Homework Assignment 3

Due: Tuesday, February 8, 2005, by 12 PM (IN CLASS)

Please, read the following instructions:

- Fill out this form with your name, student ID, email, and signature and return it as the cover page of your homework.
- Turn in your homework at the beginning of your class on the due date described at the top of this page.
- Late assignments will be penalized 25% and will not be accepted after 1:30PM of the day following the due date.
- Late assignments must be turned in to Janean Williams in room 308, 3rd floor, Levine Building.
- All writings must be neat, well-organized, and include sufficient explanations in the delineation of the solutions.
- Full credit will be given only in the case where the correct answer has been properly justified with complete explanations.
- Good luck!

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
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<tr>
<td>Max</td>
<td>20</td>
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Total: ________________________________
Problem 1 [20 points]

Consider the NFA $M = (Q, \Sigma, \delta, q_0, F)$, where $Q = \{q, r, s, t\}$, $\Sigma = \{a, b\}$, $q_0 = q$, $F = \{t\}$, and $\delta : Q \times \Sigma \to \mathcal{P}(Q)$ is the transition function such that

<table>
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<tr>
<th>$\delta$</th>
<th>$a$</th>
<th>$b$</th>
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<tbody>
<tr>
<td>$q$</td>
<td>${q, r}$</td>
<td>${q}$</td>
</tr>
<tr>
<td>$r$</td>
<td>${s}$</td>
<td>$\emptyset$</td>
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<tr>
<td>$s$</td>
<td>$\emptyset$</td>
<td>${t}$</td>
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<td>$t$</td>
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Then,

(a) Compute $\bar{\delta}(q, bbaaba)$.

(b) Describe $L(M)$. 
Problem 2 [20 points]

Use the subset construction given in class to compute a DFA $D$ such that $L(D) = L(M)$, where $M$ is the NFA defined in the previous problem.
Problem 3 [40 points]

We say that a state \( q \in Q \) of a DFA \( M = (Q, \Sigma, \delta, q_0, F) \) is reachable if there exists a string \( w \in \Sigma^* \) such that \( q = \bar{\delta}(q_0, w) \). Otherwise, we say that \( q \) is unreachable. Now, consider the following questions:

(a) Show that \( q_0 \) is a reachable state for any DFA \( M \).

(b) Is there any unreachable state in the DFA you computed for Problem 2? Justify your answer.

(c) Let \( M \) be the DFA of your answer for Problem 2 and build a DFA \( M' = (Q', \Sigma, \delta', q_0, F') \), where \( Q' = \{ q \in Q \mid q \text{ is reachable in } M \} \), \( F' = \{ q \in F \mid q \text{ is reachable in } M \} \), and \( \delta': Q' \times \Sigma \rightarrow Q' \) is the transition function defined as \( \delta'(q, a) = \delta(q, a) \), for all \( q \in Q' \) and \( a \in \Sigma \). Note that if your answer for item (b) was “no”, then \( M = M' \). Otherwise, \( M' \) can be viewed as \( M \) without the unreachable states.

(d) Show that, for every \( w \in \Sigma^* \), we must have \( \bar{\delta}(q_0, w) = \bar{\delta}'(q_0, w) \). Argue that \( L(M) = L(M') \).
Problem 4 [20 points]

Let $L \subseteq \{a, b, c\}^*$ be the language consisting of all strings $w$ over $\{a, b, c\}$ such that $w$ contains an odd number of $a$’s or an odd number of $b$’s an odd number of $c$’s; that is, $L = \{a, b, c, ab, ba, ac, ca, bc, cb, abc, \ldots\}$.

(a) Build an NFA $M$ with 7 states such that $L(M) = L$.

(b) Can you design a DFA $D$ such that $L(D) = L$ and $D$ has less than 8 states? If so, describe the five parts of such a DFA. Otherwise, argue (informally) why such a DFA cannot exist.