Machine Assistance for Programming Language Research

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Changes in PL Research

- Increasing complexity of language features being considered
  - E.g., module systems, dependently typed programming, types ensuring resource bounds or deadlock freedom, …
- Increasing concern for scale and integration
- Decreasing cycle time for tech transfer
  - Extensions to production languages
    - e.g., generics in Java/C#
  - Adoption of research languages
    - e.g. (OCaml, F#, Haskell, Scheme)

→ Big changes in the way *proofs* are communicated…

The State of the Art

Chen and Tarditi,
A Simple Typed Intermediate Language for Object-Oriented Languages,
Principles of Programming Languages (POPL), 2005

We have proved the soundness of LIL_C, in the style of [34], and the decidability of type checking. Full proofs are in the technical report.

THEOREM 1 (PRESERVATION). *If $\Sigma \vdash P : \tau$ and $P \mapsto P'$, then $\exists \Sigma'$ such that $\Sigma' \vdash P' : \tau$.*

THEOREM 2 (PROGRESS). *If $\Sigma \vdash P : \tau$, then either the main expression in P is a value, or $\exists P'$ such that $P \mapsto P'$.*

Proof sketch: by standard induction over the typing rules.

3. *If $\Theta;\bullet;\Sigma;\Gamma \vdash v : \forall tvs(\tau_1,\ldots,\tau_n) \to \tau$ ($tvs = \alpha_1 \ll u_1,\ldots,\alpha_m \ll u_m$) and $\Theta \vdash H : \Sigma$, then v is a label and $H(v) = fix\ g\langle tvs'\rangle(x_1 : \tau_1',\ldots,x_n : \tau_n') : \tau' = e_m$ and $tvs' = \alpha_1 \ll u_1',\ldots,\alpha_m \ll u_m'$.*

4. *If $\Theta;\bullet;\Sigma;\Gamma \vdash v : \exists\alpha \ll \tau_u.\ \tau$, then $v = pack\ \tau_0$ as $\alpha \ll \tau_u'$ in $(v' : \tau')$.*

5. *If $\Theta;\bullet;\Sigma;\Gamma \vdash v : Tag(\tau)$, then $v = tag(C)$ for some C.*

6. *If $\Theta;\bullet;\Sigma;\Gamma \vdash v : Tag(C)$, then $v = tag(C)$.*

7. *If $\Theta;\bullet;\Sigma;\Gamma \vdash v : C$, then $v = C(v')$ for some value v'.*

8. *If $\Theta;\bullet;\Sigma;\Gamma \vdash v : int$, then v is an integer.*

Proof: by inspection on expression typing rules, heap value rules and subtyping inversion Lemma 41. □

Theorem 57 *If $freetvs(\Sigma) = \emptyset$, and $\Theta \vdash H : \Sigma$, and $\Theta;\Sigma \vdash V : \Gamma$ and $\Theta;\bullet;\Sigma;\Gamma \vdash E : T$, then either E is a value, or E can evaluate one step, that is, $\exists H'$, V' and E' such that $(H;V;E) \mapsto (H';V';E')$.*

Proof: by induction on expression typing rules.

Case int, **Case label**, **Case tag**: all the expressions are values already.

Case var: $E = x$

By $\Theta;\Sigma \vdash V : \Gamma$, domain$(\Gamma)$ = domain(V). From $x \in$ domain(Γ), we know $x \in$ domain(V). Let $H' = H$, $V' = V$ and $E' = V(x)$. By **ev_var** $(H;V;E) \mapsto (H';V';E')$.

Case error: $E = \mathrm{error}[\tau]$: by **ev_error**, E steps to itself.

Case object: $E = C(e)$ with subderivation $\Theta;\bullet;\Sigma;\Gamma \vdash e : R(C)$.

By induction hypothesis, either e is a value or $\exists H'$, V' and e' such that $(H;V;e) \mapsto (H';V';e')$. If e is a value, then E is a value. Otherwise, let $E' = C(e')$. By the congruence rule, $(H;V;E) \mapsto (H';V';E')$.

Case c2r_c: $E = \mathrm{c2r}(e)$ with subderivation $\Theta;\bullet;\Sigma;\Gamma \vdash e : C$.

By induction hypothesis, either e is a value or $\exists H'$, V' and e' such that $(H;V;e) \mapsto (H';V';e')$. If e is a value, then by Lemma 56 $e = C(v)$. Let $E' = v$. By **ev_c2r** $(H;V;E) \mapsto (H';V';E')$. Otherwise let $E' = \mathrm{c2r}(e')$. By the congruence rule $(H;V;E) \mapsto (H';V';E')$.

Case c2r_tv: not applicable because by Lemma 17, the subderivation $\Theta;\bullet;\Sigma;\Gamma \vdash e : \alpha$ is invalid.

Case record: $E = \mathrm{new}[\tau]\{l_1 = e_1,\ldots,l_n = e_n\}$ with subderivations $\Theta;\bullet;\Sigma;\Gamma \vdash e_i : \tau_i\ \forall 1 \le i \le n$.

By induction hypothesis, each subexpression e_i either is a value or can evaluate one more step.

If all e_i are values, then let $H' = H, \ell \leadsto \{l_1 = e_1,\ldots,l_n = e_n\}$ (ℓ is a fresh label), $V' = V$ and $E' = \ell$. By **ev_record** $(H;V;E) \mapsto (H';V';E')$.

If $\exists e_i$ such that $e_1,\ldots,e_{i-1}$ are values and $\exists H',V',e_i'$ such that $(H;V;e_i) \mapsto (H';V';e_i')$ … $\mathrm{new}[\tau]\{l_1 = e_1,\ldots,l_i = e_i',\ldots,l_n = e_n\}$. By the congruence rule, $(H;V;E) \mapsto (H';V';E')$.

Case field: $E = e.l_i$ with subderivation $\Theta;\bullet;\Sigma;\Gamma \vdash e : \{l_1^{\phi_1} : \tau_1,\ldots,l_n^{\phi_n} : \tau_n\}$ and $1 \le i \le n$

By induction hypothesis, either e is a value or e can evaluate one step.

If e is a value, by canonical form Lemma 56, e is a label and $H(e) = \{l_1 = v_1,\ldots,l_n = \ldots$ $H' = H$, $V' = V$ and $E' = v_i$. By **ev_field** $(H;V;E) \mapsto (H';V';E')$.

If $\exists H',V',e'$ such that $(H;V;e) \mapsto (H';V';e')$, then let $E' = e'.l_i$ and by the congruence rule … $(H';V';E')$.

In the rest cases, we state only which evaluation rule to apply, but omit the new H', V' or …

Case assignR $E = e_1.l_i := e_2$ in e_3 with subderivations $\Theta;\bullet;\Sigma;\Gamma \vdash e_1 : \{l_1^{\phi_1} : \tau_1,\ldots,l_i^M \ldots$ $\}$ and $\Theta;\bullet;\Sigma;\Gamma \vdash e_2 : \tau_i$.

By induction hypothesis, either e_1 is a value or e_1 can evaluate one step. Similarly, either … e_2 can evaluate one step.

If both e_1 and e_2 are values, then by canonical form Lemma 56 e_1 is a label and $H(e_1) = \{l_1 = v_1,\ldots,l_n = \ldots$ $\ldots\}$, and by **ev_assignR** E can evaluate one step.

If either e_1 or e_2 can evaluate one step, by the congruence rule, E can evaluate one step.

Case array $E = \mathrm{new}[e_0,\ldots,e_{n-1}]^\tau$ with subderivations $\Theta;\bullet;\Sigma;\Gamma \vdash e_i : \tau\ \forall 0 \le i \le n-1$

If e_{t1} or e_{t2} can evaluate one step, then by the congruence rule E can evaluate one step.

Case ifTag_tv: $E = \mathrm{ifEqTag}^\tau(e_{t1}, e_{t2})$ then e_1 else e_2 with subderivation $\Theta;\bullet;\Sigma;\Gamma \vdash e_{t1} : \mathrm{Tag}(\gamma)$. Not applicable because the subderivation is invalid by Lemma 17.

Case sub: $E = e$ with subderivation $\Theta;\bullet;\Sigma;\Gamma \vdash e : \tau_1$.

By induction hypothesis, either e is a value or e can evaluate one step. That is, either E is a value or E can evaluate one step. □

Corollary 58 (progress) *If $\Sigma \vdash P : \tau$, then either the main expression in P is a value, or $\exists P'$ such that $P \mapsto P'$.*

Theorem 59 *LIL_C is Sound. Well-typed LIL_C programs do not get stuck.*

Proof: by progress and preservation. □

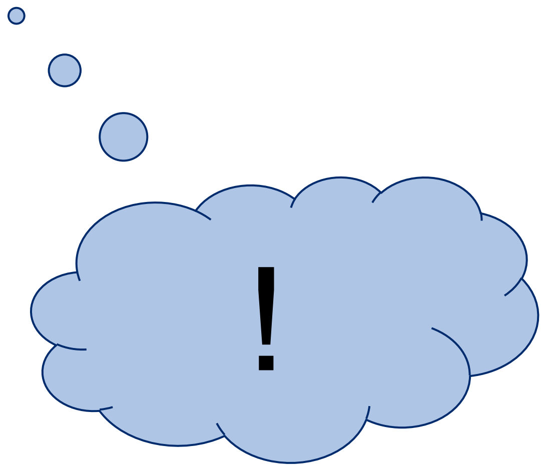
Theorem 48 *Type checking of LIL_C is decidable.*

Proof: To decide whether $\Theta;\Delta;\Sigma;\Gamma \vdash e : \tau$ holds, we can first get the minimal type τ_m of e such that $\Theta;\Delta;\Sigma;\Gamma \vdash e : \tau_m$, then test whether $\Theta;\Delta \vdash \tau_m \le \tau$. Because both minimal typing and subtyping are decidable, type checking is decidable. □

E Soundness

We’ve Got a Problem Here

- As a practical matter, a large fraction of the proofs being done about programming languages today are not possible for humans to check.



Are Proof Checkers the Answer?

- Proof checking (or "proof assistant") technology has made amazing strides in recent years
  - Several mature systems
    - E.g., Coq, HOL, Isabelle, Twelf, MetaPRL, etc., etc.
  - Some very impressive achievements in several domains

A Marriage Made in Heaven

So we've got…

A. A community with a burning need

B. A community with a great technology

Can we put them together?

Lots of Work Already Underway

- Leroy's verified C compiler
- Nipkow et al's formalization of a large part of Java
- Appel et al's Foundational Proof-Carrying Code project
- Crary et al's machine-checked development of a typed assembly language
- Harper et al's formalization of Standard ML
- Sewell et al's formalization of TCP/IP
- Etc., etc.

But We're Not There Yet!

Challenges:

- Current achievements are mostly heroic efforts by heroic individuals and small teams
- Many different proof assistants and diverse technical machinery
- Lots of black magic; high cost of entry; little sharing of knowledge across projects
- Some significant unresolved technical issues
  - Lightweight methods for reasoning about variable binding
  - Scalability and modularity of proofs

What's needed is some synergy…

Vision

A world where *every* PL paper is accompanied by a machine-checked appendix

- Plan:
  - Identify current best practices
  - Gather community consensus around them
  - Build additional tools and other infrastructure as needed
  - Address technical challenges specific to programming language research

First step…

The PoplMark Challenge

- A set of benchmark problems to help evaluate progress in the area
  - Based around metatheory of F<:, a typed lambda-calculus with polymorphism and subtyping
  - Presented at TPHOLs 2005
- Has generated tremendous interest in both PL and theorem proving communities
  - *many* solutions submitted
  - 6 different proof assistants
  - 7 different treatments of binding
- Much has been learned
  - JAR special issue CFP now open

Community Development

- Wiki & Mailing list for POPLmark challenge
  - Gathering place for news/solutions/discussion/advice
- Workshop on Mechanizing Metatheory
  - 4th Wksp, 4 Sep 2009, Edinburgh
- *Using Proof Assistants for Programming Language Research*
  or, How to write your next POPL paper in Coq
  - Tutorial for novices
  - POPL tutorial, January, 2008
  - Oregon Summer School, June 2008

Engineering Formal Metatheory

Resolving technical issues

- *Engineering Formal Metatheory*
  - Brian Aydemir, Arthur Charguéraud, Benjamin Pierce, Randy Pollack, and Stephanie Weirich
  - POPL 2008
- Describes a lightweight first-order methodology for representing binding and specifying induction principles
- Two essential components:
  - Locally nameless representation
  - Cofinite name quantification

What is so hard about binding?

- Alpha-equivalence
  - Identify "λx.x" and "λy.y"
- Barendregt convention
  - Assumption that bound variables are "sufficiently fresh"

```
Inductive exp : Set :=
 | var : atom -> exp
 | abs : atom -> exp -> exp
 | app : exp -> exp -> exp.
```

Alpha-equivalence

- Important when we need to compare terms with binding structure:
  - Type system of polymorphic language
  - Confluence for pure lambda calculus

- Formalism simpler if alpha-equals is "="

```
Lemma preservation: forall G e t,
  typing G e t -> red e e' ->
    exists t', alpha_eq t t' /\
    typing G e' t'.

Lemma preservation: forall G e t,
   typing G e t -> red e e' -> typing G e' t.
```

Barendregt Convention

```
Fixpoint subst (x:atom) (u:exp) (e:exp)
  {struct e} :=
match e with
 | var y      => if x == y then u else e
 | app t1 t2 => app (subst x u t1)
                    (subst x u t2)
 | abs z t    => abs z (subst x u t)
end.
```

- What if z == x?
- What if z in free variables of u?

Existing approaches

- No completely satisfactory mechanism for binding
  - **Name representation**: must explicitly rename terms, define alpha-equivalence
  - **de Bruijn indices**: difficult to work with as must shift indices
  - **Nominal logic**: only available in Isabelle/HOL
  - **HOAS**: exotic terms, specialized logic

Locally nameless representation

Names for free variables

de Bruijn indices for bound variables

Example: Untyped lambda calculus terms

$$t ::= \mathrm{bvar}\ i \mid \mathrm{fvar}\ x \mid \mathrm{app}\ t_1\ t_2 \mid \mathrm{abs}\ t$$

$\lambda x.\ \lambda y.\ (x\ y)\ z$ and $\lambda w.\ \lambda v.\ (w\ v)\ z$

represented as

abs (abs (app (app (bvar 1) (bvar 0)) (fvar "z"))

```
Inductive exp : Set :=
 | bvar : nat -> exp
 | fvar : atom -> exp
 | abs  : exp -> exp
 | app  : exp -> exp -> exp.
```

Basic operations

- Variable opening

  t^u replace bound index 0 with exp u

- Free variable calculation

  $\mathrm{FV}\ t$ finite set of free atoms in t

- Substitution

  $[x \mapsto u]t$ replace free variable x with u

- All operations have *simple* definitions.

Variable opening

```
Fixpoint open_rec (k : nat) (f : exp)
 (e : exp)  {struct e} : exp :=
  match e with
  | bvar i    => if k === i then f else e
  | fvar x    => fvar x
  | abs e1  =>
     abs (open_rec (1 + k) f e1)
  | app e1 e2 => app (open_rec k f e1)
                     (open_rec k f e2)
end.

Notation "e ^ u" := (open_rec 0 u e).
```

Free variable substitution

```
Fixpoint subst (z : atom) (u : exp)
  (e : exp) {struct e} : exp :=
  match e with
  | bvar i    => bvar i
  | fvar x    => if x == z then u else e
  | abs T e1  => abs T (subst z u e1)
  | app e1 e2 => app
      (subst z u e1) (subst z u e2)
end.
```

Free variable calculation

```
Fixpoint fv (e : exp)
 {struct e} : atoms :=
  match e with
   | bvar i   => {}
   | fvar x   => singleton x
   | abs e1   => (fv e1)
   | e1 e2    => (fv e1)
                  `union` (fv e2)
```

Local closure

- Not all members of type term are lambda calculus terms
  - abs (bvar 3)?
- Predicate lc picks out members datatype that are *locally-closed*
- Definitions respect local closure
  - *If* $lc\ u$ *and* $lc\ t$, *then* $lc\ ([x \mapsto u]t)$
  - *If* $t \rightarrow t'$ *then* $lc\ t$ *and* $lc\ t'$

Managing local closure

- Many theorems need not refer to local closure
  - *If* $t \to t'$ *and* $t \to t''$, *then* $t' = t''$
- Some theorems require it, tactics discharge assumptions
  - *If* $x \neq y$ *and* $lc\ u$, *then* $[x \mapsto u](t^{y}) = ([x \mapsto u]t)^{y}$

Induction principles

Definition of typing rules generates an induction principle for typing derivations

$$\frac{ok\ E \qquad (x{:}T) \in E}{E \vdash \mathrm{fvar}\ x : T}$$

$$\frac{E \vdash t_1 : S \to T \qquad E \vdash t_2 : S}{E \vdash \mathrm{app}\ t_1\ t_2 : T}$$

$$\frac{x \notin \mathrm{FV}\ t \cup dom(E) \qquad E,x{:}S \vdash t^x : T}{E \vdash \mathrm{abs}\ t : S \to T}$$

Exists vs. Cofinite Quantification

Induction hypothesis holds for some particular, unknown x

$$\frac{x \notin \mathrm{FV}\ t \cup dom(E) \qquad E,x{:}S \vdash t^x : T}{E \vdash \mathrm{abs}\ t : S \to T}$$

Induction hypothesis holds for all but some finite set of variables.

$$\frac{\forall x \notin L \qquad E,x{:}S \vdash t^x : T}{E \vdash \mathrm{abs}\ t : S \to T}$$

Weakening Lemma

If $E, G \vdash t : T$ and $ok\ (E\ ,\ F\ ,\ G)$
then $E, F, G \vdash t : T$

Proof (?):

by induction on $E,G \vdash t : T$

Abstraction case:

$$\frac{x \notin \mathrm{FV}\ t \cup dom(E,G) \qquad E,G,x{:}S \vdash t^x : T}{E,G \vdash \text{abs}\ t : S \to T}$$

WTP $E,F,G \vdash \text{abs}\ t : S \to T$

By typing rule, holds if $E,F,G,x{:}S \vdash t^x : T$
and $x \notin \mathrm{FV}\ t \cup dom(E,F,G)$

Weakening Lemma

If $E, G \vdash t : T$ and $ok\ (E\ ,\ F\ ,\ G)$
then $E, F, G \vdash t : T$

Proof: by induction on $E,G \vdash t : T$

abstraction case: $$\frac{\forall x \notin L \quad E,G,x{:}S \vdash t^x : T}{E,G \vdash \text{abs } t : S \to T}$$

WTP: $E,F,G \vdash \text{abs } t : S \to T$

By rule, holds if exists a set L', such that
$\forall x \notin L',\ \ E,F,G,x{:}S \vdash t^x : T$

IH: $\forall x \notin L,\ ok\ (E,F,G,\ x{:}S) \Rightarrow E,F,G,x{:}S \vdash t^x : T$

Choose $L' = dom\ (E,F,G) \cup L$

By definition, $\forall x \notin L',\ \ ok\ (E,F,G,x{:}S)$

Renaming lemma

- Similar reasoning proves substitution lemma
  If $E, x{:}S, F \vdash t : T$ and $E \vdash u : S$
  then $E,F \vdash [z \mapsto u]t : T$

- Important corollary of substitution and weakening

  Renaming:

  If $x \notin \mathrm{FV}\ t \cup dom\ E$ and $E, x{:}S \vdash t^x : T$
  then for all $y \notin \mathrm{FV}\ t \cup dom\ E,\quad E, y{:}S \vdash t^y{:}T$

Corollaries of renaming

- Renaming lemma gives strongest intro form:

  If exists x, s.t. $x \notin \mathrm{FV}\ t \cup dom\ E$ and
  $E, x{:}S \vdash t^x : T$ then $E \vdash \mathrm{abs}\ t : S \rightarrow T$

- And strongest inversion principle:

  If $E \vdash \mathrm{abs}\ t{:}\ S \rightarrow T$ then

  for all $x \notin \mathrm{FV}\ t \cup dom\ E,\ \ E, x : S \vdash t^x : T$

Equivalence of systems

- Renaming lemma also shows equivalence of two systems:

$E \vdash t : T$ with exists-fresh rule for abs
if and only if
$E \vdash t : T$ with cofinite rule for abs

- We are proving properties about the language we actually care about!

General Form of Development

- Def. of language syntax
- *Def. of variable opening and local closure*
- Def. of free variable substitution
- Def. of free variable function
- *Interaction lemmas*
- Def. of semantic relations *using cofinite quantification for binders*
- *Show semantic relations respect local closure*
- Substitution and weakening lemmas for each relation w/ binding
- Preservation and Progress
- *Derive renaming lemmas*

Code Distribution

- Reference *examples* & supporting experience
  - multiple calculi (STLC, F<:, CoC)
  - multiple theorems (type soundness, confluence)
- A *library* that supports this methodology
  - atoms, finite sets
  - reasoning about freshness
  - representing environments

Let's look closer at POPLmark…

Fsub_Definitions.v

syntax of types and terms, defn. of typing relation…

"The TCB"

Fsub_Lemmas.v

facts about well-formedness, environments…

"Things we should have proved on paper but didn't"

Fsub_Infrastructure.v

facts about syntax, substitution…

"The overhead"

Fsub_Soundness.v

Substitution lemma, preservation, progress…

"The meat"

Fsub_Definitions.v

syntax

open

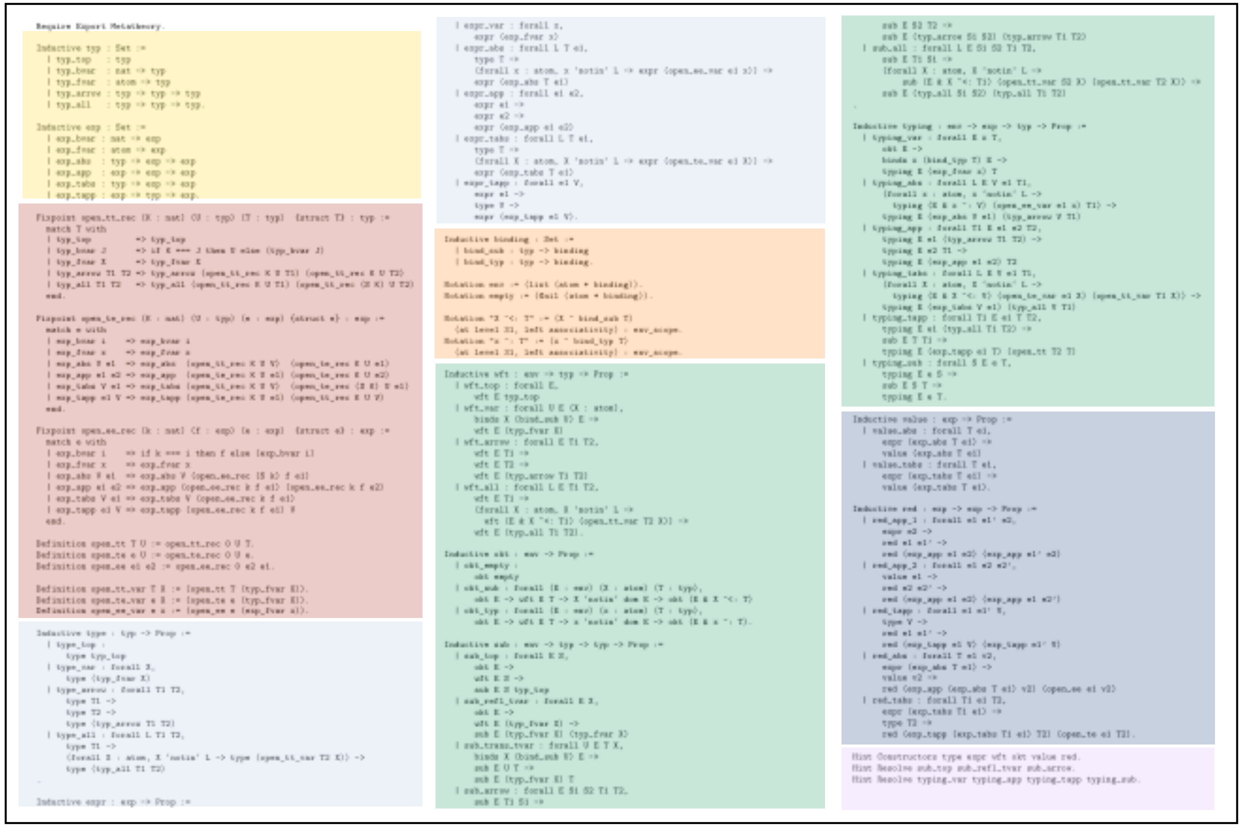
local closure

environments

typing & subtyping

operational semantics

automation hints



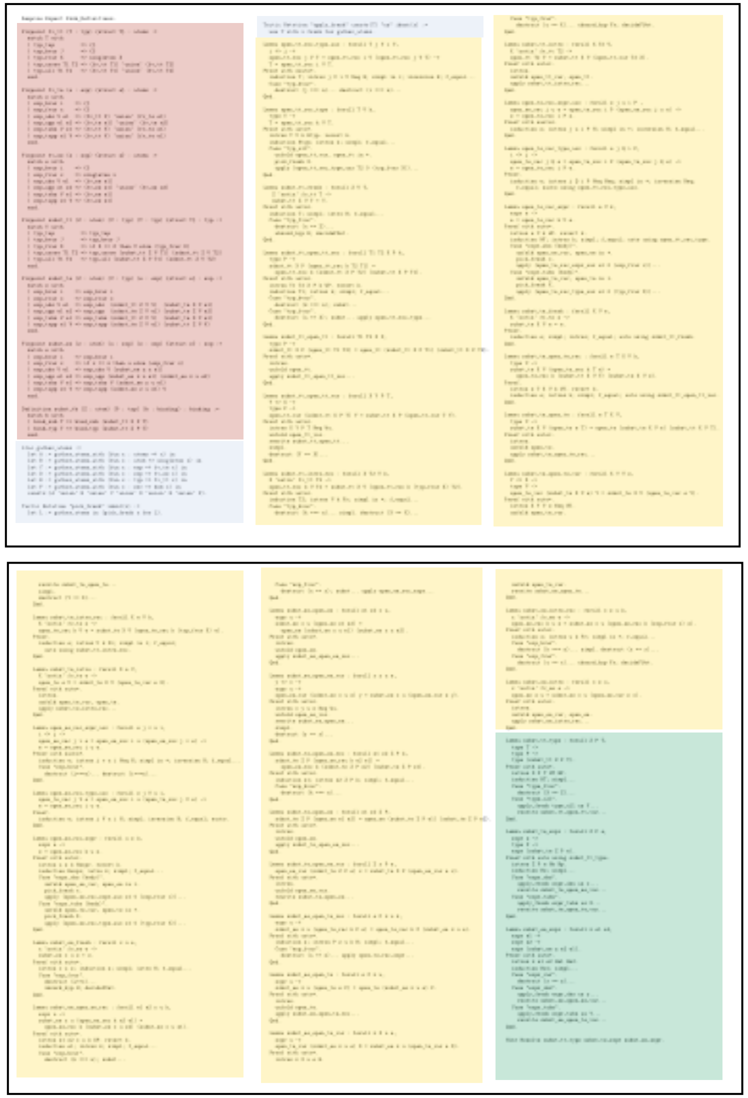
Fsub_Infrastructure.v

Basic operations (fv and subst)

Tactic to choose fresh atoms

Properties of open, fv and subst

Properties of subst and local closure



Fsub_Lemmas.v

type well-formedness

environment well-formedness

substitution in environments

regularity of relations

hints about local closure

Fsub_Soundness.v

Proofs of lemmas from the Appendix of POPLmark paper

Comparison POPLmark 1A

Author	Binding	Lemmas	Proof steps
Vouillon	de Bruijn	30	402
Leroy	Locally nameless	49	495
Stump	Levels/names	56	938
Hirschowitz & Maggesi	de Bruijn (nested datatype)	49	1574
Chlipala	Locally nameless	23	75
This work	Locally nameless	22	101

McKinna & Pollack Rule

Exists-fresh: IH holds for some fresh x

$$\frac{x \notin \mathrm{FV}\ t \cup dom(E) \qquad E, x{:}S \vdash t^x : T}{E \vdash \mathrm{abs}\ t : S \to T}$$

Cofinite: IH holds for all but some unknown set

$$\frac{\forall x \notin L \qquad E, x{:}S \vdash t^x : T}{E \vdash \mathrm{abs}\ t : S \to T}$$

Forall-fresh: IH holds for all fresh variables

$$\frac{\forall x \notin \mathrm{FV}\ t \cup dom(E) \qquad E, x{:}S \vdash t^x : T}{E \vdash \mathrm{abs}\ t : S \to T}$$

Forall vs. Cofinite

Define system with forall-fresh rules

Define swapping and show relations are stable under swapping

Show exists-fresh intro

Prove weakening and substitution

Prove type soundness

Define system with cofinite rules

Prove weakening and substitution

Show exists-fresh intro

Prove type soundness

Conclusions

- Can use Coq's standard mechanisms for reasoning (inductive defs, tactics, etc.)
- Swapping does not appear to be essential.
- Seldom need to rename during proofs. IH applies to an infinite # of suitably fresh variables.
- Specialized tactics help
  - local closure obligations
  - fresh variable introduction

Thanks to

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Geoffrey Washburn

Steve Zdancewic

Vouillon / de Bruijn indices

```
Fixpoint subst (t : term) (x : nat) (t' : term)
  {struct t} : term :=
  match t with
  | var y =>
      match lt_eq_lt_dec y x with
      | inleft (left _)  => var y
      | inleft (right _) => t'
      | inright _        => var (y - 1)
      end
  | abs T1 t2  =>
     abs T1 (subst t2 (1 + x) (shift 0 t'))
  | app t1 t2  => app (subst t1 x t')
     (subst t2 x t')
  | tabs T1 t2 => tabs T1
     (subst t2 x (shift_typ 0 t'))
  | tapp t1 T2 => tapp (subst t1 x t') T2
  end.
```