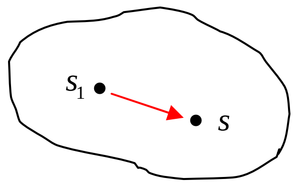
KRIGING WITH ONE PREDICTOR

Suppose we want to predict ε at location, *s*, based only on *one* other location, s_1, so that

(1) $$\hat{\varepsilon}_s = \lambda \cdot \varepsilon_1$$



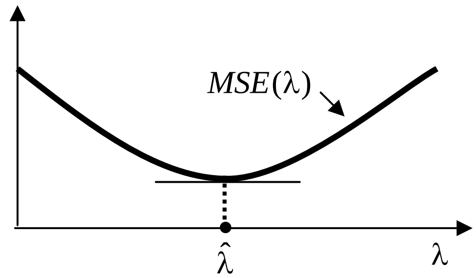
Then to find the (scalar) *minimum mean-squared error predictor*, $\hat{\lambda}$, we minimize:

(2) $$MSE(\lambda) = E\left[(\varepsilon_s - \lambda \cdot \varepsilon_1)^2\right] = E\left(\varepsilon_s^2 - 2\lambda \cdot \varepsilon_s \varepsilon_1 + \lambda^2 \varepsilon_1^2\right)$$

$$= E(\varepsilon_s^2) - 2\lambda \cdot E(\varepsilon_s \varepsilon_1) + \lambda^2 E(\varepsilon_1^2) = \sigma_s^2 - 2\lambda c_{s1} + \lambda^2 c_{11}$$

But this is a simple quadratic function in λ so that

(3) $$\tfrac{d}{d\lambda} MSE(\lambda) = -2c_{s1} + 2\lambda c_{11} \quad \text{and} \quad \tfrac{d^2}{d\lambda^2} MSE(\lambda) = 2c_{11} > 0$$



Hence the unique value of $\hat{\lambda}$ is given by the *first-order condition*:

(4) $$0 = \tfrac{d}{d\lambda} MSE(\lambda) = 2(c_{s1} - \lambda c_{11}) \Rightarrow c_{s1} = \hat{\lambda} c_{11} \Rightarrow \hat{\lambda} = (c_{11})^{-1} c_{s1}$$