EXAMPLES OF SMOOTHERS FOR SPATIAL INTERPOLATION IN GEOSTATISTICAL ANALYST

ESE 502

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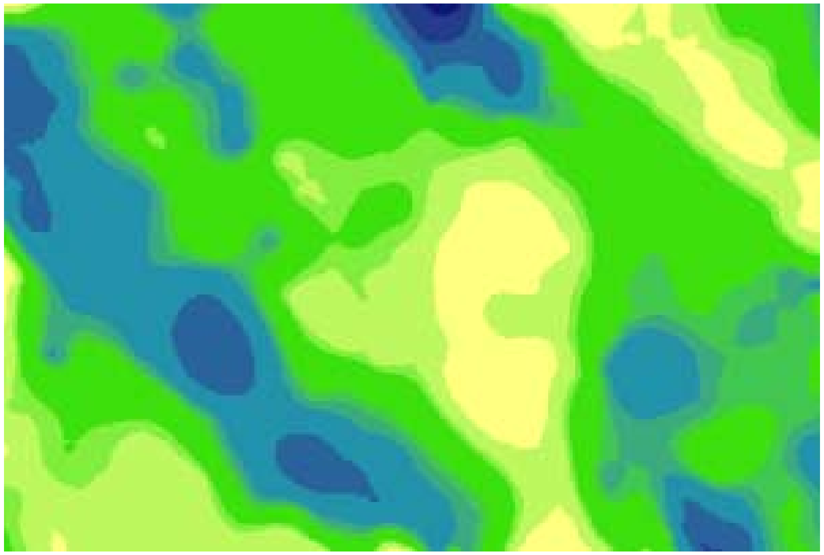
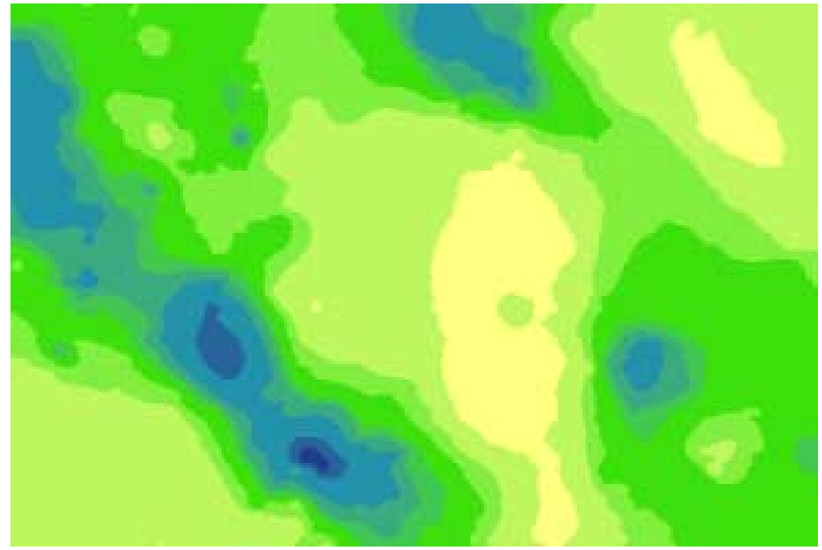
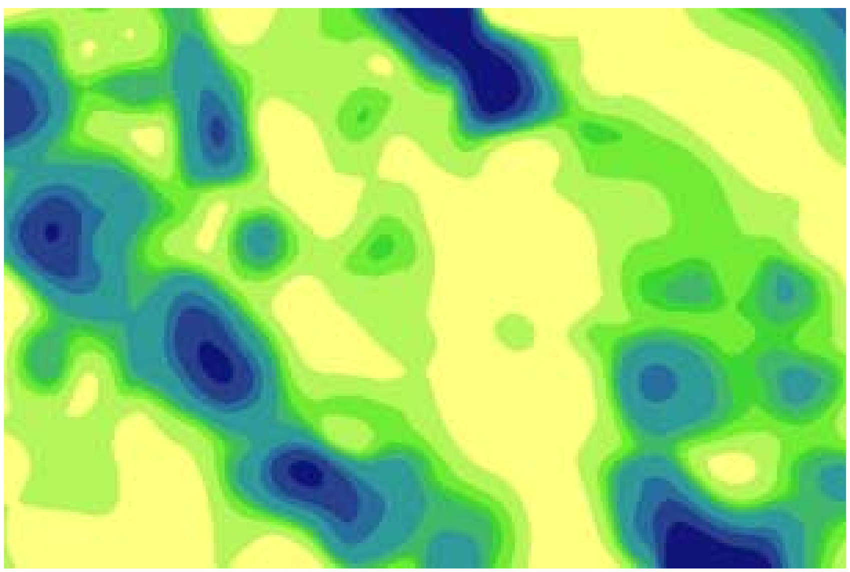


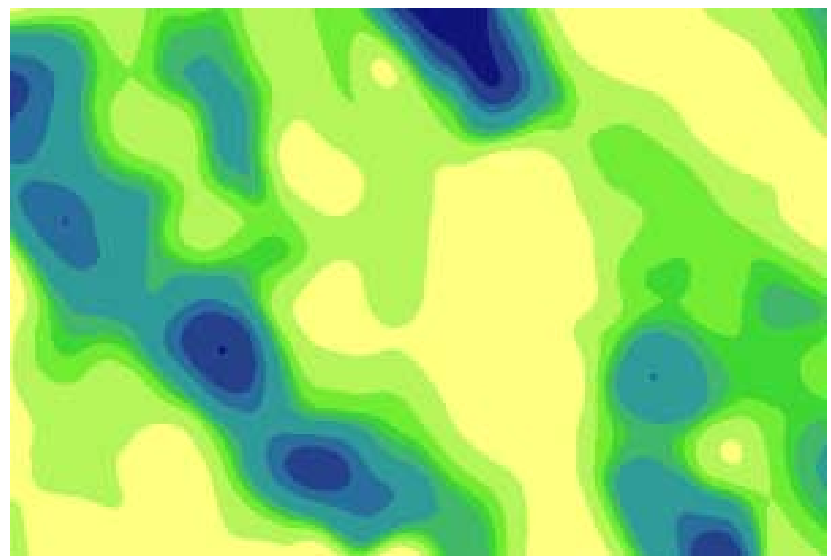
Local Polynomial Fit



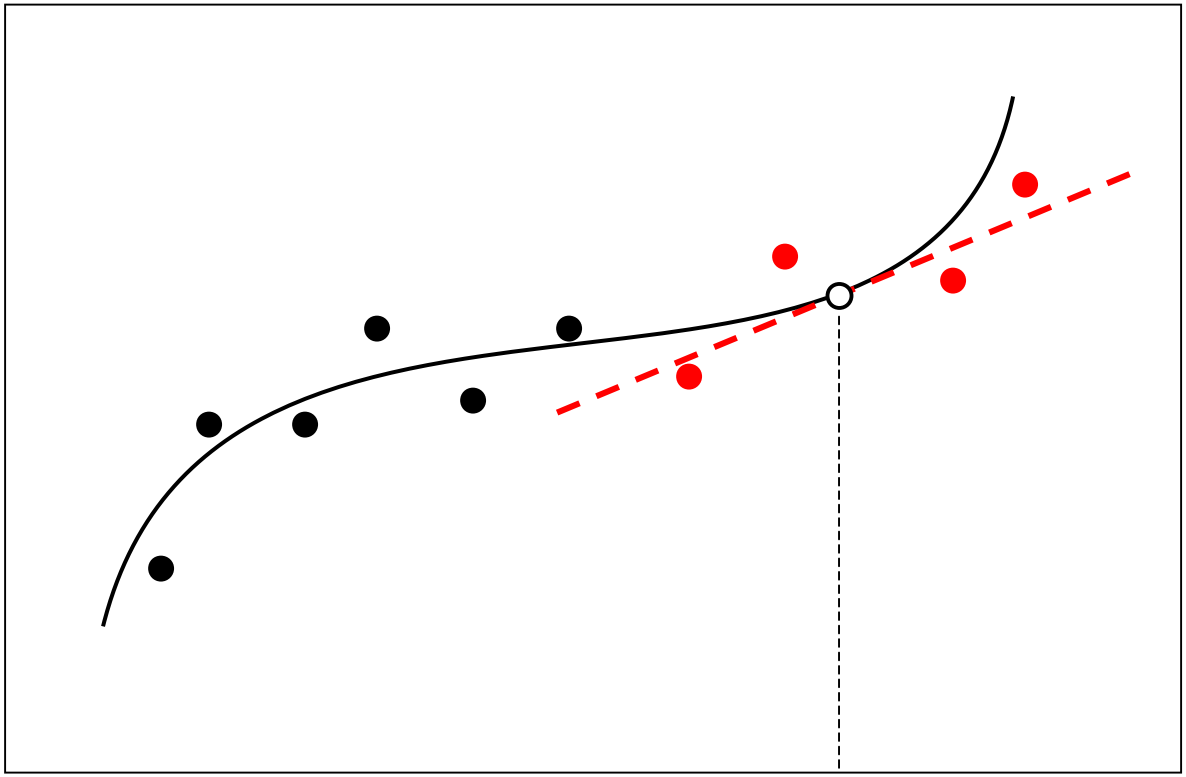
Radial Basis Fit



Spline Function Fit



Ordinary Krige Fit



LOCAL LINEAR POLYNOMIAL FIT

RADIAL BASIS FUNCTIONS

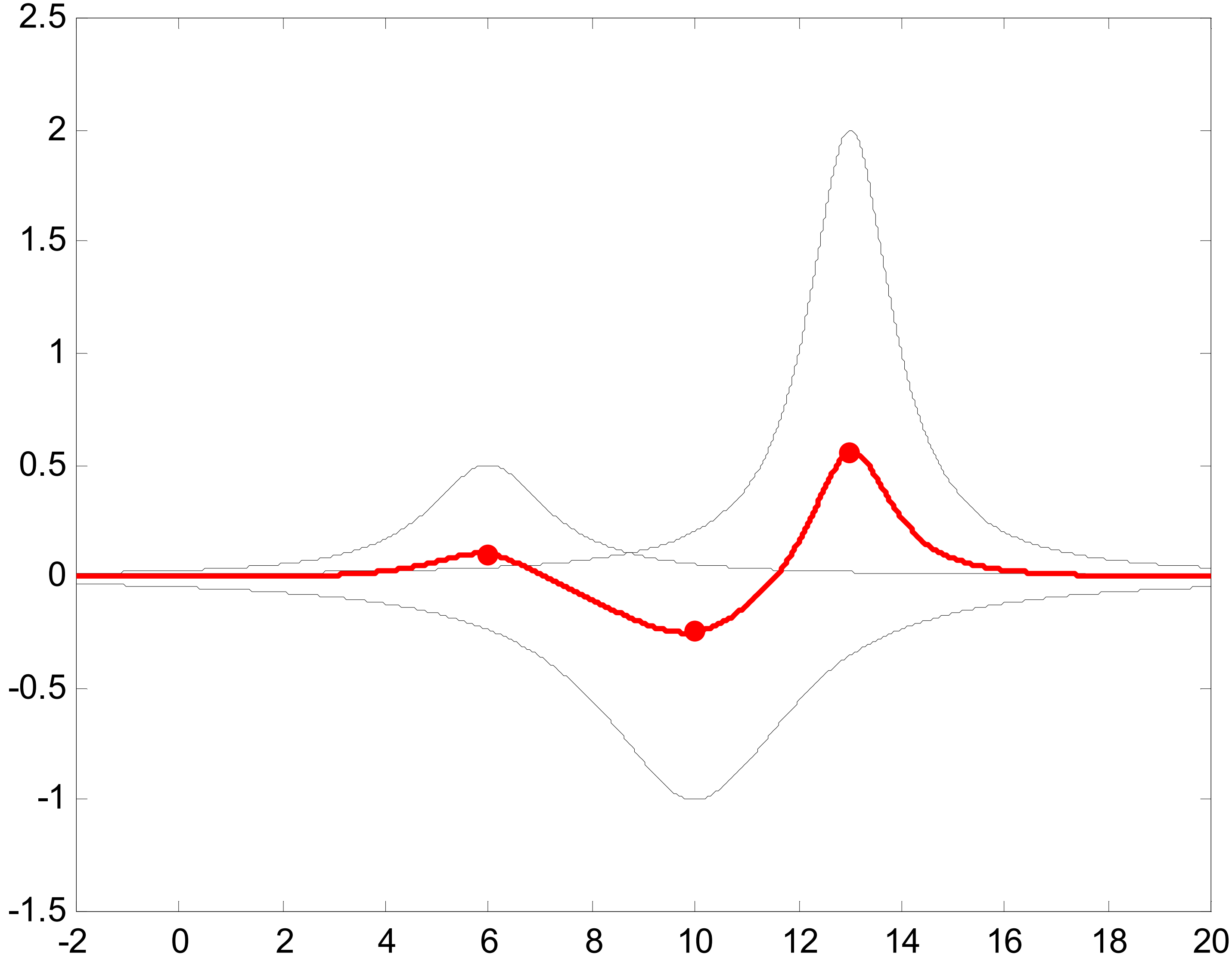
Given data (s_i, y_i) , $i = 1,..,n$,

Choose normal densities: $\phi_{i,}(s) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{(s - s_i)^2}{2} \right)$, $i = 1,..,n$

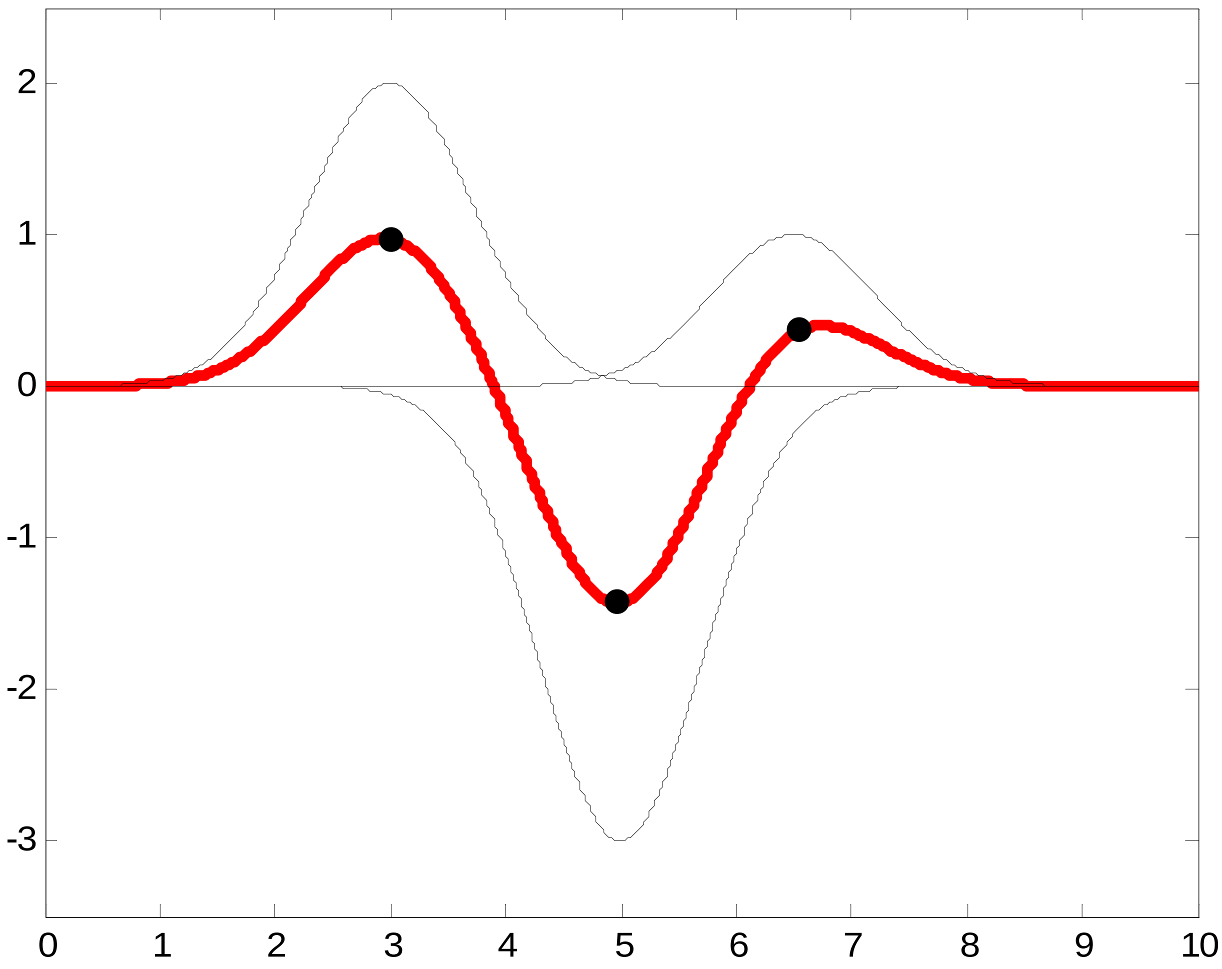
Let $y = (y_i : 1 = 1,..,n)$, $\Phi = [\phi_i(s_j) : i, j = 1,..,n]$, and

Find coefficients $a = (a_i : i = 1,..,n)$ such that $\boxed{y = \Phi a}$

$$\Rightarrow \quad \boxed{a = \Phi^{-1} y}$$



RBF with Inverse Quadratic Basis Functions



RADIAL BASIS INTERPOLATION
(Normal Density Family)

CUBIC SPLINE MODELS

- For simplicity we focus on the *one-dimensional* case, and consider a set of data points $\{(x_i, y_i) : i = 1,..,n\}$, such as the $n = 5$ points in Figure 1 below:

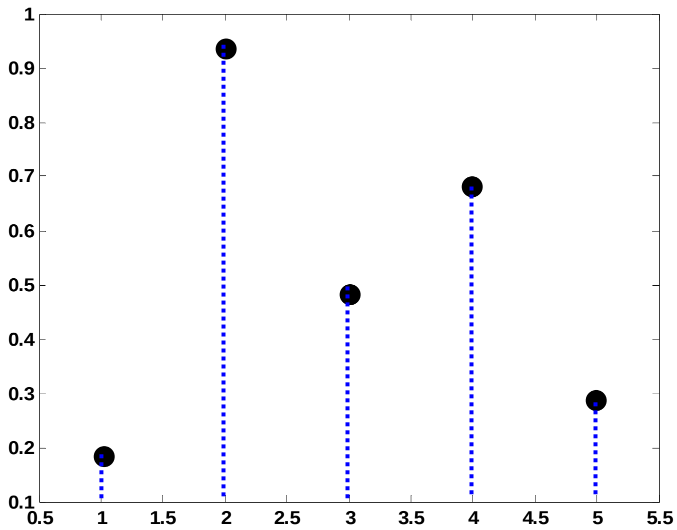


Figure 1. Data Points

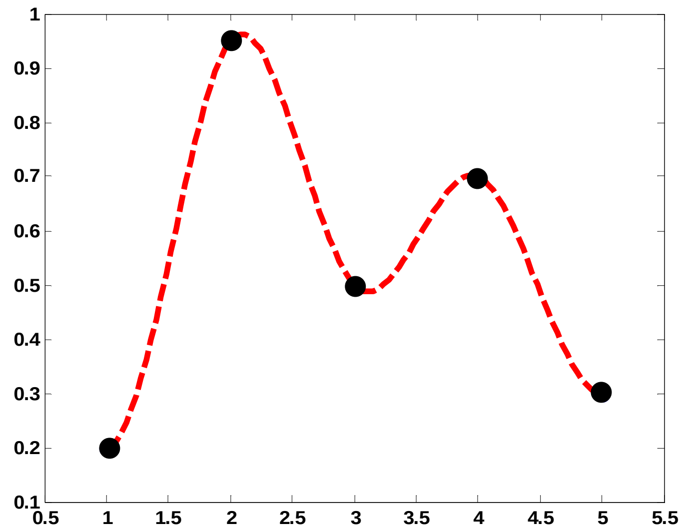


Figure 2. Smooth Interpolation

- The objective is to find a smooth curve, $y = c(x)$, that passes through these data points, such as the curve shown in Figure 2.

- If the set of *twice continuously differentiable* functions on the interval $[x_1, x_n]$ is denoted by $C[x_1, x_n]$, and if "smoothness" is taken to be a lack of sharp curvature [as measured by the second derivative, $c''(x)$, of $c(x)$], then this problem can be formalized in terms of the following *constrained minimization problem*:

(1) $$\min_{c \in C[x_1, x_n]} \int_{x_1}^{x_n} \left[c''(x)\right]^2 dx \quad \text{subject to:} \quad c(x_i) = y_i \ , \ i = 1,..,n$$

- In spite of its apparent complexity, this problem has the following remarkable solution:

THEOREM: *The unique solution,* $c_* \in C[x_1, x_n]$ *to* (1) *is described on each segment* $[x_i, x_{i+1}]$, $i = 1,..,n-1$ *by a cubic polynomial*

(2) $$c_*(x) = a_i + b_i x + c_i x^2 + d_i x^3 \ , \ x_i \le x \le x_{i+1}$$

for appropriate choices of (a_i, b_i, c_i, d_i), $i = 1,..,n-1$. This function, c_*, is *called a* ***cubic spline*** *function.*

- The actual solution to the problem in Figure 1 above is shown in Figure 3 below:

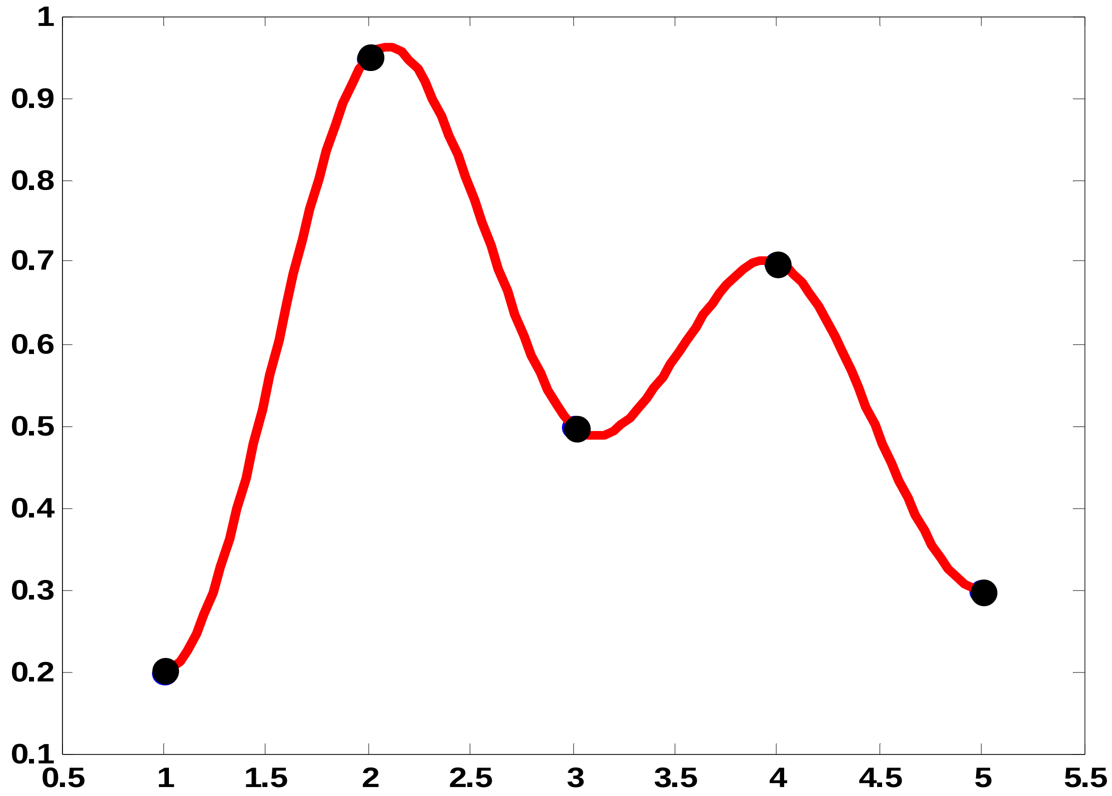


Figure 3. Cubic Spline Interpolation Example