EXAMPLES OF SMOOTHERS FOR SPATIAL INTERPOLATION IN GEOSTATISTICAL ANALYST

ESE 502

Tony E. Smith
LOCAL LINEAR POLYNOMIAL FIT
RADIAL BASIS FUNCTIONS

Given data \( (s_i, y_i) \), \( i = 1, \ldots, n \),

Choose normal densities: \( \phi_i(s) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{(s - s_i)^2}{2}\right) \), \( i = 1, \ldots, n \)

Let \( y = (y_i : 1 = 1, \ldots, n) \), \( \Phi = [\phi_i(s_j) : i, j = 1, \ldots, n] \), and

Find coefficients \( a = (a_i : i = 1, \ldots, n) \) such that \( y = \Phi a \)

\[ \Rightarrow \quad a = \Phi^{-1} y \]
RBF with Inverse Quadratic Basis Functions
RADIAL BASIS INTERPOLATION
(Normal Density Family)
CUBIC SPLINE MODELS

• For simplicity we focus on the one-dimensional case, and consider a set of data points \( \{(x_i, y_i): i = 1, \ldots, n\} \), such as the \( n = 5 \) points in Figure 1 below:

![Figure 1. Data Points](image1)

![Figure 2. Smooth Interpolation](image2)

• The objective is to find a smooth curve, \( y = c(x) \), that passes through these data points, such as the curve shown in Figure 2.
If the set of \textit{twice continuously differentiable} functions on the interval \([x_i, x_n]\) is denoted by \(C[x_i, x_n]\), and if “smoothness” is taken to be a lack of sharp curvature [as measured by the second derivative, \(c''(x)\), of \(c(x)\)], then this problem can be formalized in terms of the following \textit{constrained minimization problem}:

\[
\min_{c \in C[x_i, x_n]} \int_{x_i}^{x_n} [c''(x)]^2 \, dx \quad \text{subject to:} \quad c(x_i) = y_i, \quad i = 1, \ldots, n
\]

In spite of its apparent complexity, this problem has the following remarkable solution:

**THEOREM:** The unique solution, \(c_* \in C[x_1, x_n]\) to (1) is described on each segment \([x_i, x_{i+1}]\), \(i = 1, \ldots, n-1\) by a cubic polynomial

\[
c_*(x) = a_i + b_i x + c_i x^2 + d_i x^3, \quad x_i \leq x \leq x_{i+1}
\]

for appropriate choices of \((a_i, b_i, c_i, d_i), \ i = 1, \ldots, n-1\). This function, \(c_*\), is called a \textit{cubic spline} function.
The actual solution to the problem in Figure 1 above is shown in Figure 3 below:

Figure 3. Cubic Spline Interpolation Example