Problem 1 [15 points]: Consider a relation $R$ with four attributes $ABCD$. You are given the following dependencies: $A \rightarrow B$, $BC \rightarrow D$, $C \rightarrow AB$.

1. List all keys for $R$. (other than superkeys)

2. Is $R$ in 3NF? Why?

3. Is $R$ in BCNF? Why?

Problem 2 [30 points]: The task is to design an apartment search mash-up system. This system will integrate maps with apartment rental listings. A user will create an account, subscribe to the apartment RSS feeds, search apartments based on keywords, and view the apartments on the maps.

- Each user will have a unique numeric ID. Additionally, the first and last names, email address, and billing address will need to be stored.

- Each user is either an unregistered user or a registered user.

- Original users of the system (some still unregistered users and others registered) are further classified as beta testers.

- Each user may subscribe to one or more apartment RSS feeds. One RSS feed may be shared among multiple users.

- RSS feeds have URLs and titles.

- RSS feeds have multiple RSS posts.

- RSS posts have a title, location, price, the number of rooms and a URL.

- Each RSS post consists of a set of word occurrences and their positions. An inverted index on words is needed to support keyword search.
- The map can translate a location to several possible geocode locations (namely, latitude and longitude).

Draw an ER diagram for the apartment search mash-up system. The ER diagram should include various attributes, keys, participation constraints, overlap and covering constraints.

**Problem 3 [25 points]:** Consider a relation $R$ with six attributes $ABCDYZ$ and the FD set $F = \{AB \rightarrow Y, AC \rightarrow D, Y \rightarrow C, ZB \rightarrow D, BD \rightarrow Z\}$. Let $F^+$ denote the closure set of $F$.

1. For each of the following attribute sets, do the following: (i) write down a minimal cover of the subset of $F^+$ that holds over the set; (ii) name the strongest normal form that is not violated by the relation containing these attributes; (iii) decompose it into a collection of BCNF relations if it is not already in BCNF.
   
   (a) $ABDYZ$
   
   (b) $ABCD$

2. For each of the following decompositions of $R = ABCDYZ$, with the same set of functional dependencies $F$, say whether the decomposition is (i) dependency preserving, and (ii) lossless join.
   
   (a) \{ABYD, ABCZ\}
   
   (b) \{ACD, ABYZ, ABDZ\}

**Problem 4 [20 points]:** Suppose you are given a relation $R(A, B, C, D, E)$. For each of the following (complete) sets of FDs, (i) identify the candidate key(s) for $R$, and (ii) state whether or not the proposed decomposition of $R$ into smaller relations is a “good” decomposition and briefly explain why or why not.

1. $A \rightarrow B$, $B \rightarrow CE$, $C \rightarrow D$. Decompose into $AB$, $BCE$, and $CD$.

2. $C \rightarrow A$, $B \rightarrow D$. Decompose into $ACE$ and $BD$.

**Problem 5 [15 points]:** Why do commercial DBMSs support keys and foreign keys, but not general FDs?