REGRESSION WITH ONE PREDICTOR

The simplest possible regression involves one explanatory variable (predictor), x, with no intercept term:

(1) $Y_i = \beta x_i + \varepsilon_i\,,\ i = 1,..,n \quad \varepsilon_i \sim N(0,\sigma^2)$

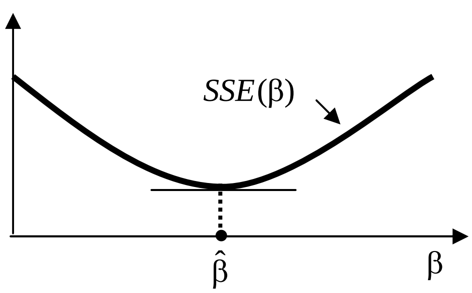
For any given data, $y' = (y_1,..,y_n), x' = (x_1,..,x_n)$, the scalar, β, is then estimated by minimizing the *sum of squared errors*:

(2) $$SSE(\beta) = \sum_{i=1}^{n} (y_i - \beta x_i)^2 = (y - \beta x)'(y - \beta x) = (y' - \beta x')(y - \beta x)$$

$$= \boxed{y'y - 2\beta(x'y) + \beta^2(x'x)}$$

But this is a simple quadratic function in β so that

(3) $$\tfrac{d}{d\beta} SSE(\beta) = -2(x'y) + 2\beta(x'x) \quad \text{and} \quad \tfrac{d^2}{d\beta^2} SSE(\beta) = 2(x'x) > 0$$



Hence the unique value of $\hat{\beta}$ is given by the *first-order condition*:

(4) $$0 = \tfrac{d}{d\beta} SSE(\beta) = 2(x'y - \beta x'x) \Rightarrow x'y = \hat{\beta}(x'x) \Rightarrow \boxed{\hat{\beta} = (x'x)^{-1} x'y}$$