CIS 540 Spring 2016: Preparing for Midterm Exam

Midterm exam will be held on Wednesday, March 2, 10.30am – 11.50am in Moore 212. The midterm will be out of 100 pts accounting 20% of the overall grade. Midterm is open book: you can consult the textbook and your own personal notes during the exam. The topics for the mid-term are Chapters 2, 3, and 4 of the textbook. You can skip Section 3.3 and the second half of Section 3.4.3 (implementation and operations on ROBDDs).

Homeworks 1, 2, and 3 should give you an idea of problems. **Midterm review session** is scheduled for Monday, Feb 29, 4.30 – 5.30 pm in Moore 212. Attached are the mid-term exams from 2014 and 2015.
CIS 540 Spring 2014: Midterm, March 5, 10.30–11.50am

You are allowed to consult your class notes and class handouts.

1. Consider the synchronous reactive component shown in the figure below. List all the possible reactions of the component. List all the await-dependencies among input/output variables. Is the component deterministic? Is the component input-enabled?

   ![Synchronous Reactive Component Diagram]

   - bool $u := 0$
   - $A_1 : u \rightarrow y$
   - $y := u$
   - $A_2 : x, u \rightarrow u, z$
   - $z := \text{choose}(x, u)$
   - $u := x$

2. Consider the Boolean formula

   $$(x \lor y) \land (\neg x \lor z) \land (\neg y \lor \neg z).$$

   Draw the ROBDD for this formula with respect to the variable ordering $x < y < z$.

3. Consider the synchronous component shown as an extended state machine in the figure below:

   ![Extended State Machine Diagram]

   - int $x := 0$
   - off
   - $x := x + 1$
   - on
   - $x := x - 1$

   Consider the property $\varphi$ given by $x \geq 0$. Show that $\varphi$ is not an inductive invariant of the system. Find a formula $\psi$ such that $\psi$ is stronger than $\varphi$ and is an inductive invariant. Prove your answer.

4. Consider a transition system $T$ with two integer variables $x$ and $y$. The transitions of the system correspond to executing the statement:

   $$\text{if } (x < y) \text{ then } x := x + y \text{ else } y := y + 1$$

   (a) Write the transition formula over the variables $x, y, x',$ and $y'$ that captures the transition relation of the system.

   (b) Consider a region $A$ of the above transition system described by the formula $0 \leq x \leq 5$. Compute the formula describing the post-image of $A$.

5. Consider an asynchronous process $P$ with two variables $x$ and $y$, both of type nat, with $x$ initialized to 0 and $y$ initialized to 1. The behavior of the process is described by two tasks. The task $A_1$ has the guard condition $x < y$ and the update code $x := x + 1$. The task $A_2$ is
always enabled, and its update code is $y := x + y$. Answer each of the questions below with a brief justification. When adding fairness assumptions, clearly specify whether you are using strong fairness, or weak fairness, and for which task.

(a) Is it guaranteed that the value of $x$ eventually exceeds 5? If not, is there a suitable fairness assumption for the two tasks under which this guarantee holds?

(b) Is it guaranteed that the value of $y$ eventually exceeds 5? If not, is there a suitable fairness assumption for the two tasks under which this guarantee holds?

(c) Is it guaranteed that at some step in the execution the values of $x$ and $y$ become equal? If not, is there a suitable fairness assumption for the two tasks under which this guarantee holds?

6. Consider the following solution to the two-process consensus problem in the asynchronous model. The processes use a shared atomic register $x$ and a shared test-and-set register $y$. The possible values for both the registers are 0 and 1, and both are initialized to 0. Each process executes the following sequence of steps:

   (a) Execute a test-and-set operation on the register $y$.

   (b) If step (a) returns 0, then write its own initial preference to the register $x$, and decide on this value.

   (c) If step (a) returns 1, then read the register $x$, and decide on the value read.

Consider the three requirements for the consensus problem: validity, agreement, and wait-freedom. Which of these requirements are satisfied by this protocol? Justify your answer.
CIS 540 Spring 2015: Midterm, March 4, 10.30–11.50am

You are allowed to consult your class notes and class handouts.

1. Answer each of the questions below briefly (in one or two sentences). 15pts
   (a) For a synchronous reactive component described using tasks, if an output variable $y$ awaits an input variable $x,$ then must the component have a task that reads $x$ and writes $y$?
   (b) Suppose $B$ is an ROBDD over $n$ Boolean variables ordered as $x_1 < x_2 < \cdots < x_n.$ At most how many vertices in $B$ can be labeled with the variable $x_n$?
   (c) Recall the leader election protocol for asynchronous processes in a ring (section 4.3.1). If the ring initially has 15 processes, what is the minimum and the maximum number of processes that are guaranteed to become “followers” after one phase of the protocol?

2. Design a synchronous reactive component $S$ with an integer input variable $x,$ an input event variable $clock,$ and an output variable $y$ with the following behavior: in every round $n,$ if the input event $clock$ is absent, then the output $y$ is also absent; if the input event $clock$ is present, then the output $y$ is present and its value equals the sum of inputs of $x$ from rounds 1, 2, $\ldots n.$ Clearly specify all the variables of the component, along with their types, initialization, reaction description, and await dependencies.

3. Consider a transition system $T$ with two integer variables $x$ and $y.$ The transitions of the system correspond to executing the statement:
   
   $$\text{if } (x > y) \text{ then } x := 2y + 1.$$  

   Consider a region $A$ of the above transition system described by the formula
   
   $$(0 \leq x \leq 5) \land (1 \leq y \leq 3).$$

   Compute the formula describing the post-image of $A.$

4. Recall the asynchronous process $\text{Split}$ from exercise 4.2: It has a single input variable $in$ of type $\text{msg}.$ Its output variables are $out_1$ and $out_2$ of type $\text{msg}.$ It maintains a single queue as its state variable with the declaration given by
   
   $$\text{queue(} \text{msg}) \ x := \text{null}.$$  

   The input task $A_i$ stores the input messages in the queue $x,$ and is specified by
   
   $$\neg \text{Full}(x) \rightarrow \text{Enqueue}(in, x).$$

   The output task $A^1_o$ is enabled when the queue $x$ is nonempty and if so, removes a message from the queue and transmits it on the output channel $out_1$:
   
   $$\neg \text{Empty}(x) \rightarrow out_1 := \text{Dequeue}(x).$$

   The output task $A^2_o$ is symmetric, and transmits messages on the output channel $out_2$:
   
   $$\neg \text{Empty}(x) \rightarrow out_2 := \text{Dequeue}(x).$$
Consider the asynchronous process

\[(\text{Split}(\text{out}_2 \rightarrow \text{temp}) \mid \text{Split}(\text{in} \rightarrow \text{temp})(\text{out}_1 \rightarrow \text{out}_3)) \setminus \text{temp}\]

obtained by connecting two instances of the process \text{Split}. Show the “compiled” version of this composite process (that is, clearly describe its input channels, output channels, state variables, initialization, input tasks, output tasks, and internal tasks).

5. Consider the synchronous component shown as an extended state machine in the figure below: 20pts

\[
\begin{align*}
\text{int } x &:= 0; y := 0 \\
\text{off} &\quad \text{on} \\
x := x + 1 &\quad y := y + 1 \\
x := y &\quad y := y + 1
\end{align*}
\]

For each of the properties specified below, state whether the property is an invariant and whether it is an inductive invariant, with a brief justification.

- (a) \(x \geq 0\).
- (b) \(y \geq 0\).
- (c) \(x \geq y\).
- (d) \((\text{mode} = \text{off}) \rightarrow (x \geq y)\).

6. Consider an asynchronous process \(P\) with two variables \(x\) and \(y\), both of type \text{nat} and both initialized to 0. The behavior of the process is described by three tasks. The task \(A_1\) is always enabled and its update code is \(x := x + 1\). The task \(A_2\) is always enabled and its update code is \(x := 0\). The task \(A_3\) has the guard condition \(x = 0\) and the update code \(y := y + 1\).

For each of the requirements below, state whether the process satisfies the requirement. If not, is there a suitable fairness assumption under which the requirement is satisfied. When adding fairness assumptions, clearly specify whether you are using strong fairness, or weak fairness, and for which tasks. Justify your answers with a brief explanation.

- (a) The value of \(x\) eventually exceeds 0.
- (b) The value of \(x\) eventually exceeds 5.
- (c) The value of \(y\) eventually exceeds 5.