Minimizing Delay in Loss-Tolerant MAC Layer Multicast

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***Abstract*— Many real-time applications require one to many (*multicast*) communication. Real time applications can gracefully accommodate some loss but require low delay. We minimize the delay in real-time MAC layer multicast by exploiting the broadcast nature of wireless medium and limited loss tolerance of the applications. We show that multiple transmissions of a packet at the MAC layer significantly reduces the delay than that when only one transmission is allowed. But each additional transmission consumes additional power and increases network load. Therefore, our goal is to design a policy that judiciously uses the limited transmission opportunities so as to deliver each packet in minimum possible time to the required number of group members. We show that the problem is an instance of the *stochastic shortest path problem*, and using this formulation obtain a computationally simple, closed form transmission strategy in important special cases. Numerical computations show that only a small number of transmissions, if used judiciously, are sufficient to minimize the delay subject to loss constraint.**

I. Introduction

In wireless networks, many real-time applications like conference meetings, emergency operation in case of a natural disaster and military operations, etc. require one to many (*multicast*) communication. Real-time applications can tolerate some packet loss but require low delay. *Our contribution is to develop transmission schemes that minimize the delay in real-time MAC layer multicast by exploiting the limited loss tolerance and the broadcast property of wireless medium.* Most of the work in wireless multicast has focused on the network and transport layers, e.g., [2], [7], [9], [10], [16], [15]. Though the performance of the network and transport layers protocols depends on the efficiency of the MAC layer strategy, MAC layer multicast has not been adequately explored. Our work is directed towards filling this void.

Now, we describe the challenges in designing optimal real-time MAC layer multicast schemes. Our aim is to minimize the delay in delivering a Head of Line (HoL) packet to the multicast group. Consider a MAC layer multicast session from a sender S_1 to receivers R_1 to R_4 in Figure 1. Let S_1 wish to deliver a packet to all the receivers $R_1, \ldots, R_4$ which are in its transmission range. Due to the broadcast property of wireless communication, all nodes in the sender's transmission range can intercept a transmission. Hence at the MAC layer, S_1 can deliver a packet to all the intended receivers using a single transmission. Apparently, this broadcast nature can be used to reduce

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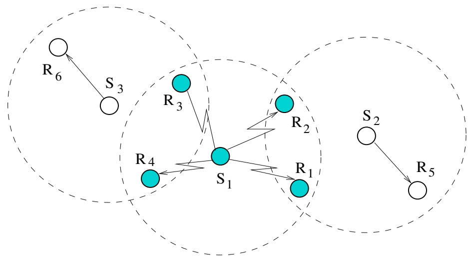

Fig. 1. An example to demonstrate the advantages and the challenges associated with wireless multicast. The figure shows three senders S_1, S_2 and S_3, and six receivers R_1 to R_6. Dashed circles indicate the communication ranges of senders.

the delay at the MAC layer. But, the broadcast nature also introduces critical challenges. A multicast specific challenge is that some but *not all* the receivers may be ready to receive due to the interference in their neighborhood and transmission quality in wireless channels. For example, when S_2 is transmitting to R_5, R_1 and R_2 can not receive a transmission from S_1 as both the transmissions will collide at these receivers (Figure 1). However, R_3 and R_4 can still receive the transmission if S_3 is not transmitting. It is not clear whether the delay would be minimized when S_1 transmits the packet only when all the receivers are ready simultaneously, or when S_1 transmits separately to receivers R_1-R_2 and R_3-R_4. The following example illustrates the point.

Example: We compare two transmission strategies in Figure 1: (a) S_1 transmits when all the receivers are ready, and (b) S_1 transmits first to R_1-R_2 and then to R_3-R_4. Let V denote the number of slots required by S_1 to transmit a packet. Let S_2 and S_3 require one slot to transmit a packet. We assume that if S_1 is not transmitting, then S_2 and S_3 transmit independently with probability (w.p.) $1-p$, and S_2 and S_3 do not transmit when S_1 is transmitting. The expected delay under the first policy ($\mathbf{E}[D_1]$) is equal to the expected time till all the four receivers become ready plus the packet transmission time, i.e., $\mathbf{E}[D_1] = 1/p^2+V$. The expected delay under the second policy ($\mathbf{E}[D_2]$) is equal to the expected time till R_1-R_2 become ready plus the expected time till R_3-R_4 become ready plus $2V$, i.e., $\mathbf{E}[D_2] = 2(1/p+V)$. Let $p = 0.1$. Now, when $V = 10$ slots, then $\mathbf{E}[D_1] = 110$ slots and $\mathbf{E}[D_2] = 40$ slots. When $V = 100$ slots, then $\mathbf{E}[D_1] = 200$ slots and $\mathbf{E}[D_2] = 220$ slots. Thus, different policies achieve smaller delays in different scenarios.

The above example shows that multiple transmissions, if utilized properly, can significantly reduce the delay in certain cases. But multiple transmissions also increase the power consumption and the network load. Hence, it is desirable to limit the number of such transmissions. Thus, one needs to design a policy that judiciously utilizes the transmissions and the broadcast nature of wireless medium to minimize the delay.

Real-time applications can accommodate some packet loss without noticeable degradation in the quality of service, e.g., for voice, depending on the encoding and transmission schemes used, 20% packet loss can be acceptable [8]. Loss tolerance is different for different applications. Hence, depending on applications, it may suffice to deliver each packet only to a certain fraction of the multicast group. The following example demonstrates that a small loss tolerance can significantly reduce the delay.

Example: Consider a MAC layer multicast session from S_1 to twenty receivers R_1 to R_{20}. Let the readiness states of the receivers be iid Bernoulli, i.e., a receiver R_j is ready in a slot w.p. 0.6 independent of the states of other receivers for $j = 1, \ldots, 20$. Let the application tolerate 5% loss. We consider two policies: (a) S_1 transmits only when all twenty receivers are ready, and (b) S_1 transmits when 19 or more receivers are ready. Let the transmission time for each packet be one slot. The mean delays under policies (a) and (b) are 27351.11 slots and 1908.22 slots respectively. The expected fraction of packets lost at a receiver under policies (a) and (b) are 0 and 0.047 respectively. Thus, the delay can be significantly reduced by exploiting the loss tolerance.

We investigate the trade-off between loss and delay for MAC layer multicast. Specifically, *we study the problem of minimizing the mean delay to deliver an HoL packet to Z receivers using at most K transmissions*. The parameters Z and K depend on loss tolerance of the application and power constraints respectively. We describe our assumptions and system model in Section II. We formulate this goal as a Stochastic Shortest Path (SSP) problem, and thereby develop a strategy to compute the policy that minimizes the expected delay (Section III). In special cases, we obtain closed form solutions for the optimal transmission strategy (Section IV). Using numerical computations we demonstrate that even with few transmissions, the proposed optimum policy attains significantly lower delay (Section V). The optimal transmission policy obtained here is threshold based. Specifically, the sender chooses a threshold τ for every transmission and transmits only when τ or more receivers that have not received the packet in previous transmissions are ready. We conclude in Section VI. The proofs are presented in the appendix.

We note that our exposition is from a theoretical perspective. Here, we aim to develop an analytically tractable model that provides important insight into various issues that one needs to address before developing a fully operational protocol. We do not consider issues like dynamic changes in group membership due to mobility, how to choose allowed number of transmissions (K) per packet, etc. We hope that our work will instigate further research in this area and will lead to the development of practical protocols.

Now, we briefly review the MAC layer protocols for multicast in ad hoc networks. The most popular MAC protocol IEEE 802.11 supports MAC layer multicast by broadcasting the packets. The control hand-shake is disabled for this broadcast transmission and hence the protocol is not reliable. Tang *et. al.* have proposed a modification that uses the capture mechanism to ensure at least one receiver is ready when the packet is broadcasted [12], [13]. Though this scheme achieves better reliability than IEEE 802.11, it cannot guarantee the required loss. Tang *et. al.* have also proposed a scheme that transmits a packet to each receiver separately in unicast mode [14]. This scheme achieves 100% reliability, but is inefficient as it does not exploit the broadcast property. We have proposed several adaptive schemes to maximize the MAC layer multicast throughput [3], [4], [5].

II. System Model

In this paper, we consider a single multicast session with G receivers. The impact of the network and the channel errors on the multicast session is that the receivers are not always ready to receive. This may happen because of a transmission in the neighborhood of a receiver, bursty channel errors, or power saving operation of a receiver. Thus, the receiver readiness states are correlated in the same time slot, and across the time slots. We model the readiness process of all the receivers as a Markov chain (MC) with an arbitrary Transition Probability Matrix (TPM) $\widehat{\mathbf{B}}$. A state of the MC is the G-dimensional *readiness* vector $\vec{j} = [j_1 \; j_2 \; \ldots \; j_G]$, where the component j_l is 1 if the l^{th} receiver is ready and 0 otherwise. Let $\mathcal{C}$ denote the state space of the MC. We assume that the $2^G \times 2^G$ TPM $\widehat{\mathbf{B}}$ is irreducible, aperiodic and time-homogeneous. We adopt this model because in a distributed environment the senders do not coordinate their transmissions, and only observe the readiness states of their receivers. Thus from the perspective of a sender the network is a stochastic disturbance which is not controllable but only partially observable. The arbitrary Markovian transitions of the readiness process allow us to consider different network loads and different inter-session interactions. This model allows us to address several open research problems that are specific to MAC layer multicast.

A sender queries the readiness states of the receivers by transmitting control packets, and decides whether to transmit a packet depending on the transmission strategy and result of the query. Every receiver maintains its readiness state throughout the transmission. This assumption is justified because the time scale of a change of transmission quality is much larger than the duration of packet transmission. Also, the level of interference does not change during a packet transmission, since in several MAC protocols (e.g., IEEE 802.11), the exchange of control messages prevents a new transmission during an ongoing transmission in the reception range of the receiver. The sender backs off for a random duration before querying the system again, irrespective of the transmission decision, so as to allow other senders to use the shared medium. The structure of the multiple access protocol described above is similar to IEEE 802.11. Note that the receiver readiness process is Markovian only when restricted to the slots in which the sender queries or backs off, e.g., in duration $[T_1, T_3] \cup X_3 \cup T_4 \cup X_4$ in Figure 2.

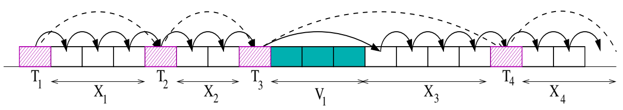

Fig. 2. Figure shows typical transitions of the receiver readiness process (solid arcs) and the sampled readiness process (dashed arcs). A box indicates a time slot. The T_i's, V_i's and X_i's denote the sample points, duration of transmission and duration of back-off, respectively.

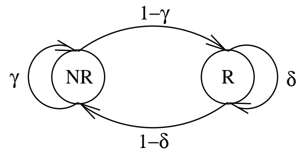

Fig. 3. Figure shows the readiness process for a receiver. NR and R indicates states *Not Ready* and *Ready* respectively.

We assume that time is slotted. The packet transmission times and back-off durations are independent and identically distributed (iid) random variables with arbitrary probability distributions and finite expected values $\mathbf{E}[V]$ and $\mathbf{E}[X]$ respectively. The slots in which the sender queries the readiness states are called *sample points*, and the readiness process observed by the sender is called *sampled readiness process*. Note that the sampled readiness process is also an irreducible, aperiodic and time-homogeneous MC. Let $\mathbf{B}$ denote the TPM for the sampled readiness process. Then, the transition probabilities of the sampled readiness process are

$$B_{\vec{j},\vec{j}_1} = \sum_{l=1}^{\infty} \widehat{B}^{(l)}_{\vec{j},\vec{j}_1} \mathbf{P}\{X = l-1\},$$

where $\widehat{B}^{(l)}_{\vec{j},\vec{j}_1}$ is the probability of being in state $\vec{j}_1$ starting from state $\vec{j}$ after l transitions of the original readiness process. At any time x, a receiver is *satisfied* if it has received the packet in the prior transmissions; otherwise it is *unsatisfied*. Initially every receiver is unsatisfied and with subsequent transmissions some receivers become satisfied.

III. Transmission Policy

Our goal is to design a transmission strategy that minimizes the expected time to deliver an HoL packet to at least Z receivers using at most K transmissions. First, we show this problem is an instance of a stochastic shortest path problem (SSP). Using this formulation we develop a technique for computing an optimal transmission policy.

The SSP formulation is as follows. Let $\vec{a} = [a_1\ a_2\ \cdots\ a_G]$ denote a vector of indicators. Here, a_i is 1 if receiver i is satisfied; otherwise it is 0. The system state is a three dimensional vector $(k, \vec{a}, \vec{j})$, where k is the number of completed transmissions and $\vec{j}$ is the readiness vector. Note that when $k = 0$, then $\vec{a} = \vec{0}$, where $\vec{0}$ is the G-dimensional vector of zeroes. In every state $(k, \vec{a}, \vec{j})$, the sender can either back-off or transmit. We now describe the resulting state transitions. If the sender backs off, then the state becomes $(k, \vec{a}, \vec{j}_1)$ w.p. $B_{\vec{j},\vec{j}_1}$. On the other hand, if the sender transmits, then the state becomes $(k+1, \vec{j}\circ\vec{a}, \vec{j}_1)$ w.p. $B_{\vec{j},\vec{j}_1}$, where $\vec{j}\circ\vec{a}$ denotes the element-wise *OR* operation of the vectors $\vec{j}$ and $\vec{a}$. The process terminates when Z or more receivers are satisfied, i.e., the states $(k, \vec{a}, \vec{j})$ such that $\sum_{i=1}^{G} a_i \geq Z$ are the termination states. If the number of receivers that are satisfied after $K-1$ transmissions is $z < Z$, then the sender transmits only if $Z - z$ or more unsatisfied receivers are ready. Thus the process always terminates. The system needs to reach a termination state in the minimum expected time.

Let $J^*(k, \vec{a}, \vec{j})$ denote the minimum expected time to terminate from $(k, \vec{a}, \vec{j})$. Also, let $J^*_T(k, \vec{a}, \vec{j})$ and $J^*_B(k, \vec{a}, \vec{j})$ denote the minimum expected time to terminate from $(k, \vec{a}, \vec{j})$ if the control decision is to transmit and back-off respectively. Then,

$$\begin{aligned}
J^*(k, \vec{a}, \vec{j}) &= \min\{J^*_T(k, \vec{a}, \vec{j}), J^*_B(k, \vec{a}, \vec{j})\}, \quad (1)\\
J^*_T(k, \vec{a}, \vec{j}) &= 1 + \mathbf{E}[V + X]\\
&\quad + \sum_{\vec{j}_1 \in \mathcal{C}} B_{\vec{j},\vec{j}_1} J^*(k+1, \vec{j}\circ\vec{a}, \vec{j}_1) \quad (2)\\
\text{and } J^*_B(k, \vec{a}, \vec{j}) &= 1 + \mathbf{E}[X] + \sum_{\vec{j}_1 \in \mathcal{C}} B_{\vec{j},\vec{j}_1} J^*(k, \vec{a}, \vec{j}_1). \quad (3)
\end{aligned}$$

If $J^*(k, \vec{a}, \vec{j}) = J^*_T(k, \vec{a}, \vec{j})$, then the optimal control decision in state $(k, \vec{j}, \vec{a})$ is to transmit; otherwise the optimal decision is to back-off. Thus, the optimal strategy can be obtained by solving the Bellman's equations (1), (2) and (3). The minimum termination time to reach at least Z receivers in at most K transmission from an initial state $\vec{j}$ of the receiver readiness process is $J^*(0, \vec{0}, \vec{j})$. Bellman's equations can be solved using the standard *value iteration* method [1] which we describe next. Let $J_l(k, \vec{a}, \vec{j})$, $J_{l,T}(k, \vec{a}, \vec{j})$ and $J_{l,B}(k, \vec{a}, \vec{j})$ be defined iteratively as follows for every $0 \leq k \leq K$ and $\vec{a}, \vec{j} \in \mathcal{C}$.

$$\begin{aligned}
J_l(k, \vec{a}, \vec{j}) &= \min\{J_{l,T}(k, \vec{a}, \vec{j}), J_{l,B}(k, \vec{a}, \vec{j})\}, \quad (4)\\
J_{l,T}(k, \vec{a}, \vec{j}) &= 1 + \mathbf{E}[V + X]\\
&\quad + \sum_{\vec{j}_1 \in \mathcal{C}} B_{\vec{j},\vec{j}_1} J_{l-1}(k+1, \vec{a}\circ\vec{j}, \vec{j}_1) \quad (5)\\
\text{and } J_{l,B}(k, \vec{a}, \vec{j}) &= 1 + \mathbf{E}[X] + \sum_{\vec{j}_1 \in \mathcal{C}} B_{\vec{j},\vec{j}_1} J_{l-1}(k, \vec{a}, \vec{j}_1) \quad (6)
\end{aligned}$$

From Proposition 2.1.2 of [1], for every $0 \leq k \leq K$ and $\vec{a}, \vec{j} \in \mathcal{C}$, $\lim_{l\to\infty} J_l(k, \vec{a}, \vec{j}) = J^*(k, \vec{a}, \vec{j})$, $\lim_{l\to\infty} J_{l,T}(k, \vec{a}, \vec{j}) = J^*_T(k, \vec{a}, \vec{j})$ and $\lim_{l\to\infty} J_{l,B}(k, \vec{a}, \vec{j}) = J^*_B(k, \vec{a}, \vec{j})$ for any initial values $J_{0,T}(k, \vec{a}, \vec{j})$ and $J_{0,B}(k, \vec{a}, \vec{j})$. Refer to [1] for other techniques to solve Bellman's equations.

A limitation of the above approach is that the number of iterations required to obtain the optimal strategy may be large. Also, size of the state space is $K2^{2G}$, which grows exponentially with G. But, we expect G to be small in practice. Also, the computation of the optimal transmission policy is a one time process, and on-line transmission decisions can be made using a look-up table.

IV. Special Cases

We now exploit the problem structure to obtain a closed form, computationally simple optimal transmission policy in important special cases. We assume that each receiver's readiness

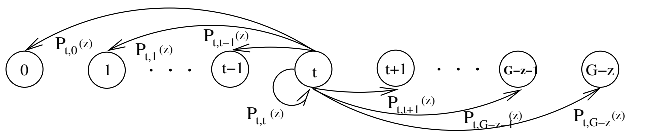

Fig. 4. Figure shows possible transitions from state t in the aggregate readiness process of $G-z$ unsatisfied receivers.

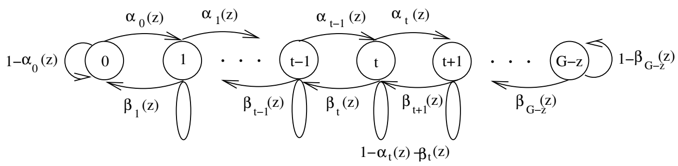

Fig. 5. Figure shows a BD process that approximates the aggregate receiver readiness process of $G-z$ unsatisfied receivers when the receivers have bursty iid Markovian readiness processes.

process evolves as per a two state Markov process (Figure 3) and the readiness states of different receivers are mutually independent. Here, the transition probability of a receiver changing its readiness state from *ready* (*not ready*) to *not ready* (*ready*) is $1-\delta$ $(1-\gamma)$. Since the readiness states are symmetric and independent, intuitively, the expected time for termination does not depend on *identity* of the satisfied or unsatisfied receivers, but rather depends only on the *number* of satisfied receivers (z) and the *number* of unsatisfied ready receivers (t). Thus, it suffices to consider the system state simply as (k, z, t). We prove this formally in appendix (Lemma 1). This observation leads to the simplification of the receiver readiness process. In particular, we only need to consider the *aggregate* readiness process of unsatisfied receivers. A state of the aggregate readiness process is the number of unsatisfied ready receivers (Figure 4). Let z receivers be satisfied, and let $P_{t,t_1}(z)$ denote the probability that t_1 receivers are ready in the current slot given that t receivers were ready in the previous slot. Then, for all $t_1 \in \{0, \ldots, G-z\}$,

$$P_{t,t_1}(z) = \sum_{u=0}^{t} \binom{t}{u} \delta^u (1-\delta)^{t-u} \times \binom{G-z-t}{t_1-u} (1-\gamma)^{t_1-u} \gamma^{G-z-t-t_1+u}. \quad (7)$$

We use a notation that $\binom{c}{\widehat{c}} = 0$ if $\widehat{c} < 0$ or $\widehat{c} > c$. If the sender backs off in state (k, z, t), then the state changes to (k, z, t_1) w.p. $P_{t,t_1}(z)$, and if the sender transmits in state (k, z, t), then the the state changes to $(k+1, z+t, t_1)$ w.p. $P_{0,t_1}(z+t)$.

Intuitively, a threshold type policy is optimal in the special case of iid Markovian readiness states because, unlike in the general case, the number and not the identity of the unsatisfied ready receivers determines the expected time for termination. Thus, the sender transmits only when the number of unsatisfied ready receivers exceeds a selected threshold. We prove this in two extreme scenarios: first when the readiness process is bursty ($1-\gamma$ and $1-\delta \approx 0$ in Figure 3) *, and second when the readiness states are iid Bernoulli ($1-\gamma = \delta$ in Figure 3). We present polynomial complexity algorithms for computing the optimal thresholds which depend only on k and z.

A. Case 1: Bursty Receiver Readiness States

We consider a case when receiver readiness states are bursty, i.e., the transition probabilities $1-\delta$ and $1-\gamma$ are close to zero. We also assume that the sender queries the receiver readiness states in every slot, i.e., there is no back-off ($\mathbf{E}[X] = 0$). From (7) we observe that if $|t - t_1| \geq 2$, then $P_{t,t_1}(z) \approx 0$ as it only contains terms with higher power of $(1-\delta)$ and/or $(1-\gamma)$ for every $z \in \{0, \ldots, G\}$. Now, for $t_1 \in \{t-1, t, t+1\}$ and $t \in \{0, \ldots, G-z\}$

$$\begin{aligned} P_{t,t+1}(z) &\overset{\text{Def}}{=} \alpha_t(z) \approx (G-t-z)\delta^t \gamma^{G-z-t-1}(1-\gamma) \\ P_{t,t-1}(z) &\overset{\text{Def}}{=} \beta_t(z) \approx t\gamma^{G-z-t}\delta^{t-1}(1-\delta) \\ P_{t,t}(z) &\approx 1 - (\alpha_t(z) + \beta_t(z)). \end{aligned}$$

Thus, the aggregate receiver readiness process can be approximated as a non-homogeneous Birth-Death (BD) process. Note that $\alpha_t(z)$ $(\beta_t(z))$ is the forward (reverse) transition probability from t to $t+1$ $(t-1)$ in the BD process when z receivers are satisfied. Using this approximation, we obtain a closed form computationally simple optimal transmission strategy (Figure 6).

Now we describe the threshold computation in detail. First, the algorithm considers states after $K-1$ transmissions, e.g., states $(K-1, z, t)$ for $z \leq G$ and $t \in \{0, 1\}$. In such states, as discussed before, the sender transmits only when at least $Z-z$ unsatisfied receivers are ready. Since the aggregate readiness process is a BD process, states with more than $Z-z$ unsatisfied ready receivers can not be reached before reaching the state with $Z-z$ unsatisfied receivers. Thus, to minimize the expected time, the sender will transmit a packet in the state with $Z-z$ unsatisfied ready receivers. Hence, the optimal time to terminate from state $(K-1, z, t)$ is the expected time to reach state $Z-z$ from state t in the aggregate readiness process of $G-z$ receivers (denoted as $s_{t,Z-z}(z)$) plus the expected duration of packet transmission. Thus, the threshold $\tau(K-1, z) = Z-z$ and $J^*(K-1, z, t) = s_{t,Z-z}(z) + \mathbf{E}[V] + 1$.

Next, the algorithm considers states after $K-2$ transmissions, e.g., states $(K-2, z, t)$ for $z \leq G$ and $t \in \{0, 1\}$. Suppose the sender decides to transmit when u unsatisfied receivers are ready. Then, the minimum expected time to terminate will be the sum of (1) the expected time to reach state u from state t in the aggregate BD process of $G-z$ receivers (2) the expected duration of the packet transmission, and (3) the minimum expected time to terminate from the states it reaches after the transmissions. After $K-1^{th}$ transmission the system is in states $(K-1, z+u, 0)$ w.p. $1-\alpha_0(z+u)$ and $(K-1, z+u, 1)$ w.p. $\alpha_0(z+u)$. Thus, the minimum time to terminate from state $(K-2, z, t)$ given that the sender transmits when u unsatisfied receivers are ready is $s_{t,u}(z) + \mathbf{E}[V] + 1 + (1-\alpha_0(z+u))J^*(K-1, z+u, 0) + \alpha_0(z+u)J^*(K-1, z+u, 1)$. Note that $J^*(K-1, z+u, 0)$ and $J^*(K-1, z+u, 1)$ have been computed before. Thus, the algorithm computes $\tau(K-2, z)$ and the minimum expected time to terminate from state $(K-2, z, t)$ as given by **(C1)**, **(C2)** and **(C3)** in Figure 6 respectively.

*Strictly speaking, in this case we have proved that the optimal strategy is threshold-type for $k > 0$. We have however obtained a closed form polynomial complexity computable optimal strategy for all values of k.

Procedure Threshold_Computation_1(K, Z)
begin

$s_{u,v}(z) \overset{\text{Def}}{=}$ the expected time to reach state v from state u for the first time in BD process of $G-z$ unsatisfied receivers.

$s_{u,v_1||v_2}(z) \overset{\text{Def}}{=}$ the expected time to reach either state v_1 or state v_2 from state u for the first time in BD process of $G-z$ unsatisfied receivers.

$r^z_{t,v_1}(\{v_1, v_2\}) \overset{\text{Def}}{=}$ is the probability of visiting state v_1 before v_2 from state t in BD process of $G-z$ unsatisfied receivers.

$\tau(k, z) \overset{\text{Def}}{=}$ the optimal threshold when z receivers are satisfied after k transmissions, for $k \geq 1$.

$\widehat{\tau}(t) \overset{\text{Def}}{=}$ a set of aggregate readiness states such that the optimal policy transmits when $u \in \widehat{\tau}(t)$ receivers are ready, if $k = 0$ and t were ready when the packet came to the HoL position.

for $(z \in \{1, 2, \ldots, G\})$ **do**
  $J^*(K-1, z, i) = s_{i,Z-z}(z) + \mathbf{E}[V] + 1$ if $z < Z$ and
    $= 0$ o.w.
  $\tau(K-1, z) = Z - z$ if $z < Z$
for $(k = K-2$ to $1)$ **do**
  for $(z = 1$ to $Z)$ **do**
    (C1) $\tau(k, z) = \arg\min_{1 \leq u \leq Z-z} \{s_{1,u}(z) + J^*_T(k, z, u)\}$
    (C2) $J^*(k, z, 1) = s_{1,\tau(k,z)}(z) + J^*_T(k, z, \tau(k, z))$
    (C3) $J^*(k, z, 0) = J^*(k, z, 1) + s_{0,1}(z)$
    Note : $J^*_T(k, z, u) = \mathbf{E}[V] + 1 + (1 - \alpha_0(z))J^*(k+1, z+u, 1)$
      $+\alpha_0(z)J^*(k+1, z+u, 0)$
for $(t \in \{0, 1, \ldots, G\})$ **do**
  $J_1(0, 0, t) = \min_{u \geq t} \{s_{t,u}(0) + J^*_T(0, 0, u)\}$
  $J_2(0, 0, t) = \min_{\substack{u_1 > t \\ u_2 < t}} \left\{ s_{t,u_1||u_2}(0) + \sum_{i=1}^{2} r^0_{t,u_i}(\{u_1, u_2\}) J^*_T(0, 0, u_i) \right\}$
  if $(J_1(0, 0, t) \leq J_2(0, 0, t))$ **then**
    (C4) $\widehat{\tau}(t) = \left\{ \widehat{u} : J_1(0, 0, t) = s_{t,\widehat{u}}(0) + J^*_T(0, 0, \widehat{u}) \right\}$
  else
    (C5) $\widehat{\tau}(t) = \left\{ \widehat{u}_1, \widehat{u}_2 : s_{t,\widehat{u}_1||\widehat{u}_2}(0) + \sum_{i=1}^{2} r^0_{t,\widehat{u}_i}(\{\widehat{u}_1, \widehat{u}_2\}) J^*_T(0, 0, \widehat{u}_i) \right\}$
end

Procedure Transmission_Strategy_1(K, Z)
begin
  if $(k = 0)$ **then**
    Transmit when $u \in \widehat{\tau}(t)$ unsatisfied receivers are ready, where t is the number of ready receivers when a packet comes to the HoL position
  else
    Transmit when $\tau(k, z)$ unsatisfied receivers are ready, if z receivers are satisfied after k transmissions
end

Fig. 6. Pseudo code for the optimal transmission policy when the receiver readiness processes are iid bursty Markovian.

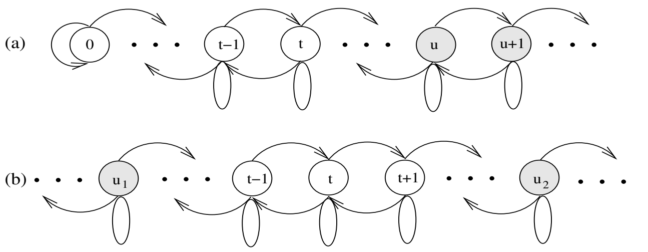

Fig. 7. Figure shows the aggregate readiness process of unsatisfied receivers. The shaded states belong to $\mathcal{T}_{k,z}$. Let the process be in state t. In case (a), $m_{k,z} = u$ which is greater than t, and in case (b) $m_{k,z} \leq u_1 < t$. So, in case (a), the optimal policy transmits when u unsatisfied receivers are ready, and in case (b), the optimal policy transmits when either u_1 or u_2 receivers are ready.

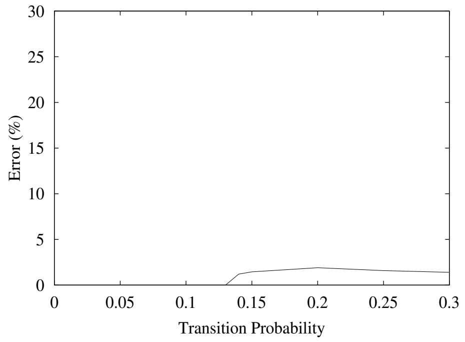

Fig. 8. Figure shows the normalized approximation error ($\frac{D_A - D_O}{D_O} \times 100\%$) introduced due to the BD approximation in the computation of the minimum expected delay as a function of the transition probability $1 - \delta$. In case 1, $\gamma = \delta$. In case 2, $\gamma = 0.9$. Here, $G = Z = 10$, $EV = 100$ slots and $K = 5$.

Similarly, the algorithm computes $\tau(k, z)$ and $J^*(k, z, t)$ for $0 < k \leq K-3$ and $z \leq G$ sequentially, i.e., first $\tau(K-3, z)$ and $J^*(K-3, z, t)$, then $J^*(K-4, z, t)$ and $\tau(K-4, z)$ and so on.

The transmission policy at $k = 0$ differs from that at other values of k. Let t_k be the number of ready, unsatisfied receivers right after kth transmission, for $k \geq 1$, and t_0 be the number of ready, unsatisfied receivers when the packet reaches the HoL position. Note that $t_0 \in \{0, \ldots, G\}$, while $t_k \in \{0, 1\}$, for $k \geq 1$. Now, we explain how this difference affects the optimal strategy. Let z receivers be satisfied after k transmissions and let $\mathcal{T}_{k,z}$ denote the set of aggregate readiness states in which the optimal decision is to transmit:

$$\mathcal{T}_{k,z} = \{t : J^*_T(k, z, t) < J^*_B(k, z, t)\}.$$

Note that the optimal strategy transmits when $u \in \mathcal{T}_{k,z}$ unsatisfied receivers are ready. Let $m_{k,z}$ denote the smallest member of $\mathcal{T}_{k,z}$. If $t_k \leq m_{k,z}$, then the optimal policy transmits when $m_{k,z}$ unsatisfied receivers are ready, as then the states with $m_{k,z} + 1$ or more unsatisfied ready receivers cannot be reached before $m_{k,z}$ due to the birth-death nature of the readiness process (Figure 7(a)). Clearly, $m_{k,z} \geq 1$ for every k and z. Since $t_k \in \{0, 1\}$ when $k \geq 1$, the optimal policy transmits when $m_{k,z}$ unsatisfied receivers are ready (Figure 7(a)). This explains the existence of optimal thresholds in this case. But, t_0 can exceed 1. Hence, $m_{0,z}$ may be less than t_0. Then, the optimal policy transmits when either u_1 or u_2 unsatisfied receivers are ready, where u_1 is the largest element of $\mathcal{T}_{0,z}$ such that $u_1 < t_0$ and u_2 is the smallest element of $\mathcal{T}_{0,z}$ such that $u_2 > t_0$ (Figure 7(b)). Thus, strictly speaking, for $k = 0$, the optimal strategy may not be threshold-type. If $m_{0,z} \geq t_0$, the optimal policy can be computed as for $k \geq 1$ (Figure 6). Finally, the optimal termination time is the minimum of the termination times $J_2(0, 0, t)$ and $J_1(0, 0, t)$ in the two cases.

Theorem 1: The transmission strategy proposed in Figure 6 minimizes the expected time required to deliver a packet to at least Z receivers in at most K transmissions when aggregate receiver readiness processes of unsatisfied receivers is a BD process. The threshold computation algorithm has complexity $O(KG^3)$.

Note that the BD modeling is an approximation for iid Markovian processes. We now evaluate the error due to this approximation. Let D_O denote the minimum expected delay obtained using the optimal policy in Section III. Let D_A denote the expected delay obtained using the policy in Figure 6. In Figure 8, we plot the normalized approximation error

Procedure Threshold_Computation_2(K, Z)
begin

$\widehat{s}_u(z) \stackrel{\text{Def}}{=}$ the expected time to reach states with u or more unsatisfied ready receivers in the group of $G - z$ unsatisfied receivers.

$q_{u,u_1}(z) \stackrel{\text{Def}}{=}$ the probability that u_1 out of $G - z$ unsatisfied receivers are ready given that $u_1 \geq u$.

For every $z \in \{1, 2, \ldots, G\}$
$J^*(K-1, z) = \widehat{s}_{Z-z}(z) + \mathbf{E}[V + X] + 1$ if $z < Z$ and
$= 0$ o.w.
$\tau(K-1, z) = Z - z$ if $z < Z$;
for ($k = K - 2$ to 1) **do**
for ($z = 1$ to Z) **do**
(D1) $J^*(k, z) = \min_{0 \leq u \leq Z-z} \Big\{ \widehat{s}_u(z) + \sum_{v=u}^{G-z} q_{u,v}(z) J^*(k+1, z+v) \Big\}$
$+\mathbf{E}[V + X] + 1,$
(D2) $\tau(k, z) = \arg\min_{0 \leq u \leq Z-z} \Big\{ \widehat{s}_u(z) + \sum_{v=u}^{G-z} q_{u,v}(z) J^*(k+1, z+v) \Big\}$
end

Procedure Transmission_Strategy_2(K, Z)
begin
Initialize the system state $(k, z) = (0, 0)$.
while ($z < Z$) **do**
Transmit when the number of unsatisfied ready receivers (say r) is greater than or equal to $\tau(k, z)$.
Update the system state after transmission as follows. $k = k + 1$ and $z = z + r$.
end

Fig. 9. Pseudo code for the optimal transmission policy when the receiver readiness processes are iid Bernoulli.

$\frac{D_A - D_O}{D_O} \times 100\%$ as a function of $1 - \delta$. This normalized approximation error turns out to be 0 for small values of $1 - \delta$ and it is less than 2% for $1 - \delta \leq 0.3$. This validates the BD approximation. Note that when the readiness states are generated by a Raleigh fading channel which is good for 99% of the time and has mean fade duration of 10 slots, then $1 - \delta = 0.001$ and $1 - \gamma = 0.1$ [11].

B. Case 2: Bernoulli Readiness States

Now, we assume that the receiver readiness states are iid Bernoulli, i.e., in a slot, a receiver is ready w.p. p. We now allow arbitrary back-off, i.e., $\mathbf{E}[X]$ need not be 0. We first point out the differences with case 1. Now, $P_{t_1,t_2}(z)$ does not depend on t_1 as the readiness states are independent across the slots. This simplifies the optimal strategy, as the decisions do not depend on the readiness states in the previous slots. Thus, it suffices to maintain a two dimensional system state (k, z). Also, now the aggregate readiness process is not a BD process, and can make arbitrary transitions (Figure 4). The optimal transmission algorithm (Figure 9) is, however, still threshold based. But, due to these differences, we need a different algorithm for threshold computation and a different proof for the optimality.

Now, we explain the algorithm for computing the thresholds. Consider a state (k, z). Let the threshold be u. So, the sender will transmit a packet when u or more unsatisfied receivers are ready. Hence the expected time to transmit in this case is the expected time to reach states with u or more unsatisfied ready receivers ($\widehat{s}_u(z)$). After transmission the system state can be $(k+1, z+u), (k+1, z+u+1), \ldots, (k+1, G)$ with probabilities $q_{u,u}(z), q_{u,u+1}(z), \ldots, q_{u,G-z}(z)$ respectively. Here, $q_{u,u_1}(z)$ denotes the probability that u_1 unsatisfied receivers are ready when the sender transmits and the threshold is u. So, for threshold u, the minimum expected time to terminate from the state (k, z) is given by equation (D1) in Figure 9. Now, there are various possible threshold values that a sender can choose in the state (k, z). The optimal threshold $\tau(k, z)$ is the one that provides the minimum expected time for termination (equation (D2) in Figure 9).

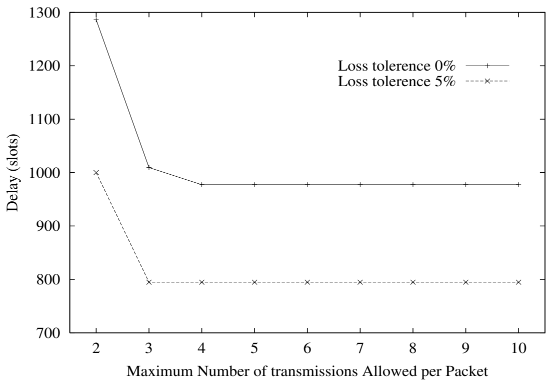

Fig. 10. Figure shows the delay attained by the optimal transmission strategy when the readiness states are iid Markovian. Here, $\mathbf{E}[V] = 100$, $G = 10$ and $\delta = \gamma = 0.99$.

Theorem 2: The transmission strategy proposed in Figure 9 minimizes the expected time required to deliver a packet to at least Z receivers in at most K transmissions when receiver readiness processes are iid Bernoulli. The threshold computation algorithm has complexity $O(KG^2)$.

V. PERFORMANCE COMPARISON

We use numerical computations and simulation to evaluate the performance of the proposed policies. Here, we only present the results for the iid Markovian readiness states (Figure 10). The results for the iid Bernoulli readiness states follow similar trends [6]. Figure 10 shows that the expected delay significantly decreases as K increases for small K. But this performance advantage saturates with further increase in K. This indicates that a small number of transmissions are sufficient to achieve the minimum expected delay. Figure 10 also shows that the minimum expected delay is significantly lower for 5% loss tolerence than that for zero loss tolerance. When $K = 1$, the delays for 0% and 5% loss tolerence are 6,726,479 and 380,463.969 slots respectively. But for $k \geq 2$, these delays are less than 1300 and 1000 slots respectively. This demonstrates the significant reduction in delay with only a limited number of transmissions per packet than that when only one transmission is allowed.

Using simulations, we aim to demonstrate that the intuition and performance trends obtained from the analytically tractable models, which make simplifying assumptions on receiver readiness process, carry over to the actual network scenario where these assumptions may not hold. Specifically, we want to evaluate the performance of the strategy that is optimal under Markovian assumption on readiness states in actual network. We consider a simple symmetric topology shown in Figure 11. We assume that the unicast sender U_i generates packets as per Poisson process with rate λ_U. The multicast sender M always has a packet. The multicast sender M needs to deliver each packet

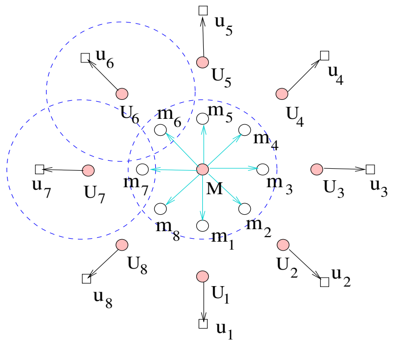

Fig. 11. Figure shows a topology we use to study the performance of the optimal policy using simulations. The topology has a multicast session with sender M and 8 receivers m_1 to m_8 and 8 unicast sessions. Unicast session i has sender U_i and receiver u_i, $1 \leq i \leq 8$.

to all the receivers. The mean packet size for the multicast session is 20 slots, and that for the unicast sessions is 1 slot. For larger packet sizes the results differ only in magnitude, but the trends remain the same. We study the impact of high network load on the multicast session by increasing λ_U. Now, the readiness states are generated by packet transmissions. For every $i \in \{1, \ldots, 8\}$, U_i is ready except when it is backing off or m_i is receiving a packet. Also, m_i is ready except when U_i is transmitting a packet to u_i. The multicast sender M is not ready when it backs off, while u_i is always ready.

We model the readiness process for each receiver as a Markov process and estimate γ and δ from the readiness states. We subsequently obtain the transmission policy using these estimates and the algorithm proposed in Figure 6. For moderate values of λ_U this policy can significantly reduce the delay for multiple transmissions ($K = 3$) than when only one transmission ($K = 1$) is allowed (Figure 12(a)). When λ_U is small, the receivers are ready most of the time and hence transmitting only once when all the receivers are ready is optimal. Hence, the delays for $K = 1$ and $K = 3$ are the same. Though the retransmissions when used judiciously can reduce the delay, they also increase the network load. Hence, the stability region for unicast sessions when $K = 3$ is smaller than that when $K = 1$ (Figure 12(b)). Finally, we compare the performance of the proposed policy with a naive heuristic. In this heuristic, which we refer to as *threshold-1* policy, M transmits when at least one unsatisfied receiver is ready for the first $K - 1$ transmissions. For the K^{th} transmission, M transmits when all the unsatisfied receivers are ready. Figure 12(c) shows that the policy proposed in Figure 6 achieves significantly smaller delay than the threshold-1 policy.

VI. Conclusion

We study the problem of minimizing the MAC layer delay to deliver an HoL packet to the multicast group. We present the optimal policy using SSP formulation. We prove that the optimal policy can be computed in polynomial complexity for important special cases. Using numerical computations and simulation, we demonstrate that the optimal policy provides significant performance benefits. Specifically, we show that a small loss tolerance of real-time applications and broadcast nature of wireless communication can be exploited to significantly reduce the MAC layer delay. Further, a limited number of packet retransmissions, if used judiciously, can provide significant reduction in the delay.

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Appendix

First, we derive some properties of the optimal solution (Lemmas 1 to 5), and using these we prove Theorems 1 and 2. Consider a system state $(k, \vec{a}, \vec{j})$. Let $z_{\vec{a}}$ denote the number of satisfied receivers, and let $t_{\vec{a},\vec{j}}$ denote the number of unsasified ready receivers in state $(k, \vec{a}, \vec{j})$, i.e.,

$$z_{\vec{a}} = \sum_{i=1}^{G} a_i \qquad \text{and} \qquad t_{\vec{a},\vec{j}} = \sum_{i=1}^{G} \max\{j_i - a_i, 0\}.$$

Lemma 1: Let $(k, \vec{a}, \vec{j})$ and $(k, \vec{a}_1, \vec{j}_1)$ be the system states such that

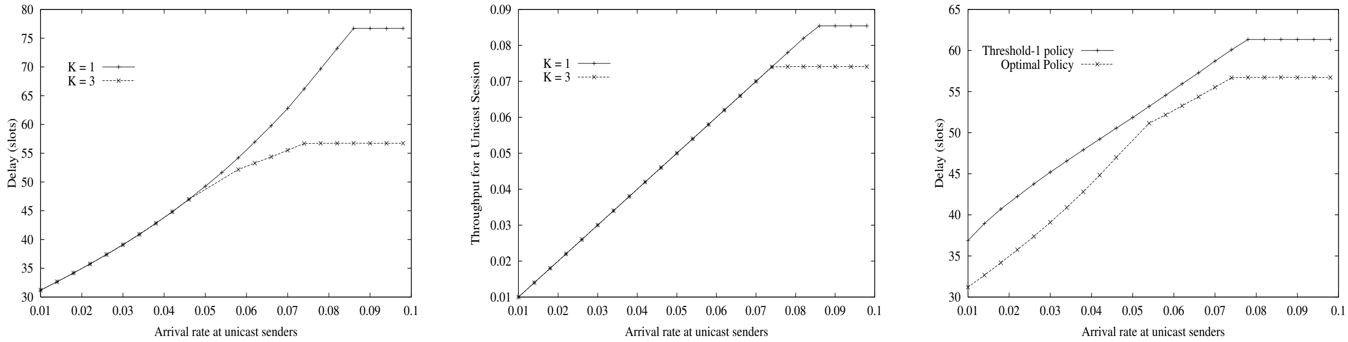
$$z_{\vec{a}} = z_{\vec{a}_1} \text{ and } t_{\vec{a},\vec{j}} = t_{\vec{a}_1,\vec{j}_1}. \tag{8}$$

Then, for iid Markovian receiver readiness states

$$J^*(k, \vec{a}, \vec{j}) = J^*(k, \vec{a}_1, \vec{j}_1) \text{ for every } k \in \{0, \ldots, K-1\}. \tag{9}$$

Proof: We prove the required using the value iteration approach described in Section III. In particular, we prove (9) by showing that for every l,

$$J_l(k, \vec{a}, \vec{j}) = J_l(k, \vec{a}_1, \vec{j}_1) \quad \forall\, k \in \{0, \ldots, K-1\}. \tag{10}$$



(a) Retransmission Gain

(b) Throughput for Unicast Sessions

(c) Comparison with Threshold-1 Heuristic

Fig. 12. Figure (a) shows the average delay of the multicast session for various values of λ_U. Figure (b) shows the throughput of a unicast session. Figure (c) shows the delays of the multicast session under optimal and threshold-1 policies.

Note that (9) follows after taking limits as l goes to ∞ on both sides of equation (10). To prove (10), it suffices to show the following for every l.

$$J_{l,B}(k,\vec{a},\vec{j}) = J_{l,B}(k,\vec{a}_1,\vec{j}_1) \quad \text{and} \tag{11}$$

$$J_{l,T}(k,\vec{a},\vec{j}) = J_{l,T}(k,\vec{a}_1,\vec{j}_1). \tag{12}$$

We prove (11) and (12) using induction. Recall that the convergence of $J_{l,T}(k,\vec{a},\vec{j})$ and $J_{l,B}(k,\vec{a},\vec{j})$ for every readiness vector $\vec{j}$, k and $\vec{a}$ is guaraneteed for any intial values ($l=0$) of these quantities. We initialize each quantity to zero. Thus, (11) and (12) hold vaccuously for $l=0$. By induction hypothesis we assume that (11) and (12) hold for every $l \le L-1$.

Consider system state $(k,\vec{a},\vec{j})$. From (6),

$$J_{L,B}(k,\vec{a},\vec{j}) = 1 + \mathbf{E}[X] + \sum_{\vec{j}' \in \mathcal{C}} B_{\vec{j},\vec{j}'} J_{L-1}(k,\vec{a},\vec{j}').$$

Now, we partition the state space $\mathcal{C}$ for the readiness process as follows. Let $\mathcal{C}_u(\vec{a},\vec{j}) = \left\{\vec{j}' \,:\, t_{\vec{a},\vec{j}'} = u\right\}$ for $u \in \{0,\ldots,G - z_{\vec{a}}\}$. Thus,

$$J_{L,B}(k,\vec{a},\vec{j}) = 1+\mathbf{E}[X]+\sum_{u=0}^{G-z_{\vec{a}}} \sum_{\vec{j}' \in \mathcal{C}_u(\vec{a},\vec{j})} B_{\vec{j},\vec{j}'} J_{L-1}(k,\vec{a},\vec{j}').$$

Now we consider, $J_{L,B}(k,\vec{a},\vec{j}) - J_{L,B}(k,\vec{a}_1,\vec{j}_1)$

$$\begin{aligned}
&= \sum_{u=0}^{G-z_{\vec{a}}} \sum_{\vec{j}' \in \mathcal{C}_u(\vec{a},\vec{j})} B_{\vec{j},\vec{j}'} J_{L-1}(k,\vec{a},\vec{j}') \\
&\quad - \sum_{u=0}^{G-z_{\vec{a}_1}} \sum_{\vec{j}' \in \mathcal{C}_u(\vec{a}_1,\vec{j}_1)} B_{\vec{j}_1,\vec{j}'} J_{L-1}(k,\vec{a}_1,\vec{j}') \\
&= \sum_{u=0}^{G-z_{\vec{a}}} \left[\sum_{\vec{j}' \in \mathcal{C}_u(\vec{a},\vec{j})} B_{\vec{j},\vec{j}'} - \sum_{\vec{j}' \in \mathcal{C}_u(\vec{a}_1,\vec{j}_1)} B_{\vec{j}_1,\vec{j}'} \right] J_{L-1}(k,\vec{a},\vec{j}') \\
&\qquad \text{(from (8) and induction hypothesis).}
\end{aligned}$$

Now, $\sum_{\vec{j}' \in \mathcal{C}_u(\vec{a},\vec{j})} B_{\vec{j},\vec{j}'}$ is the probability that any u unsatisfied receivers are ready from state $(k,\vec{a},\vec{j})$. Hence, $\sum_{\vec{j}' \in \mathcal{C}_u(\vec{a},\vec{j})} B_{\vec{j},\vec{j}'} = \sum_{\vec{j}' \in \mathcal{C}_u(\vec{a}_1,\vec{j}_1)} B_{\vec{j}_1,\vec{j}'}$ for every $u \in \{0,\ldots,G - z_{\vec{a}}\}$ whenever the receiver readiness states are iid and (8) holds. Relation (12) follows from similar arguements. ■

Thus from here onwards we consider a system state to be (k,z,t), where z is the number of satisfied receivers, t is the number of unsatisfied ready receivers after k transmissions. Corresponding to a doublet (k,z) we define a set $\mathcal{T}_{k,z}$ as follows.

$$\mathcal{T}_{k,z} = \{u \,:\, 0 \le u \le G - z \text{ and } J_B^*(k,z,u) > J_T^*(k,z,u)\}. \tag{13}$$

In other words, $\mathcal{T}_{k,z}$ is the set of aggregate readiness states in which "to transmit" is the optimal decision if z receivers are satisfied after k transmissions.

Lemma 2: The set $\mathcal{T}_{k,z}$ is unique and noempty.

Proof: Uniqueness of $\mathcal{T}_{k,z}$ follows from the unique solution to the Bellman's equations (1) to (3). Further, $\mathcal{T}_{k,z}$ is non-empty as every $u \ge Z - z$ belongs to the set. ■

Now we define some notation. Let $\mathcal{A}(z) = \{0,\ldots,G-z\}$, and let $\mathcal{P}(z)$ demote a power set of $\mathcal{A}(z)$. Note that $\mathcal{A}(z)$ is the state space of aggregate readiness process of $G-z$ unsatisfied receivers. Consider a set $A \in \mathcal{P}(z)$. Let $r_{v,u}(A)$ denote the probability that the first state visited in A is u starting from state v in the aggregate readiness process of $G-z$ unsatisfied receivers. Also, let $x_v(A)$ denote the expected time to reach any of the states $u \in A$ for the first time starting from state v in the aggregate readiness process of $G-z$ unsatisfied receivers.

Lemma 3:

$$J^*(k,z,t) = \min_{A \in \mathcal{P}(z)} \left\{ x_t(A) + \sum_{u \in A} r_{t,u}(A) J_T^*(k,z,u) \right\}. \tag{14}$$

Proof: We note that $x_t(A) + \sum_{u \in A} r_{t,u}(A) J_T^*(k,z,u)$ is the expected termination time if a policy transmits when it reaches any of the states of aggregate readiness process of unsatisfied receivers that belong to A for the first time. Any policy has to choose some A as it has to transmit at least one more time in order to terminate when $z < Z$. Thus the optimal cost is the minimum over all $A \in \mathcal{P}(z)$. ■

Lemma 4:

$$J^*(k,z,t) = x_t(\mathcal{T}_{k,z}) + \sum_{u\in\mathcal{T}_{k,z}} r_{t,u}(\mathcal{T}_{k,z}) J_T^*(k,z,u). \quad (15)$$

Proof: Since, the optimal policy obtained by solving the Bellman's equation transmits when a state in $\mathcal{T}_{k,z}$ is reached for the first time, the result follows. ∎

Lemma 5: Let $m_{k,z}$ denote the smallest member of $\mathcal{T}_{k,z}$. The smallest member $m_{k,z}$ is greater than 0 for every $k \in \{0,\ldots,K-1\}$ and $z \in \{0,\ldots,G\}$.

Proof: The proof is by contradiction. Suppose, there exists k and z such that $0 \in \mathcal{T}_{k,z}$. Note that

$$\begin{aligned} J_T^*(k,z,0) &= E[V+X] + 1 + (1-\alpha_0(z))J^*(k+1,z,0) \\ &\quad +\alpha_0(z)J^*(k+1,z,1) \quad \text{and} \\ J_B^*(k,z,0) &= E[X] + 1 + (1-\alpha_0(z))J^*(k,z,0) \\ &\quad +\alpha_0(z)J^*(k,z,1). \end{aligned}$$

Now we argue that

$$J^*(k+1,z,0) \geq J^*(k,z,0) \text{ and } J^*(k+1,z,1) \geq J^*(k,z,1). \quad (16)$$

Consider the first inequality in (16). Let policy π_1 be the optimal transmission policy from state $(k+1,z,0)$. Consider policy π_2 that takes the same transmission decisions as π_1 from state $(k,z,0)$. Note that π_1 terminates. Hence, it delivers the packet to atleast $Z-z$ unsatisfied receivers in at most $K-k-1$ transmissions. Since, π_2 makes the same transmission decisions, it will also deliver the packet to at least $Z-z$ unsatisfied receivers, and thus π_2 terminates aswell. Also, the number of total transmissions under π_2 equals the number of transmissions utilized under π_1 minus 1 which is less than K. Thus, π_2 terminates from $(k,z,0)$ in the same time as the optimal policy π_1 from state $(k+1,z,0)$. Since optimal termination time from $(k,z,0)$ is less than or equal to that achieved by π_2, the first inequality in (16) follows. Using similar arguements we can show other inequality in (16).

Now, since $E[V]$ is greater than 0 the result follows. ∎

A. *Proof of Theorem 1*

Proof: We consider two cases, namely, $k \geq 1$ and $k = 0$. Let us first consider $k \geq 1$. Let z receivers be satisfied after k transmissions. Then the initial state states observed by the sender are $(k,z,0)$ w.p. $1-\alpha_0(z)$ and $(k,z,1)$ w.p. $\alpha_0(z)$. Since, $m_{k,z} \geq 1$ (by Lemma 5) and the aggregate readiness process of $G-z$ receivers is a BD process, the optimal policy will transmit for $k+1^{th}$ time when $m_{k,z}$ unsasfied receivers are ready. Thus, we need to show that $\tau(k,z)$ obtained by (**C1**) in Figure 6 is equal to $m_{k,z}$. We show using the induction on k that (*i*) $\tau(k,z) = m_{k,z}(z)$ and (*ii*) the termination costs obtained in (**C2**) and (**C3**) in Figure 6 are the optimal termination costs.

First, we note that if $k = K-1$, then $m_{k,z} = Z-z$. Thus, the optimal policy will transmit when $Z-z$ unsatisfied receivers are ready. Thus, statements (*i*) and (*ii*) hold for $k = K-1$. By induction hypothesis, we assume that (*i*) and (*ii*) hold for all $k \in \{K-1,\ldots,\widehat{k}+1\}$.

Now we prove (*i*) and (*ii*) for $k = \widehat{k}$. Note that by Lemma 4, for $t \in \{0,1\}$

$$J^*(\widehat{k},z,t) = x_t(\mathcal{T}_{k,z}) + \sum_{u\in\mathcal{T}_{k,z}} r_{t,u}(\mathcal{T}_{k,z}) J_T^*(\widehat{k},z,u). \quad (17)$$

But since the aggregate readiness process of $G-z$ receivers is a BD process and $t \leq m_{k,z}$,

$$\begin{aligned} x_t(\mathcal{T}_{k,z}) &= s_{1,m_{k,z}}(z) \quad \text{and} \\ r_{t,u}(\mathcal{T}_{k,z}) &= 1 \quad \text{for } u = m_{k,z} \\ &= 0 \quad \text{o.w..} \end{aligned}$$

$$\begin{aligned} \text{Thus, } J^*(\widehat{k},z,t) &= s_{t,m_{\widehat{k},z}}(z) + J_T^*(\widehat{k},z,m_{\widehat{k},z}) \\ &\geq \min_{u\geq 1}\left\{s_{t,u}(z) + J_T^*(\widehat{k},z,u)\right\}. \end{aligned}$$

Since $J^*(\widehat{k},z,t)$ is the optimal temination time.

$$J^*(\widehat{k},z,t) = \min_{u\geq 1}\left\{s_{t,u}(z) + J_T^*(\widehat{k},z,u)\right\}.$$

Thus, (*i*) and (*ii*) hold for $\widehat{k}$. Hence, (*i*) and (*ii*) are true for $k \geq 1$ by induction.

Now we consider a case $k = 0$. Here, we aim to show that (**C4**) and (**C5**) in Figure 6 obtain optimal transmission states. Unlike $k \geq 1$ case, here the initial state observed by the sender is $(0,0,t)$, where t can be any integer from 0 to G. Recall that for $k \geq 1$, the system state was (k,z,t) with $t \in \{0,1\}$. We consider two cases. First, $m_{0,0} \geq t$ or $t \in \mathcal{T}_{0,0}$. Second, $m_{0,0} < t$ and $t \notin \mathcal{T}_{0,0}$. In the first case, by the similar arguements as in $k \geq 1$ case, it follows that

$$\begin{aligned} J^*(0,0,t) &= \min_{u\geq t}\{s_{t,u}(z) + J_T^*(0,0,u)\} \\ &= J_1(0,0,t) \text{ as defined in Figure 6.} \end{aligned}$$

Now, in the second case, we define $m_{0,0}^1$ and $m_{0,0}^2$ as follows.

$$\begin{aligned} m_{0,0}^1 &= \max_{u<t}\{u \,:\, u \in \mathcal{T}_{0,0}\} \\ m_{0,0}^2 &= \min_{u>t}\{u \,:\, u \in \mathcal{T}_{0,0}\}. \end{aligned}$$

Note that $m_{0,0}^1$ ($m_{0,0}^2$) is the largest (smallest) element of $\mathcal{T}_{0,0}$ that is smaller (greater) than t. Since the aggregate readiness process is a BD process, and the optimal policy transmits when it reaches a state in $\mathcal{T}_{0,0}$, we conclude that the optimal policy will transmit in $m_{0,0}^1$ or $m_{0,0}^2$. Thus, by Lemma 4,

$$\begin{aligned} J^*(0,0,t) &= s_{t,m_{0,0}^1||m_{0,0}^2}(0) + \sum_{u=1}^{2} r_{t,m_{0,0}^u} J_T^*(0,0,m_{0,0}^u) \\ &\geq \min_{\substack{v_1<t \\ v_2>t}}\left\{s_{t,v_1||v_2}(0) + \sum_{u=1}^{2} r_{t,v_u} J_T^*(0,0,v_u)\right\} \\ &= J_2(0,0,t) \text{ as defined in Figure 6.} \end{aligned}$$

But since $J^*(0,0,t)$ is the optimal termination time, we conclude that $J^*(0,0,t) = J_2(0,0,t)$.

Now, we note that for t such that case 1 is true, $J_1(0,0,t) \leq J_2(0,0,t)$ as $J_1(0,0,t)$ is the optimal termination time. Likewise, for t such that case 2 is true, $J_1(0,0,t) \geq J_2(0,0,t)$. Thus, (**C4**) and (**C5**) in Figure 6 obtain optimal transmission states. ■

B. Proof for Theorem 2

Proof: We use value iteration approach decribed in Section III to prove the result. The proof is by induction on number of iterations. For each iteration we will prove that

1) if $J_{l,T}(k,z,t) \leq J_{l,B}(k,z,t)$, then $J_{l,T}(k,z,t+1) \leq J_{l,B}(k,z,t+1)$. This shows that the threshold based decision is optimal.
2) $J_l(k,z+1,t) \leq J_l(k,z,t)$.
3) $J_l(k,z+1,t-1) \leq J_l(k,z,t)$.

We intialize $J_{0,T}(k,z,t)$ and $J_{0,T}(k,z,t)$ to 0 for every state (k,z,t). Hence 1,2 and 3 hold vaccuously for $l=0$. By induction hypothesis we assume that 1,2, and 3 hold till l^{th} iteration. As a final step, we prove these three statements for $l+1^{th}$ iteration. Let $\overline{V} = V + X$.

Proof of statement 1): Let $p_i(z)$ be the probability that i out of $G-z$ unsatisfied receivers are ready. We observe from equation (6) that

$$\begin{aligned} J_{l+1,B}(k,z,t) &= \mathbf{E}[X] + 1 + \sum_{i=0}^{G-z} p_i(z) J_l(k,z,i) \\ &= J_{l+1,B}(k,z,t+1), \end{aligned} \tag{18}$$

If the policy decides to transmit from state $(k,z,t+1)$, then the resulting system state is $(k+1,z+t+1,i)$ with probability $p_i(z+t+1)$.

Hence from (5), $J_{l+1,T}(k,z,t+1)$

$$= \mathbf{E}[\overline{V}] + 1 + \sum_{i=0}^{G-z-t-1} p_i(z+t+1) J_l(k+1,z+t+1,i) \tag{19}$$

Now, $J_{l+1,T}(k,z,t)$

$$\begin{aligned} &= \mathbf{E}[\overline{V}] + 1 + \sum_{i=0}^{G-z-t} p_i(z+t) J_l(k+1,z+t,i) \\ &= \mathbf{E}[\overline{V}] + 1 + \sum_{i=0}^{G-z-t-1} p_i(z+t+1) p J_l(k+1,z+t,i+1) \\ &\quad + \sum_{i=0}^{G-z-t-1} p_i(z+t+1)(1-p) J_l(k+1,z+t,i) \end{aligned} \tag{20}$$

(since the receiver readniess states are iid).

Now we compare each term in the summation of equations (19) and (20) separately. By induction hypothesis, we note that $J_l(k+1,z+t+1,i) \leq J_l(k+1,z+t,i+1)$ and $J_l(k+1,z+t+1,i) \leq J_l(k+1,z+t,i)$. Hence, $J_l(k+1,z+t+1,i) \leq pJ_l(k+1,z+t,i+1) + (1-p)J_l(k+1,z+t,i)$. This proves statement 1) for iteration $l+1$.

Proof of statement 2): We need to prove that $J_{l+1}(k,z+1,t) \leq J_{l+1}(k,z,t)$. To prove the desired result we will show that $J_{l+1,B}(k,z+1,t) \leq J_{l+1,B}(k,z,t)$ and $J_{l+1,T}(k,z+1,t) \leq J_{l+1,T}(k,z,t)$. Now,

$$\begin{aligned} &J_{l+1,B}(k,z+1,t) \\ &\quad = \mathbf{E}[X] + 1 + \sum_{i=0}^{G-z-1} p_i(z+1) J_l(k,z+1,i). \end{aligned} \tag{21}$$

Furthermore, as discussed in the proof of statement 1),

$$\begin{aligned} &J_{l+1,B}(k,z,t) \\ &\quad = \mathbf{E}[X] + 1 + \sum_{i=0}^{G-z-1} pp_i(z+1) J_l(k,z,i+1) \\ &\quad\quad + \sum_{i=0}^{G-z-1} (1-p) p_i(z+1) J_l(k,z,i). \end{aligned} \tag{22}$$

Again comparing individual terms in the summation of equations (21) and (22) and using the induction hypothesis as in the proof of statement 1) to obtain the desired result that $J_{l+1,B}(k,z+1,t) \leq J_{l+1,B}(k,z,t)$.

Now, we consider

$$\begin{aligned} &J_{l+1,T}(k,z+1,t) \\ &\quad = \mathbf{E}[\overline{V}] + 1 + \sum_{i=0}^{G-z-t-1} p_i(z+t+1) J_l(k+1,z+t+1,i) \\ &\quad = J_{l+1,T}(k,z,t+1). \end{aligned}$$

Hence, using statement 1), we conclude that $J_{l+1,T}(k,z+1,t) \leq J_{l+1,T}(k,z,t)$.

Thus we obtained the desired result $J_{l+1}(k,z+1,t) \leq J_{l+1}(k,z,t)$.

Proof of statement 3): We need to show that $J_{l+1}(k,z+1,t-1) \leq J_{l+1}(k,z,t)$. To show the desired result, we will show that $J_{l+1,B}(k,z+1,t-1) \leq J_{l+1,B}(k,z,t)$ and $J_{l+1,T}(k,z+1,t-1) \leq J_{l+1,T}(k,z,t)$.

We observe that

$$\begin{aligned} &J_{l+1,B}(k,z+1,t-1) \\ &\quad = \mathbf{E}[X] + 1 + \sum_{i=0}^{G-z-1} p_i(z+1) J_l(k,z+1,i) \\ &\quad = J_{l+1,B}(k,z+1,t). \end{aligned}$$

In the proof of statement 2), we have shown that $J_{l+1}(k,z+1,t) \leq J_{l+1,B}(k,z,t)$. Hence, $J_{l+1,B}(k,z+1,t-1) \leq J_{l+1}(k,z,t)$.

Furthermore, $J_{l+1,T}(k,z+1,t-1) = J_{l+1,T}(k,z,t)$. This is because after transmission, system makes transition from both the states to the same state with equal probability.

Thus, statement 3) is proved for $l+1^{th}$ iteration. ■