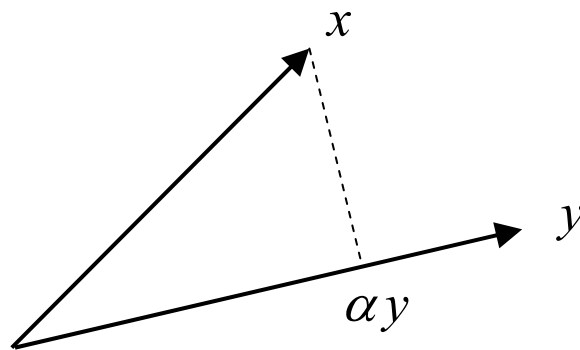


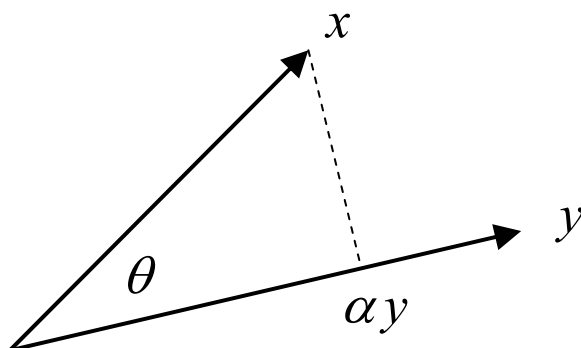
VECTOR COSINES AND ORTHOGONALITY



$$\begin{aligned}\min_{\alpha} \|x - \alpha y\|^2 &= (x - \alpha y)'(x - \alpha y) \\ &= x'x - 2\alpha x'y + \alpha^2 y'y\end{aligned}$$

$$0 = \frac{\partial \|x - \alpha y\|^2}{\partial \alpha} = -2x'y + 2\alpha y'y$$

$$\Rightarrow \alpha = \frac{x'y}{y'y} = \frac{x'y}{\|y\|^2}$$



$$\cos(\theta) = \frac{\|\alpha y\|}{\|x\|} = \frac{\alpha \|y\|}{\|x\|} = \frac{x'y \|y\|}{\|y\|^2 \|x\|} = \frac{x'y}{\|y\| \|x\|}$$