)$ is continuous in the continuous subspace X for any given $y \in Y$. Then, the optimal value of the objective function of the extended dual problem $\max_{\lambda \geq 0} \Big(\min_{z \in X \times Y} f(z) + \lambda g^+(z) \Big)$ equals that of the primal problem, i.e., there in no duality gap for the extended dual[13].

Now formulate the extended dual problem, as introduced in [36]. Let $\tau \in \mathbb{R}^{\mathcal{C}_{\mathcal{S}} \times \Omega}$ and $\varphi \in \mathbb{R}^{\mathcal{C}_{\mathcal{S}}}$. Define
$h_k^{\mathcal{S}}(\tau, \varphi) \quad = \quad \max_{\substack{\alpha_{jl} \in [0,\alpha] \\ c_l, b_k \in \{0,1\}}} \Big(\; - \; f_k b_k \; - \; \sum_{l \in \mathcal{C}_l} g_l c_l \; +$

[12]Thus, L is the number of accuracy digits of the generated solution. Often, the computation time results are stated in units of L, e.g., $O(C^{3/2} V^2)$ per accuracy digit in the algorithm output. Note that Karmarkar's algorithm generates an ϵ-solution, that is a solution that (i) attains an objective value that is at most ϵ less than the maximum value and (ii) satisfies the feasibility constraints within an error margin of ϵ. The error margin ϵ decreases with increase in the number of iterations.

[13]$A^+ \triangleq \max[A, 0]$

$\sum_{\substack{j\in\mathcal{M}_\mathcal{S}\\ l\in\mathcal{C}_k\\ \omega\in\Omega}} \mathbb{P}(\omega)\alpha_{jl}(\omega)u_{jl}(\omega) - \sum_{\substack{l\in\mathcal{C}_k\\ \omega\in\Omega}} \tau_l(\omega)(\sum_{j\in\mathcal{M}_\mathcal{S}} \alpha_{jl}(\omega) - c_l)^+ - \sum_{l\in\mathcal{C}_k}(c_l - b_k)^+\Big)$

The extended dual $\mathrm{D_f}(\mathcal{S})$ will be as follows.

$\mathrm{D_f}(\mathcal{S}) : \min \sum_{k\in\mathcal{B}_\mathcal{S}} h_k^\mathcal{S}(\tau,\varphi)$

subject to: $\tau,\varphi \geq 0$

Formulate $\mathrm{D_f}(\mathcal{N})$ by defining vectors τ and φ appropriately. Let $\mathcal{D}_\mathrm{f}$ constitute the set of optimal solutions of $\mathrm{D_f}(\mathcal{N})$. Note that $\mathcal{D}_\mathrm{f} \neq \emptyset$. Now let $\mathcal{I}_\mathrm{f} = \{\mathbf{x}^* \in \mathbb{R}^\mathcal{N} : x_i^* = \sum_{k\in\mathcal{B}_i} h_k^\mathcal{N}(\tau^*,\varphi^*),\ \text{for some } (\tau^*,\varphi^*) \in \mathcal{D}_\mathrm{f}\}$.

Here is the main result.

Theorem V.7. *$\mathcal{I}_\mathrm{f} \neq \emptyset$, and $\mathcal{I}_\mathrm{f} \subseteq \mathcal{C}_\mathrm{f}$.*

Proof: According to Theorem V.6, $\mathrm{D_f}(\mathcal{S})$ has zero duality gap. Note that since $\mathrm{D_f} \neq \emptyset$, it is clear that $\mathcal{I}_\mathrm{f} \neq \emptyset$. Now let $\mathbf{x}^*$ be an arbitrary vector in $\mathcal{I}_\mathrm{f}$ corresponding to vectors $\tau^*, \varphi*$. We show that $\mathbf{x}^* \in \mathcal{C}_\mathrm{f}$.

$x^*(\mathcal{N}) = \sum_{i\in\mathcal{N}} x_i^*$ is the optimal value of the optimization $\mathrm{D_f}(\mathcal{N})$ and by strong duality equals $v_\mathrm{f}(\mathcal{N})$. Now we only need to show that $x^*(\mathcal{S}) \geq v_\mathrm{f}(\mathcal{S})$ for all $\mathcal{S} \subset \mathcal{N}$. We have $x^*(\mathcal{S}) = \sum_{k\in\mathcal{B}_\mathcal{S}} h_k^\mathcal{N}(\tau^*,\varphi^*)$. It is easy to check that $h_k^\mathcal{N}(\varphi) \geq h_k^\mathcal{S}(\tau,\varphi)$ for all τ and φ. Therefore we have $x^*(\mathcal{S}) = \sum_{k\in\mathcal{B}_\mathcal{S}} h_k^\mathcal{N}(\tau^*,\varphi^*) \geq \sum_{k\in\mathcal{B}_\mathcal{S}} h_k^\mathcal{S}(\tau^*,\varphi^*)$. On the other hand, since $v_\mathrm{f}(\mathcal{S})$ is equal to the optimal value of $\sum_{k\in\mathcal{B}_\mathcal{S}} h_k^\mathcal{S}(\tau,\varphi)$, it follows that $v_\mathrm{f}(\mathcal{S}) \leq \sum_{k\in\mathcal{B}_\mathcal{S}} h_k^\mathcal{S}(\tau^*,\varphi^*)$. Thus, $x^*(\mathcal{S}) \geq v_\mathrm{f}(\mathcal{S})$ and the claim follows. ■

VI. Cooperation In Multi-hop Networks

We have so far investigated cooperation in single hop networks. Next, we proceed to study cooperation among providers in multi-hop networks. Intuitively, cooperation in multi-hop networks has all the advantages of that in single hop ones, which is sharing the service units and spectrum. In addition, it has another benefit via what we call *power sharing*. That is, when the providers cooperate, they can redirect their traffic through possibly better multi-hop routes, which in turn could reduce their transmission power consumption. In this section, we generalize our model to incorporate multi-hop networks. Subsequently, we examine the coalitional game in this model and show that its core is nonempty.

Consider a network in which customers can communicate with service units via potentially multi-hop routes, that is, via other customers which act as relays. However, when a customer relays others' packets, it uses its time and energy without contributing to its own utility. In order to motivate customers, providers agree to discount their charges based on how much they relay. Nevertheless, a customer might want to have a *maximum relaying agreement* with its provider. In this type of networks, providers must decide the allocation of service units as well as the communication routes. If now a set of providers agree to cooperate by pooling their service units and customers, not only can they benefit from sharing others' service units, but they also enjoy a larger set of relay nodes. This, in turn, can increase the capacity of the network, as well as its power efficiency[14]. Therefore, cooperation in multi-hop networks has even higher potential than in single hop networks. We now present a framework that captures all these issues.

As in Section III, let $\mathcal{N}$ be the set of providers. Let $\mathcal{B}_i$ and $\mathcal{M}_i$ be the sets of provider i's service units and customers, respectively. As before, we consider uplink communications. We assume that the locations of service units and the set of channels they have access to are determined a priori. The service rate of a customer j is defined as the total rate at which j's packets are delivered to any service unit, via either a single or multi-hop routes. Let m_j be the minimum rate requirement of customer j. We assume that each service unit (likewise, each customer) has access to a single channel (for transmission). In addition, we assume that no two service units in a vicinity have access to the same channel. We also assume that a pair of customers can communicate with each other (to relay packets) without interfering with the communications of other customer-customer or customer-service unit pairs (owing to appropriate channel allocation for example). Therefore, the necessary and sufficient condition for the simultaneous transmissions to be successful is that the set of transmitter-receiver pairs form a matching. Similar transmission models have extensively been assumed in related contexts [37], [38]. We discuss how this assumption can be relaxed at the end of this section.

A sufficient condition for a schedule to be feasible is that the fraction of time each service unit or customer communicates be below θ, where θ is a constant in $(0, 1]$ and depends on the network topology. For bipartite networks, for instance, $\theta = 1$, which is also a necessary condition [22]. It has been shown that in general, $\theta = \frac{2}{3}$ is a sufficient but not a necessary condition [22]. We assume that the network operates in a way that this condition always holds. This assumption can be motivated by the fact that operating the network at full capacity raises the delay which is not desirable.

Suppose now that a customer j can transmit to a service unit or another customer k at a rate equal to r_{jk}, a random variable which is a function of the location of customer j and the state of channel k. Let Ω be the state space of the channels' states and customers' locations. We assume $|\Omega|$ is finite. Let ω be an outcome in this state space and $\mathbb{P}(\omega)$ be its probability.

A customer and a service unit, or two customers, can communicate only when both are associated with the same provider or the providers associated with them are in a coalition. Let random variable $\beta_{j_2k}^{j_1} \in [0, 1]$ be the fraction of time, customer j_2 transmits packets of customer j_1, to service unit or customer k. Without loss of generality we can assume that $\beta_{j_1k}^{j_2} = 0$ for $k = j_1$ or $k = j_2$. $\beta_{j_1k}^{j_2}$s are determined by the allocation scheme.

We now discuss the mechanism which determines the payoffs providers receive and the costs they incur by serving the customers. Let τ_j be the maximum fraction of

[14]Note that for certain customers, the increase in the power usage may not be proportional to that in their service rates, but cooperation increases the power efficiency of the network as a whole.

time customer j spend as a relay. Consider a coalition $\mathcal{S}$. When the provider associated with customer j is in $\mathcal{S}$ and the network realization is ω, j receives a service rate $y_j(\omega) = \sum_{j_1 \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}} \beta^j_{j_1 k}(\omega) r_{j_1 k}(\omega)$. Besides, j relays the traffic for t_j fraction of time, where $t_j(\omega) = \sum_{j_1, j_2 \in \mathcal{M}_\mathcal{S} \setminus j, k \in \mathcal{B}_\mathcal{S}} (\beta^{j_1}_{j_2 j}(\omega) + \beta^{j_1}_{j j_2}(\omega) + \beta^{j_1}_{jk}(\omega))$. Suppose when a customer j receives a service rate y_j and relays traffic a fraction of time equal to t_j, it pays the associated provider, an amount of $U_j(y_j, t_j)$, where $U_j(y_j, t_j)$ is a concave function increasing in y_j and decreasing in t_j. Let random variables $p_{jk}(\omega)$ represent the power usage of customer j when it transmits to service unit or customer k. Then a customer j in a coalition $\mathcal{S}$, has a total power usage of $z_j(\omega) = \sum_{j_1 \in \mathcal{M}_\mathcal{S}, k \in \mathcal{M}_\mathcal{S} \cup \mathcal{B}_\mathcal{S},} \beta^{j_1}_{jk}(\omega) p_{jk}(\omega)$. This in turn inflicts a cost equal to $V_j(z_j)$ on coalition $\mathcal{S}$, where $V_j(\cdot)$ is an increasing convex function.

The aggregate payoff available to providers in a coalition is the difference between their utilities and costs. Therefore, in order to maximize their aggregate payoff, providers in a coalition must decide the routes along which they communicate with each node, and schedule the service units to those routes based on the locations of customers, and payoff and cost functions, subject to minimum rate, maximum relaying, and allocation constraints.

Let $v(\mathcal{S})$ denote the maximum aggregate payoff achievable by a coalition $\mathcal{S}$. Then, $v(\mathcal{S})$ is the optimal value of the objective function of the following convex optimization:

$$\mathrm{P_m}(\mathcal{S}) : \max \sum_{\substack{j \in \mathcal{M}_\mathcal{S} \\ \omega \in \Omega}} \mathbb{P}(\omega) \Big(U_j(y_j(\omega), t_j(\omega)) - V_j(z_j(\omega)) \Big)$$

subject to:

1) $y_j(\omega) = \sum_{\substack{j_1 \in \mathcal{M}_\mathcal{S} \\ k \in \mathcal{B}_\mathcal{S}}} \beta^j_{j_1 k}(\omega) r_{j_1 k}(\omega), \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
2) $t_j(\omega) = \sum_{\substack{j_1, j_2 \in \mathcal{M}_\mathcal{S} \setminus j \\ k \in \mathcal{B}_\mathcal{S}}} \Big(\beta^{j_1}_{j_2 j}(\omega) + \beta^{j_1}_{j j_2}(\omega) + \beta^{j_1}_{jk}(\omega) \Big), \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
3) $z_j(\omega) = \sum_{\substack{j_1 \in \mathcal{M}_\mathcal{S} \\ k \in \mathcal{B}_\mathcal{S} \cup \mathcal{M}_\mathcal{S}}} \beta^{j_1}_{jk}(\omega) p_{jk}(\omega), \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
4) $\sum_{k \in \mathcal{M}_\mathcal{S} \cup \mathcal{B}_\mathcal{S}} \beta^{j_1}_{j_2 k}(\omega) r_{j_2 k}(\omega) = \sum_{j \in \mathcal{M}_\mathcal{S}} \beta^{j_1}_{j j_2}(\omega) r_{j j_2}(\omega), \quad j_1 \neq j_2 \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
5) $t_j(\omega) + \sum_{k \in \mathcal{B}_\mathcal{S} \cup \mathcal{M}_\mathcal{S}} \beta^j_{jk}(\omega) \leq \theta, \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega.$
6) $\sum_{j_1, j_2 \in \mathcal{M}_\mathcal{S}} \beta^{j_1}_{j_2 k}(\omega) \leq \theta, \quad k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$
7) $\sum_{\omega \in \Omega} \mathbb{P}(\omega) y_j(\omega) \geq m_j, \quad j \in \mathcal{M}_\mathcal{S}.$
8) $\sum_{\omega \in \Omega} \mathbb{P}(\omega) t_j(\omega) \leq \tau_j, \quad j \in \mathcal{M}_\mathcal{S}.$
9) $\beta^{j_1}_{j_2 k}(\omega) \geq 0, \quad j_1, j_2 \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S} \cup \mathcal{M}_\mathcal{S}, \omega \in \Omega$

Constraints (4) ensure that the set of $\beta^{j_1}_{j_2 l}$s satisfy the flow feasibility constraints, while constraints (5) and (6) guarantee that they constitute a feasible allocation. Constraints (7) and (8) impose minimum rate and maximum relaying guarantees, respectively.

We argue that the core of the coalitional game $< \mathcal{N}, v >$ is nonempty. Similar to the proof of Theorem IV.1, one can formulate the dual problem of the optimization $\mathrm{P_m}(\mathcal{N})$ (which is always feasible) and subsequently, define the set $\mathcal{I}$ appropriately. The same proof technique, then shows that $\mathcal{I}$ belongs to the core. Hence, nonemptiness of the core follows. Furthermore, solving the dual problem provides an imputation in the core, which can be obtained in polynomial time.

Now assume that simultaneous scheduling of some sets of links without common nodes is infeasible due to the interference constraints. This arises when for example same channels have been allocated to units in a vicinity. Nevertheless, it is still possible to find a set of constants $\{\theta_j, \theta_k \in (0,1], j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}\}$, such that for any scheduling to be feasible, it suffices that every node j (service unit or customer) in the network communicates for less than θ_j fraction of time. If, as before, we assume that this condition always holds in the network, replacing θ in constraints (5) and (6) in $\mathrm{P_m}(\mathcal{S})$ with the appropriate θ_j, θ_k, leads to the optimization problem that gives $v(\mathcal{S})$ for all coalitions. Analogous results extend similarly.

Finally, similar formulations may be used to model cooperation among internet service providers (ISPs) in the same tier. Specifically, peer ISPs may form coalitions where the providers in the same coalition route traffic to the customers (i.e., end users or the ISPs in lower tiers) through each others routers (analogous to service units in our terminology) and links. The characteristic function $v(S)$ now represents the total profit of the ISPs in a coalition S, and can be obtained as the objective function of a concave maximization with linear constraints, similar to $\mathrm{P_m}(\mathcal{S})$ - the differences in this optimization are that (i) there is only one ω as the link qualities will not vary randomly in wireline networks (ii) the utility functions $U_i(.)$ depend only on the rates provided to the customers (iii) cost functions $V_j(.)$ are zero as the routers belong to the ISPs (iv) constraint 6 on the fraction of time each service

e unit and relay is used must be replaced by link capacity constraints. The duality gap continues to be zero. Hence, it can be shown similar to the proof of Theorem IV.1 that the core is non-empty and an allocation in the core can be obtained in polynomial time.

VII. Other Solution Concepts: Nucleolus And Shapley Value

In Section III-B we defined the core of a coalitional game, and observed that sharing mechanism based on an imputation in the core stabilizes the grand coalition. This fact then motivated us to examine whether the cores of the games we formulated are nonempty. The core of an arbitrary coalitional game, however, consists of multiple imputations and we have so far, presented techniques for computing one of them. But, it is not clear how to select an appropriate imputation among all the available ones. One approach to this problem is to run an optimization over the set of imputations in the core with an appropriate set of constraints, so as to satisfy some additional selection criteria. Another approach is to use other well known sharing mechanisms (also known as solution concepts) in coalitional games that best suit the application. Each of these solution concepts has properties that make it an interesting candidate for a sharing mechanism. However, not all of them are guaranteed to stabilize the grand coalition, in the sense that an imputation in the core does.

We next proceed to investigate two well known solution concepts in coalitional games, nucleolus and Shapley value, and examine whether these belong to the core.

A. Nucleolus

Definition VII.1. *The excess of a coalition $\mathcal{S}$ under an imputation $\mathbf{x}$ is $e_{\mathcal{S}}(\mathbf{x}) = v(\mathcal{S}) - x(\mathcal{S})$. Let $E(\mathbf{x}) = (e_{\mathcal{S}}(\mathbf{x}), \mathcal{S} \in 2^{\mathcal{N}})$ be the vector of excesses arranged in monotonically decreasing order. The nucleolus is the set of imputations x for which the vector $E(x)$ is lexicographically minimal.*

One can think of $e_{\mathcal{S}}(\mathbf{x})$ as a measure of dissatisfaction of $\mathcal{S}$ under $\mathbf{x}$. For example, if the share allocated to coalition $\mathcal{S}$ by $\mathbf{x}$ is less than what $\mathcal{S}$ can make on their own (i.e., $v(\mathcal{S})$), then $e_{\mathcal{S}}(\mathbf{x})$ is positive and its value reflects $\mathcal{S}$'s level of dissatisfaction. If instead, $\mathcal{S}$ receives what it can earn by itself, $e_{\mathcal{S}}$ is negative, which suggests a relative satisfaction. Then the nucleolus is basically the set of imputations that maximizes the levels of coalitions' satisfactions in a max-min fair fashion. That is, it maximizes the minimum satisfaction. Subject to that, it maximizes the next minimum satisfaction, and so forth. In Example IV.1, the nucleolus is $(\frac{Q+P}{2}, \frac{3Q-P}{2})$.

The nucleolus of any coalitional game with transferable payoff is a singleton [23, pp. 288]. Whenever the core of a coalitional game is nonempty, the nucleolus belongs to the core. This is because, for any imputation $\mathbf{x}$ in the core, $E(\mathbf{x}) \leq 0$ i.e., the maximum excess is negative. Hence for any imputation $\mathbf{x}^*$, which leads to lexicographically minimal excess vector among all the imputations, the corresponding maximum excess will be negative. Hence, $E(\mathbf{x}^*) \leq 0$. Thus, from (1), $\mathbf{x}^*$ belongs to the core.

Thus, in those games where we proved that the core is nonempty, the nucleolus belongs to the core, and hence renders the grand coalition stable.

B. Shapley value

Definition VII.2. *For any i, and $\mathcal{S} \subset \mathcal{N}$ such that $i \notin \mathcal{S}$, let $\Delta_i(\mathcal{S}) = v(\mathcal{S} \cup \{i\}) - v(\mathcal{S})$. The Shapley value is the imputation $\mathbf{x}$ for which*

$$x_i = \frac{1}{n!} \sum_{U \in \mathcal{U}} \Delta_i(\mathcal{S}_i(U)), \tag{2}$$

where $\mathcal{U}$ is the set of all orderings of the set of players, and $\mathcal{S}_i(U)$ is the set of players preceding i in ordering U.

In Example IV.1, $\Delta_i(\varnothing) = v(\{i\}), \Delta_1(\{2\}) = Q, \Delta_2(\{1\}) = 2Q - P$, and the Shapley value is $\big((Q+P)/2, (3Q-P)/2\big)$.

The significance of the Shapley value is that it is the unique imputation that attains the following properties [23, pp. 292]. (a) If i and j are interchangeable, i.e., $\Delta_i(\mathcal{S}) = \Delta_j(\mathcal{S})$ for each $\mathcal{S}$ such that $i, j \notin \mathcal{S}$, then the imputation allocates equal shares to both i and j (*symmetry*). (b) If $\Delta_i(\mathcal{S}) = v(\{i\})$ for each $\mathcal{S}$ such that $i \notin \mathcal{S}$, then the imputation allocates revenue $v(\{i\})$ to i (*dummy player allocation*). (c) Consider two coalitional games with the set of players $\mathcal{N}$, and the characteristic functions $w_1(\cdot), w_2(\cdot)$, and a third coalitional game with the same set of players, and the characteristic function $w_1(\cdot) + w_2(\cdot)$. Then, the imputation that constitutes the Shapley value for the third coalitional game equals the sum of those for the first two (*additivity*).

We next provide an example to demonstrate that the Shapley value need not be in $\mathcal{C}$.

Example VII.1. *Let $\mathcal{N} = \{1, 2, 3\}$. Let $\mathcal{B}_i = \{i\}$ and $\mathcal{M}_i$ be arbitrary nonempty disjoint sets for each provider i. Let $r_{j2} = 1, j \in \mathcal{M}_\infty \cup \mathcal{M}_\ni, r_{j1} = r_{j3} = 1, j \in \mathcal{M}_\in$ and $r_{jk} = 0$ otherwise. Also, let $m_j = 0$, $\forall j \in \mathcal{M}_{\mathcal{N}}$. Suppose utility functions are the sum of the customers' service rates and there is no cost.*

Clearly, $v(\{i\}) = 0 \ \forall \ i$, $v(\{1,2\}) = v(\{2,3\}) = v(\{1,2,3\}) = 2, v(\{1,3\}) = 0$. Table I shows all possible orderings U of the providers, and $\Delta_i(U)$ for each provider i and ordering U. From (2) *and Table I, the Shapley value of the providers is $\mathbf{x} = (\frac{2}{6}, \frac{8}{6}, \frac{2}{6})$. Note that $x_1 + x_2 = \frac{10}{6} < v(\{1,2\})$. Hence $\mathbf{x} \notin \mathcal{C}$.*

Remark VII.1. *Although unlike the nucleolus, the Shapley value need not be in the core, its useful properties, such as uniqueness and tractability, makes it an interesting solution concept in cooperation analysis among many researchers in different fields of study. Finally, note that finding a polynomial time algorithm to compute the nucleolus and Shapley value in our settings, remains an open problem.*

TABLE I: All possible orderings and marginal contributions of the players.

U	$\Delta_1(U)$	$\Delta_2(U)$	$\Delta_3(U)$
123	0	2	0
132	0	2	0
213	2	0	0
231	0	0	2
312	0	2	0
321	0	2	0

VIII. Optimum Selection of Service Level Agreements

In previous sections we assumed that the set of customers subscribed to each provider is given a priori, and so is not part of the decision variables. We now investigate cooperation in the case where providers can in fact decide which customers to accept as subscribers.

We first illustrate the impact of provider cooperation on the customers and why the customers negotiate service level agreements. Cooperation enhances providers' aggregate payoffs which are increasing functions of service rates of the customers. Thus, intuitively, the rates of most of the customers increase when the providers cooperate. Cooperation may however decrease the rates of some of the customers. Consequently, it may induce unfairness to the customers and may also reduce the customer base of individual operators. In Example IV.1 when the providers do not cooperate, all customers may receive non-zero rates; yet, the customers of provider 1 receive no service when the providers cooperate.

The unfairness is however mitigated when the providers' benefit functions are strictly concave - a choice allowed by our framework. For example, if the benefit function in Example IV.1 is logarithmic (instead of linear), i.e., $U_i(\mathbf{y}_i) = \sum_{j\in\mathcal{M}_i}\log(1+y_j)$, then each customer of each provider is served half of the time if the providers do not cooperate. When providers cooperate, together they solve the following optimization problem.
$\max\sum_{j=1}^{2}\log(1+\alpha_j P)+\sum_{j=3}^{4}\log(1+\alpha_j Q)$
subject to: $\sum_{j=1}^{4}\alpha_j \le 2$,
where α_j is the fraction of time customer j is served. We have used the symmetry of service units to get a reduced optimization problem. We can further reduce it to the following problem.
$\max\log(1+\alpha_1 P)+\log(1+\alpha_3 Q)$
subject to:

1) $\alpha_1+\alpha_3 \le 1$
2) $\alpha_2 = \alpha_1$
3) $\alpha_4 = \alpha_3$

Consequently, it can be seen that each customer of provider 1 is served $[1-(1/P-1/Q)]/2$ fraction of time while each customer of provider 2 is served $[1+(1/P-1/Q)]/2$ fraction of time while (assuming $1/P-1/Q<1$ which for example happens if $P>1$). Note that when both $P>>1$ (since $Q>P$, then $Q>>1$ as well), then each customer of provider 1 (and of provider 2 as well) is served approximately 50% of time. Thus, cooperation does not induce any unfairness in this case[15]. The benefit functions may be chosen during negotiations between providers and the customers and may also be controlled by regulatory bodies (e.g., FCC in USA).

Our coalitional game framework also allows the customers to mitigate this unfairness (even in presence of linear benefit functions) by imposing minimum rate constraints through SLAs (Example IV.1 had no SLAs), e.g., all the customers in Example IV.1 may ask for a minimum rate $\frac{P}{2}$. Then, $v(\{1\}) = P, v(\{2\}) = Q, v(\{1,2\}) = P+Q$, and each customer receives the same rate irrespective of cooperation. But, then, the core has the unique imputation of (P,Q) which provides no payoff gain to any provider as compared to when they do not cooperate. The question then is whether provider 1 should accept the above SLA? More generally, should providers accept any SLA? The following discussion suggests that the providers ought to accept SLAs, but selectively.

Consider the network model in Section III. Now let $\mathcal{M}_i$ be the set of potential customers of provider i. Each customer $j\in\mathcal{M}_i$ negotiates an SLA (equivalently, minimum service guarantee) with provider i, denoted by m_j. If provider i does not accept the SLA, customer j will subsequently leave the network. The following example elucidates some counter-intuitive phenomena.

[15]Under logarithmic benefit functions, cooperation does not enhance the providers payoffs in this case either. This happens since each customer has the same rate from all the service units. However, when customers have rate-diversity, i.e., have potentially different rates from different service-units, cooperation substantially enhances the payoffs of individual providers for logarithmic and several other strictly concave benefit functions (Section IX)

Example VIII.1. *Consider Example IV.1. Assume that providers have agreed to cooperate and divide the payoffs according to the nucleolus. Recall that the nucleolus is* $(\frac{Q+P}{2},\frac{3Q-P}{2})$. *Also, the customers of provider* 1 *do not receive any service. Now, suppose they demand minimum rate guarantees, operator* 1 *does not agree to these demands, and these users subsequently leave. Then,* $\mathcal{M}_1=\phi$. *Thus,* $v(\{1\})=0, v(\{2\})=Q, v(\{1,2\}=2Q$, *and the nucleolus is* $(\frac{Q}{2},\frac{3Q}{2})$*; operator* 1 *gets less revenue than in the earlier case. Thus, although the users of an operator do not generate any revenue, their mere presence enhances the revenues earned by the operator.*

If a provider can not honor accepted SLAs by himself, he is penalized drastically, should he not cooperate with others. It affects provider's standing in negotiations for dividing the payoffs. However, as the following example shows, accepting such SLAs may increase provider's share in the coalition.

Example VIII.2. *Let* $\mathcal{N}=\{1,2,3\}$. *Let* $\mathcal{B}_i=\{i\}$ *and* $\mathcal{M}_i=\{i\}$, *for each provider* i. *Let* $r_{j1}=P, j=2,3$, $r_{1k}=P, k=2,3$, *and* $r_{jk}=0$, *otherwise. Also, let* $0<m_1<P$ *and* $m_j=0, j=2,3$. *Let the utility functions be the sum of the customers' service rates. Since operator* 1 *can not provide rate* m_1 *to user* 1, $v(\{1\})=-\infty$. $v(\{2\})=v(\{3\})=v(\{2,3\})=0$ *and* $v(\{1,2\}=v(\{1,3\})=v(\{1,2,3\})=2P$. *It is straightforward to verify that* $\mathcal{C}=\{(2P,0,0)\}$. *Now, if operator* 1 *refuses the SLA of user* 1, $\mathcal{M}_1=\phi$ *and* $v(\{1\})=0$. $v(\{2\})=v(\{3\})=v(\{2,3\}=0$ *and* $v(\{1,2\}=v(\{1,3\})=v(\{1,2,3\})=P$. *Again, we can see that* $\mathcal{C}=\{(P,0,0)\}$. *Thus, the revenue of operator* 1 *decreases when it refuses the SLA of its user.*

Accepting an SLA does not necessarily lead to a higher share of payoff for a provider, even when it increases the aggregate payoff of a coalition. Conversely, it is possible that accepting an SLA by a provider decreases the aggregate payoff, but increases that provider's share. The following example illustrates these situations.

Example VIII.3. *Again consider Example IV.1, with the difference that each customer of provider* 1 *requests an SLA equal to* $\frac{P}{2}$. *Moreover, customers in* $\mathcal{M}_1$ *do not require service rates above* $\frac{3P}{4}$, *and as a result will not pay for any extra service*[16]. *We assume providers have agreed to cooperate and divide the payoffs according to the nucleolus. If provider* 1 *rejects both SLAs, customers in* $\mathcal{M}_1$ *leave and we have:* $v(\{1\})=0, v(\{2\})=Q$, *and* $v(\{1,2\})=2Q$. *Consequently, providers' shares will be* $(x_1,x_2)=(\frac{Q}{2},\frac{3Q}{2})$. *On the other hand, if provider* 1 *accepts one of the SLAs and rejects the other, we have:* $v(\{1\})=\frac{3P}{4}, v(\{2\})=Q$, *and* $v(\{1,2\})=\frac{P}{2}+\frac{3Q}{2}$, *which lead to payoffs* $(x_1,x_2)=(\frac{5P+2Q}{8},\frac{10Q-P}{8})$. *Finally, if provider* 1 *accepts both SLAs, we have:* $v(\{1\})=P, v(\{2\})=Q$, *and* $v(\{1,2\})=P+Q$, *and therefore,* $(x_1,x_2)=(P,Q)$. *Now suppose* P *and* Q *satisfy* $\frac{3P}{2}<Q<2P$ *(this is more restrictive than* $P<Q$ *required*

[16]This can be captured in our framework by simply choosing appropriately upper bounded utility functions

in Example IV.1). Then it is optimal for provider 1 to accept only one of the SLAs.

Remark VIII.1. *Although accepting both SLAs increases $v(\{1\})$, it decreases provider 1's share, for it decreases the aggregate payoff. On the other hand, rejecting both SLAs increases the aggregate payoff, but decreases provider 1's share, since it decreases $v(\{1\})$. Also, note that if provider 1 does not have any customer, even without any upper-bound on the benefit functions, $v(\{1\}) = 0, v(\{2\}) = Q$, and $v(\{1,2\}) = 2Q$ and the nucleolus is $(\frac{Q}{2}, \frac{3Q}{2})$, whereas in Example IV.1 (i.e., when the customers did not have SLAs and provider 1 had 2 customers) provider 1's payoff share as per the nucleolus was $\frac{Q+P}{2}$ (last paragraph of Section VII-A). Thus, although when the providers cooperate, the customers of provider 1 do not receive any service, and therefore do not generate any revenue, their mere presence enhances the payoff of provider 1 (from $Q/2$ above to $\frac{Q+P}{2}$)!*

An interesting question now is: what is the optimum strategy for providers in accepting the SLAs? We propose two approaches to address this question.

A. Cooperative SLA Selection

As was shown in Example VIII.3, the preference of a provider in accepting SLAs is not necessarily in line with that of the coalition as a whole. One way to deal with this scenario is to have providers select SLAs cooperatively. Clearly, the optimal cooperation strategy then involves selecting a set of SLAs that maximize the aggregate payoff. In Example VIII.3, for instance, providers 1 and 2, respectively, select no SLA and both SLAs (note that we can assume provider 2's customers request SLAs equal to zero). However, as discussed before, if the characteristic function $v(\cdot)$ defined in Section IV is used as the reference to compute each provider's share, provider 1 is likely to refuse this selection of SLAs. In fact, we observed in Example VIII.3 that for the Shapley value computed using function $v(\cdot)$, provider 1 prefers to accept one SLA, instead of no SLA. Therefore, it is imperative to design an appropriate payoff sharing mechanism, so as to make cooperation practical.

Let us redefine the characteristic function that is used to characterize coalitions' values as follows. Define $\hat{v}(\mathcal{S})$ to be the maximum payoff achievable by coalition $\mathcal{S} \subset \mathcal{N}$, among all the possible choices of SLAs available to $\mathcal{S}$[17]. In other words, for a coalition $\mathcal{S} \subseteq \mathcal{N}$, $\hat{v}(\mathcal{S})$ is given by the following optimization problem.

$\mathrm{P}_{SLA}(\mathcal{S}) : \max \sum_{\substack{i\in\mathcal{S}\\ \omega\in\Omega}} \mathbb{P}(\omega)\Big(U_i(\mathbf{y}_i(\omega)) - V_i(\mathbf{z}_i(\omega))\Big)$

subject to:

1) $y_j(\omega) = s_j \sum_{k\in\mathcal{B}_\mathcal{S}} \alpha_{jk}(\omega) r_{jk}(\omega), \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega$
2) $z_l(\omega) = \sum_{j\in\mathcal{M}_\mathcal{S}} \alpha_{jk}(\omega), \quad k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$
3) $\sum_{k\in\mathcal{B}_\mathcal{S}} \alpha_{jk}(\omega) \le 1, \quad j \in \mathcal{M}_\mathcal{S}, \omega \in \Omega$
4) $\sum_{j\in\mathcal{M}_\mathcal{S}} \alpha_{jk}(\omega) \le 1, \quad k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega$
5) $\sum_{\omega\in\Omega} \mathbb{P}(\omega) y_j(\omega) \ge s_j m_j, \quad j \in \mathcal{M}_\mathcal{S}$
6) $s_j \in \{0,1\}, \alpha_{jk}(\omega) \in [0,\alpha], \quad j \in \mathcal{M}_\mathcal{S}, k \in \mathcal{B}_\mathcal{S}, \omega \in \Omega,$

where $s_j = 1$ if customer j's SLA is accepted and zero otherwise. Note that for any customer j, the minimum rate constraint is nontrivial, only if $s_j = 1$ (5). Also, for customers with $s_j = 0$, the service rate $y_j = 0$ by (1). These two conditions ensure that the customers whose SLAs are rejected do not receive any service. Thus, in any optimal solution of the above optimization problem, only customers with accepted SLAs are served. For instance, in Example VIII.3, $\hat{v}(\{1\}) = P, \hat{v}(\{2\}) = Q$, and $\hat{v}(\{1,2\}) = 2Q$.

Notice that if $\hat{v}(\cdot)$ is used as the reference in payoff sharing, instead of $v(\cdot)$, a provider's share does not depend on the set of SLAs he accepted, but rather on the set of SLAs accepted by his coalition. Therefore, providers' only concern is to maximize the aggregate payoff. Also note that in this setup, towards maximizing the aggregate payoff, a provider may have to accept a set of SLAs, it can not honor by itself. That is, the optimization problem in Section IV becomes infeasible for a coalition $\mathcal{S}$. However, $\mathcal{S}$ will not be penalized for that, since $\mathrm{P}_{SLA}(\mathcal{S})$ is feasible for all coalitions, and therefore $\hat{v}(\mathcal{S}) \ge 0$ for all $\mathcal{S} \subseteq \mathcal{N}$. In Example VIII.3, for instance, suppose that both customers of provider 2 request a minimum rate of Q. Note that this does not change the optimum selection of SLAs by the grand coalition, since the grand coalition can in fact deliver the minimum rate requested. However, by accepting both SLAs, provider 2 can no longer honor the minimum rate guarantees of his customers by himself and totally depends on provider 1. This fact can drastically weaken provider 2's bargaining power in deciding individuals' share, provided that $v(\cdot)$ is the reference function in payoff sharing. But since the function $\hat{v}$ remains unchanged (particularly, $\hat{v}(\{2\}) = Q$), provider 2 is not penalized under this sharing policy and therefore continues to accept both SLAs as part of the optimal strategy.

Remark VIII.2. *It remains an open problem, whether the coalitional game $<\mathcal{N}, \hat{v}>$ has a nonempty core.*

B. Competitive SLA Selection

Another approach to the SLA selection problem is to let providers select their SLAs competitively. That is, first each provider selects his SLAs according to some optimal strategy without coordinating with other providers. Then, they cooperatively allocate the service units and channels to their customers as already studied (e.g., as given by the optimization problem in Section IV) and divide the payoffs according to an appropriate sharing rule (e.g., the core). It is evident that in this scenario, the SLAs chosen by each provider, directly affects his payoff. Thus in this scenario, each provider selects the SLAs so as to maximize his share, rather than the aggregate payoff. In Example VIII.3, for instance, provider 1 accepts one SLA, although it does not maximize the aggregate payoff.

Let $\mathcal{A}(\mathcal{S})$ denote the set of joint actions of providers in coalition $\mathcal{S}$. That is, $\mathcal{A}(\mathcal{S})$ contains all the possible selections of SLAs by providers in $\mathcal{S}$. Now suppose providers form the grand coalition and decide to divide the aggregate payoff as per an already agreed on sharing rule (e.g., the nucleolus).

[17] In the original formulation, $v(\mathcal{S})$ was the payoff of coalition $\mathcal{S}$, when the set of SLAs selected by $\mathcal{S}$ was the corresponding subset of the set of SLAs selected by the grand coalition.

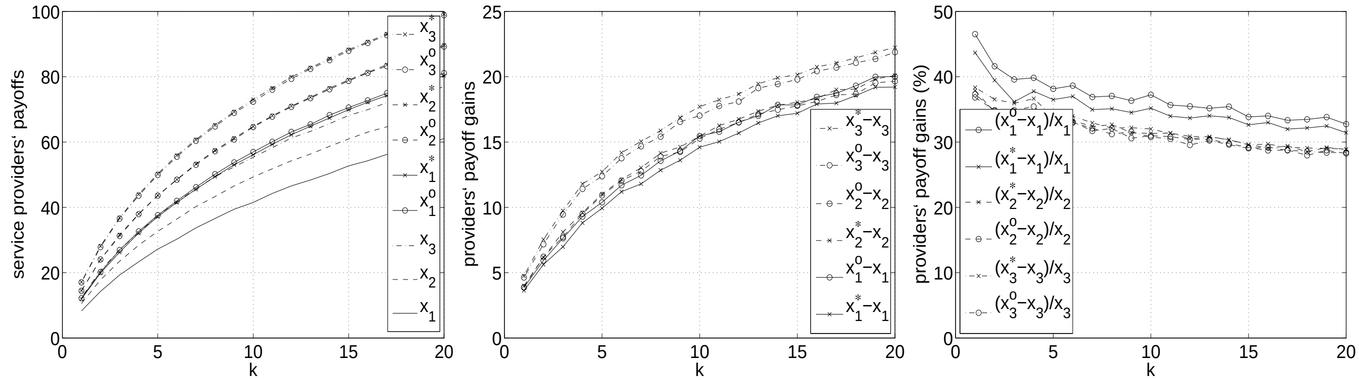

Fig. 2: The left, middle and right sub-plots respectively show providers' payoffs, payoff gains and percentage payoff-gains as functions of the number of customers: the three providers have $3k, 4k$ and $5k$ customers, respectively.

Then, for a joint action $\alpha \in \mathcal{A}(\mathcal{N})$, providers receive payoffs according to an imputation $\mathbf{x}(\alpha)$, where $\mathbf{x}(\cdot)$ is the payoff function defined by the sharing rule. We consider only joint actions $\alpha \in \mathcal{A}(\mathcal{N})$, for which the optimization problems solving $v(\{i\})$ are feasible for all $i \in \mathcal{N}$ (this implies that the optimization problems solving $v(\mathcal{S})$ are feasible for all $\mathcal{S} \subset \mathcal{N}$ and so all $v(\mathcal{S})$ are well defined). Now $\big(\mathcal{A}(\mathcal{N}), \mathbf{x}(\cdot)\big)$ defines a noncooperative game. A Nash equilibrium of this game can be used by providers as their strategy for selection of SLAs. In Example VIII.3, for instance, the following joint action is trivially a Nash equilibrium of the game: provider 1 selects one SLA and provider 2 selects both. However, determining whether the game $\big(\mathcal{A}(\mathcal{N}), \mathbf{x}(\cdot)\big)$ has a Nash equilibrium in general remains an open problem.

C. Comparison

Cooperative selection of SLAs has the advantage of maximizing the aggregate payoff, and so is Pareto-optimal. However, should the core of the game be empty, motivating providers to form a coalition is definitely not a forte of this approach. On the other hand, the competitive SLA selection has the advantage that the core of the game $< \mathcal{N}, v >$, as shown in Section IV, is always nonempty. Thus, for any possible selection of SLAs, we can find a sharing rule that stabilizes the grand coalition and hence justifies the cooperation. The drawback is that the outcome then could be significantly suboptimal which is not globally desirable.

IX. Quantitative Evaluations

In the context of the resource pooling game (Section IV), we evaluate the benefits of cooperation and compare different payoff sharing schemes such as the dual-based payoff shares (Section IV) and the nucleolus (Section VII) for a range of benefit functions.

We first consider logarithmic revenue (benefit) functions $U_i(\mathbf{y}_i) = \sum_{j\in\mathcal{M}_i} \log(1+y_j)$ and zero cost functions $V_i(\mathbf{z}_i) = 0$. $U_i(\mathbf{y}_i)$ are strictly concave functions and assumes positive values except when $\mathbf{y}_i$ is the zero vector and in this case the revenue is 0. Note that logarithmic functions have been widely used as satisfaction functions of customers and therefore constitute good candidates for the revenues they pay (and hence for the benefits the providers incur). The cost functions are zero when the providers acquire the resources (spectrum, base-stations) apriori by paying fixed (licensing or deployment) fees and do not incur subsequent usage based costs[18]. Also, we assume that the customers do not have SLAs as is typically the case for elastic transfers from the Internet (e.g., file transfers). We allow the service-unit-customer rates r_{jk} to be uniformly distributed over the set $\{0, 100, 200\}$Kbps, and to be independent across service-unit-customer pairs j, k. The characteristic functions $v(\mathcal{S})$ for different coalitions $\mathcal{S}$ and the dual based imputation in the core can now be obtained by solving the concave optimization P($\mathcal{S}$), D($\mathcal{S}$) (Section IV) once the number of providers N, and the number of service-units B_i and customers M_i of the different providers are specified. The nucleolus can subsequently be computed using Definition VII.1. We denote the payoff of a provider i (i) in absence of cooperation as x_i (note that $x_i = v(\{i\})$, (ii) in the grand coalition as x_i^o (as per the nucleolus) or x_i^* (via solving the dual problem) or x_i^+ (as per the Shapley value). Owing to large state spaces we use *Monte Carlo simulations* in our evaluations.

We first consider 3 providers, and $M_1 = 3k, M_2 = 4k, M_3 = 5k$ and $B_1 = B_2 = B_3 = 1$ where k ranges from 1 to 20 (Figure 2). The plots show that cooperation leads to substantial payoff improvements for all providers, and the payoff-gains increase as number of customers increase. As expected (from Definition VII.1), *the nucleolus distributes the payoff gains more equitably than the dual based profit-share* which allocates payoff gains in increasing order of the number of customers (wealth generated), reserving the highest payoff gain for the provider with the highest number of customers. Nevertheless, the payoffs of each provider are similar under both payoff sharing rules, and also to those under the Shapley

[18]Recall that the fixed service-unit deployment and acquisition fees need to be considered explicitly only when the deployment and acquisition of service-units constitute optimization decision variables as in the resource deployment game in Section V, and not when these are decided apriori as in the resource pooling game of Sections IV.

value allocations (Figure 3) Furthermore, *the percentage gains in payoffs due to cooperation are quite significant (in the range of $30\% - 40\%$) for each provider*.

Henceforth, for simplicity, we focus on 2 providers. Note that the Shapley value is the same as the nucleolus in this case (Section VII - paragraph before Example VII.1). We investigate the impact of varying only the (i) demands (number of customers) and (ii) assets (number of service units) of one provider while keeping the other's demand and asset fixed. First, let $N = 2$, $B_1 = B_2 = 1$, $M_1 = 20$ and vary M_2 the number of customers of provider 2 (Figure 4). Next, we let $N = 2$, $M_1 = M_2 = 20$, $B_1 = 5$, and vary the number of service units B_2 of provider 2 (Figure 5). As the demand (or assets) of the second provider is increased, the payoff of the second provider increases under both the nucleolus and dual-based payoff sharing rules, but that of the first may either increase (Figure 4) or decrease (Figure 5), depending on whether its importance in the cooperation increases or decreases due to the increase in the demands (or assets) of the second. Mathematically, $x_1^o = v(\{1,2\}) + v(\{1\}) - v(\{2\})/2$, and as the demands (assets) of the second increases, $v(\{1,2\}), v(\{2\})$ increase but $v(\{1\})$ does not change. Thus, the difference $v(\{1,2\}) - v(\{2\})$ may either increase, or decrease. Nevertheless, the payoff of the first still remains significantly above that when it does not cooperate with the second. Also, in both cases the provider with the larger demand or asset obtains higher payoffs under both sharing rules.

We now investigate how the choice of the revenue function affects providers' payoff gains. In particular, we consider the generalized $\alpha-$fair revenue function [39]: $U_i(\mathbf{y}_i) = \sum_{j \in \mathcal{M}_i} \frac{(y_i)^{1-\alpha}}{1-\alpha}$, where $0 < \alpha < 1$. Note that for each j $\frac{\partial^2}{\partial^2 y_j} U_i(\vec{y}) = -\alpha$ and thus intuitively the "concavity" of the revenue function increases with increase in α (the function is linear at one extreme: $\alpha = 0$). We plot the providers' percentage payoff gains as a function of α, for $N = 2$, $B_1 = B_2 = 1$ and $M_1 = 10$ and $M_2 = 20$ (Figure 6). Payoff gains are very similar under the nucleolus and the dual based sharing rules. More importantly, *the percentage payoff gains for both providers increase significantly with increase in α - thus, higher the concavity, the more beneficial cooperation is.* This can be explained as follows. For small α (i.e., nearly linear benefit functions), at any network state ω, the aggregate revenue is maximized by allocating each service unit to one customer. Next, given that the number of customers (10 or 20) significantly exceeds the number of service units (1) of each provider, usually (i.e., for most ω) each provider's service unit has excellent transmission conditions to at least one customer. Thus, cooperation can not enhance the aggregate customers' rates, nor the providers' aggregate and hence individual payoffs. As α increases, the aggregate payoff is maximized by allowing the providers to time-share among, and provide more equitable rates to, the customers at each ω. When not in coalition, in order to roughly equalize the rates of all the customers, each provider's service unit must therefore serve customers with poor transmission quality r_{jk} for considerable fractions of

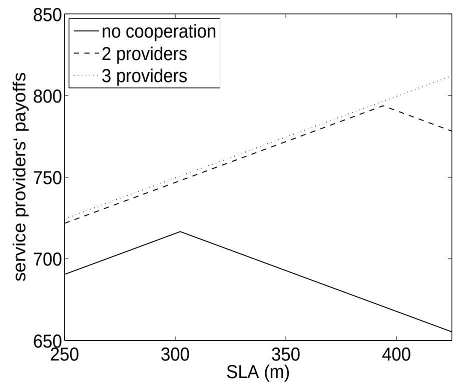

Fig. 7: Providers' payoffs as a function of the guaranteed rate to the premium customers

time. When the providers cooperate, usually, most (or more) of the customers have high transmission rates from at least one service unit (rate-diversity) - thus equitable rates can also be provided by allowing each service unit to time-share among the customers (not necessarily those of the same provider) that have good transmission quality from it. Thus, equity is attained through good match between customers and service units and without compromising the overall customer rates and providers' revenues. Thus, cooperation substantially enhances aggregate, and therefore individual, payoffs. Cooperation turns out to be beneficial even for low α when a large number of customers have statistically better transmission qualities from other provider's service units than those of the one they have subscribed to.

Finally, we illustrate the benefits of cooperation and compare the nucleolus, Shapley value and dual based payoff shares in presence of SLAs. We consider 3 providers each with 3 service units and 10 customers. Now, $r_{jk} = 100$ Kbps (200Kbps, resp.) with probability 0.8 (0.2, resp.). Each provider has 3 *premium* and 7 *best effort* customers: the former have negotiated SLAs which guarantee a minimum average rate m. We consider linear revenue functions:

$$U_i(\mathbf{y}_i) = \sum_{j=1}^{3} (\beta m + \alpha(y_j - m)) + \sum_{j=4}^{10} \alpha y_j \qquad (3)$$

where $\beta > \alpha$ captures the higher payoff per Kbps for the service guarantees to the premium customers. We choose $\alpha = 1$ and $\beta = 1.5$. The revenue $\alpha \sum_{j=1}^{10} y_j$ is denoted as "usage based revenue" and the rest $(\beta - \alpha)3m$ is the fixed fee associated with SLAs. Due to symmetry, providers receive equal payoffs under both dual and nucleolus based shares. As Figure 7 reveals, cooperation enhances each provider's revenue: the increase is significant when the size of each coalition increases from 1 to 2, and somewhat less when the size increases to 3. For small m, a provider does not need to compromise on the efficient usage of resources (i.e., it preferentially serves the customers with high transmission rates). Each provider's payoff increases linearly with m in this region due to the increase of the fixed fees associated with m. However, beyond a certain threshold, each provider needs to

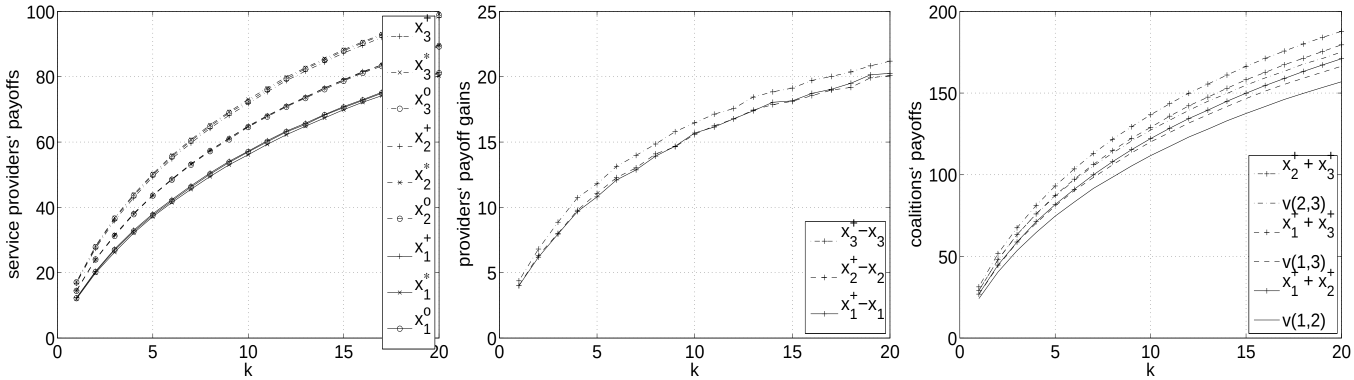

Fig. 3: The left and middle sub-plots respectively show providers' payoffs and payoff gains (corresponding to the Shapley value) as functions of the number of customers: the three providers have $3k, 4k$ and $5k$ customers, respectively. The last plot shows that $\sum_{i\in\mathcal{S}} x_i^+ \geq v(\mathcal{S})$ for all $\mathcal{S}\subset\mathcal{N}$, thus implying that, for the chosen parameter values, the Shapley value lies in the core.

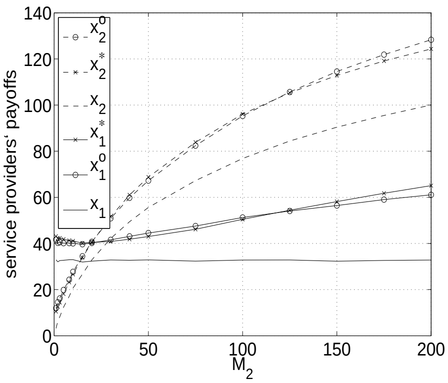

Fig. 4: Providers' payoffs as functions of number of customers: the first provider has 20 customers while the number of customers of the second, M_2, is varied.

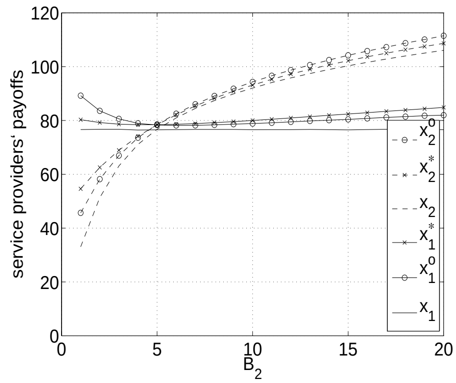

Fig. 5: Providers' payoffs as functions of number of base stations: The first provider owns 5 service units while the number of service units, B_2, of the second is varied.

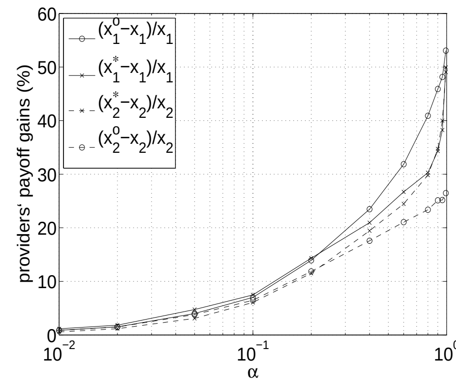

Fig. 6: The percentage payoff gains of the providers are plotted as functions of α.

schedule a few lower rate links to the premium customers (instead of the higher rate links to the best effort customers) to satisfy the SLAs. This lowers the aggregate service rates, and each provider's payoff decreases linearly with increase in m. Cooperation increases this threshold and also the aggregate rate of all the customers by allowing the scheduling of higher rate links more often.

Next, we consider an asymmetric scenario where each provider has 10 customers as before, but they respectively have $3, 0, k$ premium customers; k is varied from 1 to 7. All the premium customers demand a minimum guaranteed rate of 125Kbps. It turns out that a provider alone cannot guarantee 125Kbps to more than 3 customers. Similarly, any two providers can support at most 8 premium customers together. Thus, $\mathrm{P}(\{3\})$ is not feasible for $k > 3$, and assumption IV.1 no longer holds. For $k > 3$, we define $v(\{3\})$ as the objective function of $\mathrm{P}(\{3\})$ with 3 premium customers, for $k > 5$, $v(\{1,3\})$ is the objective function of $\mathrm{P}(\{1,3\})$ with 5 premium customers, and for $k > 8$, $v(\{2,3\})$ is the objective function of $\mathrm{P}(\{2,3\})$ with 8 premium customers. It turns out that the dual and nucleolus payoff shares are in the core, and hence the core is non-empty. In the left and middle subfigures of Figure 8, we show the providers' payoffs as functions of the number of premium customers of the third provider. We have plotted the providers' payoffs that correspond to the dual based allocation, the nucleolus and the Shapley value (the middle subfigure) in the grand coalition. We have also plotted the providers' payoffs when they do not cooperate. In the right subfigure, we plot the maximum attainable payoffs of all the coalitions. The dual based allocation equally divides the total usage based payoffs among all providers, and allocates the fixed fees of each provider's customers to the provider. Thus, the payoffs of providers $1, 2$ do not change with increase in k, but that of provider 3 increases linearly with increase in k. The nucleolus however transfers a part of the fixed fees provider 3 earns to other providers - intuitively such transfer is warranted as provider 3 can not support all its premium customers by itself for $k > 3$. Thus, payoff shares of all providers change

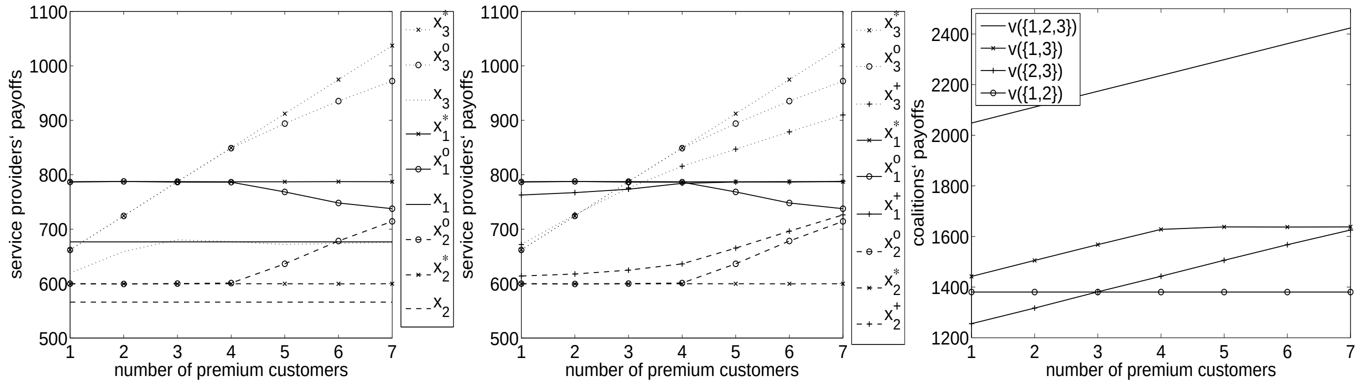

Fig. 8: The left and middle sub-figures show providers' payoffs as functions of the number of the premium customers of the third provider. x_1^o, x_2^o, x_3^o, x_1^*, x_2^*, x_3^* and x_1^+, x_2^+, x_3^+ are payoffs corresponding to the dual allocation, the nucleolus and the Shapley value respectively. x_1, x_2, x_3 are the providers' payoffs if they do not cooperate. The right sub-figure shows the maximum attainable payoffs of all the coalitions.

with increase in k, and evidently, the nucleolus based payoff gains are more equitable than the dual based ones. In all the allocations, a provider with larger number of premium customers gets a larger payoff share, and each provider's payoff increases substantially due to cooperation.

Calculation of Characteristic Functions: Recall that in presence of SLAs $\mathrm{P}(\mathcal{S})$ can not be solved by solving separate convex optimizations (Section IV-A). Consequently, the computation times can be large since $|\Omega|$, typically, is large. Let us focus on a scenario where there are two classes of customers premium and best-effort ones. The premium customers require a minimum guaranteed rate m while the best-effort customers are not guaranteed any minimum rate. We assume that (i) the channels are symmetric across all the service units and customers, i.e., all service unit-customer pairs see identical channel statistics, (ii) functions $U_i(\cdot)$ are piecewise linear and identical for all the service providers (e.g., see Equation (3)) (iii) functions $V_i(\cdot) = 0$ for all i and (iv) there are more customers than service units. Under these assumptions, we show that the characteristic functions can still be solved by solving separate optimization problems, one for each $\omega \in \Omega$. The following algorithm is for the case when each channel can be in two states: good (rate H) or bad (rate L). It can be suitably modified for the cases when each channel can be in more than two states.

Step 1: Generate a network state ω, using the distribution on the service unit-customer rates. For each ω, schedule as many good channels as possible while giving preference to the premium customers. If there are unmatched service units left, schedule as many bad channels as possible again giving preference to the premium customers. Subsequently, compute the average rates of all the premium and best-effort customers over a large number of runs.

If the average rates of premium customers (these will be same for all premium customers) exceed the minimum guaranteed rates, then the aggregate payoff of all the providers corresponding to the above schedules gives the characteristic function. Otherwise, we proceed to the following Step 2.

Step 2: If the average rates of the premium customers are less than the minimum guaranteed ones, determine the minimum fraction of time that must be shifted from the good channels of best-effort customers to the bad channels of the premium customers in order to meet their minimum rate requirements. For this fraction of time, reduce the service to the best-effort customers from H to 0 and enhance the service to the premium customers from 0 to L. The aggregate payoff of all the providers corresponding to the resulting schedules gives the characteristic function.

X. Conclusion and Future work

We studied cooperation among service providers in wireless networks. If service providers cooperate, they can pool their resources (e.g., service units and spectrum) and allocate them to the joint pool of customers in an optimal fashion. We formulated the problem as a transferable utility coalitional game. We showed nonemptyness of cores in various scenarios (see Theorems IV.1, V.5, V.7 etc.) implying that cooperation is not only globally optimal, but also makes each of the providers better off. Our proof technique is constructive and also yields an optimal resource allocation and corresponding profit shares. We also discussed two other profit sharing rules, nucleolus and Shapley value. Cooperative game framework provides insights into a number of service providers' decision problems, e.g., where should they place service unit (Section V) and which service level agreements they should accept (Section VIII)

In practice, coalition formation can incur overheads, e.g., it can lead to increased loads on the call processors and billing systems. Investigating the stability of the grand coalition considering the coalition formation overhead constitutes an open problem. Moreover, the computation time for an allocation in the core will be high since it depends polynomially on the number of possible channel state and mobile location realizations ($|\Omega|$), which is large. Obtaining near-optimal solutions with low computation time remains open. Finally, we considered a system where the customer subscriptions and the provider's revenue function have already been determined. Investigating cooperation among the providers when the customers dynamically decide their subscription based on the revenue functions, and how providers can dynamically and optimally select the revenue functions so as to enhance their individual share of the overall profit remain open.

We show that a concave function, that has a bounded second derivative, can be approximated as the minimum of a set of linear functions. Consequently, a concave optimization problem can be solved with arbitrary precision in polynomial time since linear optimizations are polynomial time solvable using interior point methods.

Lemma .1. *Consider a concave, double-differentiable function $g : R \to R$ such that $|g''(x)| \leq B$ for $x \in [a, b]$. Then, given any $\epsilon > 0$, for every integer $l \geq \lceil (b-a)^2 B/\epsilon \rceil$ there exists l linear functions of the form $c_t + d_t x$ such that $g(x) \leq c_t + d_t x$ for each $x \in [a, b], t = 1, 2, \ldots, l$ and $g(x) \geq \min_{t=1}^{l} (c_t + d_t x) - \epsilon$.*

Proof: Consider $a = 0$ without loss of generality. Let $\Delta = b/l$, $d_i = g'((i-1)\Delta)$, $i = 1, \ldots, l$, $c_1 = g(0)$, $\{c_i\}$ is such that $c_{i+1} + d_{i+1}(i\Delta) = c_i + d_i(i\Delta)$.

First, we prove using induction, that $g(x) \leq c_t + d_t x$ for each $x \in [0, b], t = 1, 2, \ldots, l$. Let $t = 1$. $g(x) \leq g(0) + g'(0)x$ for $x \geq 0$ (follows from the definition of concavity, Equation 3.2, p. 69, Boyd). Thus, $g(x) \leq c_1 + d_1 x$ for all $x \geq 0$. Now, let the claim hold for $t = 1, \ldots, m$. We prove the claim for $t = m+1$. First, let $x \leq m\Delta$. Note that $c_{m+1} + d_{m+1}x \geq c_m + d_m x$. This is because at $x = m\Delta$, $c_{m+1} + d_{m+1}x = c_m + d_m x$ and $d_{m+1} \leq d_m$. Now, the claim follows for $t = m$. Now, let $x > m\Delta$. Again, from the definition of concavity,

$$\begin{aligned} g(x) &\leq g(m\Delta) + g'(m\Delta)(x - m\Delta) \\ &\leq c_m + g'((m-1)\Delta) m\Delta + g'(m\Delta)(x - m\Delta). \end{aligned}$$

The last inequality follows from the induction hypothesis for $t = m$ at $x = m\Delta$. Now, $c_{m+1} = c_m + [g'((m-1)\Delta) - g'(m\Delta)]m\Delta$. Thus,

$$g(x) \leq c_{m+1} + g'(m\Delta)x.$$

The claim holds for $t = m+1$.

Note that for $t = 1, \ldots, l$, $(t-1)\Delta \leq x \leq t\Delta$ $\arg\min_{m=1}^{l} (c_m + d_m x) = t$. Thus, for the second part of the lemma, we just need to show that for $t = 1, \ldots, l$, $(t-1)\Delta \leq x \leq t\Delta$ $g(x) \geq c_t + d_t x - tB\Delta^2$. The result follows since $l\Delta = b$. We prove this by induction. First, let $t = 1$, and $x \in (0, \Delta)$. By twice application of the mean value theorem, for some $0 < \kappa' < \kappa < x$,

$$\begin{aligned} g(x) &= g(0) + g'(0)x + g''(\kappa')x\kappa \\ &\geq c_1 + d_1 x - B\Delta^2. \end{aligned}$$

Now, let the claim hold for $t = 1, \ldots, m$. Let $t = m+1$, and let $x \in (m\Delta, (m+1)\Delta)$. First note that

$$\begin{aligned} c_{m+1} + d_{m+1}x &= c_{m+1} + d_{m+1}m\Delta + d_{m+1}(x - m\Delta) \\ &= c_m + d_m m\Delta + d_{m+1}(x - m\Delta) \\ &\leq g(m\Delta) + mB\Delta^2 + g'(m\Delta)(x - m\Delta). \end{aligned}$$

By twice application of the mean value theorem, for some $m\Delta < \kappa' < \kappa < x$,

$$\begin{aligned} g(x) &= g(m\Delta) + g'(m\Delta)(x - m\Delta) \\ &\quad + g''(\kappa')(x - m\Delta)(\kappa - m\Delta) \\ &\geq c_{m+1} + d_{m+1}x - (m+1)B\Delta^2. \end{aligned}$$

Thus, the claim holds for $t = m+1$. ■

Now, consider a maximization with the objective function $\sum_{i=1}^{Q} g_i(x_i)$ where $\mathbf{x}$ is a $Q-$dimensional vector in a bounded set $\mathcal{G}$ where $\mathcal{G}$ is specified by C linear constraints involving W variables ($W \geq Q$), and $g_i(\cdot)$ is a double-differentiable concave function with $g_i''(\cdot)$ upper bounded by B. Then the optimum value of this maximization is at most $Q\epsilon$ less than a linear program with Q additional variables and $Q\lceil \frac{(b-a)^2 B}{\epsilon} \rceil$ additional constraints, where a, b are the minimum and maximum values of any component of $\mathbf{x} \in \mathcal{G}$. The objective function of the linear program is $y_1 + \ldots y_Q$ where the additional constraints are y_i less than or equal to each of the above linear functions for the $g_i(\cdot)$ function. Thus, given any $\epsilon > 0$, b, a, B, the concave maximization can be computed within an error margin of ϵ in $O(W^2(C + \frac{Q^2(b-a)^2 B}{\epsilon})^{3/2})$ computation time using Karmarkar's algorithm [40, Chapter 10].

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