

# NONNEGATIVE MATRICES

**Definition:** A matrix  $A = (a_{ij} : i, j = 1, \dots, n)$  is *connected* iff for each  $i \neq j$  there is a path  $(a_{i v_1}, a_{v_2 v_3}, \dots, a_{v_k j})$  of nonzero cells from  $i$  to  $j$ .

**Perron-Frobenius Theorem.** *If matrix  $A$  is a nonnegative and connected, then the maximum eigenvalue of  $A$  is positive and has a unique positive eigenvector of length one.*

**Geometric Proof:** ( $n = 2$ )

