REGRESSION WITH ONE PREDICTOR

The simplest possible regression involves one explanatory variable (predictor), \( x \), with no intercept term:

\[
Y_i = \beta x_i + \varepsilon_i, \quad i = 1, \ldots, n \quad \varepsilon_i \sim N(0, \sigma^2)
\]

For any given data, \( y' = (y_1, \ldots, y_n), x' = (x_1, \ldots, x_n) \), the scalar, \( \beta \), is then estimated by minimizing the sum of squared errors:

\[
SSE(\beta) = \sum_{i=1}^{n} (y_i - \beta x_i)^2 = (y - \beta x)'(y - \beta x) = (y' - \beta x')(y - \beta x)
\]

\[
= y'y - 2\beta(x'y) + \beta^2(x'x)
\]

But this is a simple quadratic function in \( \beta \) so that

\[
\frac{d}{d\beta} SSE(\beta) = -2(x'y) + 2\beta(x'x) \quad \text{and} \quad \frac{d^2}{d\beta^2} SSE(\beta) = 2(x'x) > 0
\]

Hence the unique value of \( \hat{\beta} \) is given by the first-order condition:

\[
0 = \frac{d}{d\beta} SSE(\beta) = 2(x'y - \beta x'x) \quad \Rightarrow \quad x'y = \hat{\beta}(x'x) \quad \Rightarrow \quad \hat{\beta} = (x'x)^{-1} x'y
\]