MAXIMUM LIKELIHOOD ESTIMATION

General Estimation for Coin Toss Problem:

- Given only that the true probability, p, satisfies $0 \leq p \leq 1$, what is a best estimate of p given that k of n tosses are *Heads* ?

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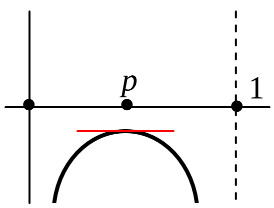

$$\equiv\; \max_{0\le p\le 1}\left[const. + k\ln(p) + (n-k)\ln(1-p)\right]$$

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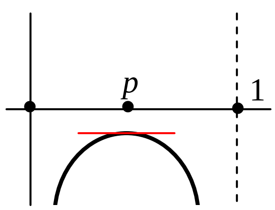

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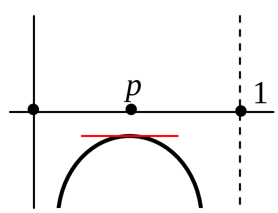

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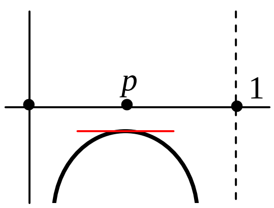

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SMALL SAMPLES: $k = n = 3 \Rightarrow \hat{p}_3 = 1$ (**??**)