PLEASE, READ THE FOLLOWING INSTRUCTIONS:

• Fill out this form with your name, student ID, email, and signature and return it as the cover page of your homework.

• Turn in your homework at the beginning of your class on the due date described at the top of this page.

• Late assignments will be penalized 25% and will not be accepted after 1:30PM of the day following the due date.

• Late assignments must be turned in to Janean Williams in room 308, 3rd floor, Levine Building.

• All writings must be neat, well-organized, and include sufficient explanations in the delineation of the solutions.

• Full credit will be given only in the case where the correct answer has been properly justified with complete explanations.

• Good luck!

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TOTAL: ____________________________
Problem 1 [15 points]

Let $\Sigma = \{a, b, c\}$. Then,

(a) Give a regular expression that describes the set of all strings $w$ in $\Sigma^*$ such that $|w|$ is odd and $w$ ends with a $b$.

(b) Give a regular expression that describes the set of all strings $w$ in $\Sigma^*$ such that $w$ do not contain two consecutive $c$’s.
Problem 2 [25 points]

We saw in class that regular languages are closed under complementation, union, concatenation, Kleene closure, and intersection. Use one of these facts and the given fact that the language

$$L_1 = \{ w \in \{0, 1\}^* \mid w = 0^n1^n \text{ for some non-negative integer } n \}$$

is not regular to prove that the language

$$L_2 = \{ w \in \{0, 1\}^* \mid w = 0^j1^k \text{ for some non-negative integers } j, k \text{ such that } j \neq k \}$$

is also not regular.
Problem 3 [20 points]

Let $\Sigma$ be an alphabet, and let $L$ be any language over $\Sigma$. Then, we define the unary operator $E$ as

$$E(L) = \{ w \in \Sigma^* \mid w = xy, \text{ where } x \in \Sigma^+ \text{ and } y \in L \}.$$ 

(a) Given a DFA accepting $L$, describe how to modify this DFA to create another DFA (or an NFA, or an $\epsilon$-NFA) accepting $E(L)$.

(b) Assuming that your construction for (a) is correct, argue that the set of all regular languages closed under the operator $E$. 
Problem 4 [25 points]

Let $L$ be a language over an alphabet $\Sigma$, and let $a$ be a symbol of $\Sigma$. We define the quotient $L/a$ of $L$ and $a$ to be the language of all strings $w$ in $\Sigma^*$ such that $wa$ is in $L$. For example, if $\Sigma = \{0, 1\}$ and $L = \{0, 001, 100\}$ then $L/0 = \{\epsilon, 10\}$.

(a) Given a DFA accepting $L$, describe how to modify this DFA to create another DFA (or an NFA, or an $\epsilon$-NFA) accepting $L/a$.

(b) Prove that the finite state machine you provided as an answer for (a) is correct.
Problem 5 [15 points]

Let $\mathcal{N}_\Sigma$ be the set of all languages over an alphabet $\Sigma$ that are not regular. Is $\mathcal{N}_\Sigma$ closed under union? If your answer is yes, prove it. Otherwise, give a counterexample.