Generative type abstraction and type-level computation
(Wrestling with System FC)

Stephanie Weirich, Steve Zdancewic
University of Pennsylvania
Dimitrios Vytiniotis, Simon Peyton Jones
Microsoft Research, Cambridge

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Type generativity is useful

- **Module implementor:**

  ```haskell
  module MImpl (Tel, ...) 
  ...
  newtype Tel = MkTel String 
  ...
  ```
  
  Inside MImpl:
  
  \[
  \text{Tel} \sim \text{String}
  \]

  We can also lift this equality:
  
  \[
  \text{List Tel} \sim \text{List String}
  \]

- **Module consumer:**

  ```haskell
  module MCons
  import MImpl 
  ...
  f :: Tel -> Tel 
  f x = "0030" ++ x
  ```
  
  Inside MCons:
  
  \[
  \text{Tel} \sim \text{String}
  \]

- Well-explored ideas found in various forms in modern languages [e.g. see papers on ML modules by Harper, Dreyer, Rossberg, Russo, …]
Type-level computation is useful

In the Glasgow Haskell Compiler, type-level computation involves type classes and families:

```haskell
module MImpl (Tel)
...
class LowLevel a where
  type R a
  toLowLevel :: a -> R a

instance LowLevel String where
  type R String = ByteArray
  toLowLevel x = strToByteArray x

instance LowLevel Tel where
  type R Tel = Int64
  toLowLevel x = ...

...
```

R is a “type function”

R String ~ ByteArray

R Tel ~ Int64
But there’s a problem!

module MImpl (Tel, ...)  

newtype Tel = MkTel String  

class LowLevel a where  
  type R a  
  ...  

instance LowLevel String where  
  type R String = ByteArray  
  ...  

instance LowLevel Tel where  
  type R Tel = Int64  
  ...  

In the rest of the module:  
  Tel ~ String  
Hence by lifting  
  R Tel ~ R String  
Hence ...  
  ByteArray ~ Int64
This paper

- Type generativity and type functions are both and simultaneously useful!
- But it’s easy to lose soundness [e.g. see GHC bug trac #1496]
- So, what’s some good solution that combines these features?

**System FC2**

A novel, sound, strongly-typed language with type-level equalities

1. Stages the use of the available equalities, to ensure soundness
2. Distinguishes between “codes” and “types” as in formulations of Type Theory [e.g. see papers by Dybjer] and intensional type analysis [e.g. see papers by Weirich, Crary]
3. Improves GHC’s core language [System FC, Sulzmann et al.]
4. Soundness proof w/o requiring strong normalization of types

This talk. The rest is in the paper.
Recap

<table>
<thead>
<tr>
<th>newtype Tel = MkTel String</th>
<th>-- Tel ~ String</th>
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<tbody>
<tr>
<td>type instance R String = ByteArray</td>
<td>-- R String ~ ByteArray</td>
</tr>
<tr>
<td>type instance R Tel = Int64</td>
<td>-- R Tel ~ Int64</td>
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</table>

R String **MUST NOT BE EQUATED TO** R Tel

(List String) **OK TO BE EQUATED TO** (List Tel)
A non-solution

- So **lifting** is(?) the source of all evil:
  \[
  \Gamma \vdash \tau \sim \sigma \\
  \Rightarrow \\
  \Gamma \vdash T\tau \sim T\sigma
  \]

- Possible solution: disallow lifting if T is a type function

- Seems arbitrary, and restrictive, **and** does not quite work

```haskell
data TR a = MkTR (R a)

to :: ByteArray -> TR String
to x = MkTR x

from :: TR Tel -> Int64
from (MkTR x) = x
```

JUST AS BAD, BECAUSE THEN:

from.to :: ByteArray -> Int64
Type Theory to the Rescue: Roles

- As is common in Type Theory, distinguish between a code (a "name") and a type (a "set of values").

  newtype Tel
    = MkTel String

- Newtype definitions introduce type equality:

  \[ \Gamma \vdash \text{Tel} \sim \text{String} : \ast/	ext{TYPE} \]
  \[ \Gamma \vdash \text{Tel} \sim \text{String} : \ast/	ext{CODE} \]

- A code (such as Tel) can improve upon a type, e.g.
  \[ (\lambda x : \text{Tel}. \text{Var} \text{Tel} t) \]

- Importantly, codes and types have different notions of equality: code-equality and type-equality
Code vs Type Equality

- If $\tau$ and $\sigma$ are equal as codes then they are equal as types:

\[
\frac{\Gamma \vdash \tau \sim \sigma : */\text{CODE}}{\Gamma \vdash \tau \sim \sigma : */\text{TYPE}}
\]

- But two different codes may or may not be equal as types.

\[
\begin{align*}
\text{newtype } \text{Tel} &= \text{MkTel String} \\
\text{newtype } \text{Address} &= \text{MkAddr String}
\end{align*}
\]

\[
\begin{align*}
\Gamma \vdash \text{Tel} \sim \text{Address} : */\text{TYPE} \\
\Gamma \vdash \text{Tel} \not\sim \text{Address} : */\text{CODE}
\end{align*}
\]
Using the FC2 kind system to track roles

- Key idea:
  - Type-level computations dispatch on codes, not types
- Use the kind system of FC2 to track codes

<table>
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<tr>
<th><code>Fω:</code></th>
<th><code>FC2:</code></th>
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<tbody>
<tr>
<td>( \kappa ::= * \mid \kappa \rightarrow \kappa )</td>
<td>( \eta ::= * \mid \kappa \rightarrow \eta )</td>
</tr>
<tr>
<td>( \kappa ::= &lt;\eta/\text{TYPE}&gt; \mid &lt;\eta/\text{CODE}&gt; )</td>
<td></td>
</tr>
</tbody>
</table>

Source `FC2` axioms:

- `type family R a`  
- `type instance R String = ByteArray`
- `type instance R Tel = Int64`

```
R : (<*/CODE> \rightarrow *)/CODE  
R String ~ ByteArray : */CODE  
R Tel ~ Int64 : */CODE
```
Look ma, no special lifting!

- Lifting equalities must simply be kind respecting:

\[(T : \langle */\rho \rangle \Rightarrow *) \in \Gamma\]

\[\Gamma \vdash \tau \sim \sigma : */\rho\]

\[\Gamma \vdash T \tau \sim T \sigma : */\text{TYPE}\]

- Actual rule is more general but the above simplification conveys the intentions!
Why does that fix the problem?

YOUR TAKEAWAY #2

\[ \frac{\Gamma \vdash \pi : \tau}{\pi \vdash \tau \sim \sigma : \tau \sim \Gamma \vdash \sigma : \tau \sim \Gamma} \]

Impossible to derive

R String \sim R Tel : */TYPE

... because R expects a CODE equality!

Tel \sim String : */TYPE
Tel \sim String : */CODE

R : (<*/CODE> \Rightarrow *) \in \Gamma
Lifting over type constructors

\[(T : \langle */\rho \rangle \Rightarrow *) \in \Gamma \]  
\[\Gamma \vdash \tau \sim \sigma : */\rho \]  
\[\Gamma \vdash T\tau \sim T\sigma : */\text{TYPE}\]

Similarly:

\[\text{TR} : (\langle */\text{CODE} \rangle \rightarrow *)\]

Hence:

\[\text{TR Tel} \sim \text{TR String} : */\text{TYPE}\]

BUT:

\[\text{List} : (\langle */\text{TYPE} \rangle \rightarrow *)\]

Hence:

\[\text{List Tel} \sim \text{List String} : */\text{TYPE}\]

data \(\text{TR} \ a = \text{MkTR} \ (\text{R} \ a)\)

data \(\text{List} \ a = \text{Nil} \mid \text{Cons} \ a \ (\text{List} \ a)\)
FC2: The formal setup

- Source program
- Source declarations

- FC2 constants and axioms $\Gamma$

- FC2 program

- Elaborated program contains **explicit types** and **proof terms**
- Easy to typecheck
- By elaborating to a type safe language we establish type soundness of the source and soundness of type inference and soundness of compiler transformations
FC2 typing judgements

- All equalities have explicit proof witnesses. Three judgements:
  \[ \Gamma \vdash e : \tau \]
  \[ \Gamma \vdash \tau : \eta/\rho \]
  \[ \tau ::= a \mid T\bar{\tau} \mid \forall a : \kappa. \tau \mid \tau \sim \sigma \Rightarrow \varphi \]
- Role \( \rho ::= \text{TYPE} \mid \text{CODE} \)
- Coercions \( \gamma \): Equality proof witnesses
  \[ \gamma ::= \text{id}_\tau \mid \text{sym} \gamma \mid c \mid C \mid \gamma_1 ; \gamma_2 \mid T \gamma \mid \text{nth} \ i \ \gamma \]
- Coercion abstractions
- Typing rule that connects typing and coercions in FC2:
  \[ \begin{array}{c}
  \Gamma \vdash e : \tau \\
  \Gamma \vdash \gamma : \tau \sim \sigma : * / \text{TYPE}
  \end{array} \]
  \[ \Gamma \vdash (e \triangleright \gamma) : \sigma \]
Type-soundness via consistency

- Based on progress and subject reduction, using a semantics that “pushes” coercions:

\[
\gamma_1 = nth \ 1 \ \gamma \quad \quad \quad \gamma_0 = nth \ 0 \ \gamma
\]

\[
(\lambda x: \tau. e_1) \triangleright \gamma \quad e_2 \quad \rightarrow \quad (\lambda x: \tau. e_1 \triangleright \gamma_1) (e_2 \triangleright sym \ \gamma_0)
\]

- Progress is proven with the assumption of consistency:

A context \( \Gamma \) is consistent iff whenever \( \Gamma \vdash \gamma : \tau \sim \sigma : \eta /\text{TYPE} \) is derived and \( \tau, \sigma \) are value types, and \( \tau \) is a datatype application (\( T \ \varphi \)) then \( \sigma \) is also the same datatype application (\( T \ \varphi' \))
Establishing consistency

- **Step 1**
  - Define a *role-sensitive* type rewrite relation
  - [Novel idea: don’t require strong normalization of axioms, but require instead more determinism]

- **Step 2**
  - Prove soundness and completeness of the type rewrite relation wrt the coercibility relation

- **Step 3:**
  - Show that rewriting preserves head value constructors

See paper and extended version for the gory details
More interesting details in the paper

- I’ve talked about coercion lifting, but when is coercion decomposition safe? And under which roles?

\[
\Gamma \vdash T \varphi \sim T \psi : * / \text{TYPE} \\
\Gamma \vdash \varphi \sim \psi : ????
\]

- FC2 typing rules are not formulated with only two universes (TYPE / CODE) but allow a semi-lattice of universes – perhaps a nice way to incorporate safely many notions of equality?
Is this all Haskell specific?

No, though no other existing language demonstrates the same problem today so Haskell is a good motivation

But:
- Type generativity via some mechanism is useful
- Type-level computation is independently useful
- GHC happened to arrive at this situation early

Sooner or later, as soon as both these features are in your type system you have to look for a solution
Lots of exciting future directions

- Present a **semantics** that justifies the proof theory of FC2
- Shed more light into **coercion decomposition:**
  - Injectivity of constructors admissible in Fω but not derivable (conj.)
  - Hence in need of semantic justification for the decomposition rules
  - Direction: Extend the kinds of Fω with roles and type functions, and encode equalities as Leibniz equalities. Can this shed any more light? What are the parametric properties of that language?
- **Enrich the universe of codes with term constructors**
- Investigate **other interesting** equalities (e.g. syntactic, β)
  - Can roles help in security and information flow type systems where different equalities may arise from different confidentiality levels?
- **Develop source language technology** to give programmers control over the kinds of their declarations
Thank you for your attention

Questions?