CIT 596 – Theory of Computation  
Spring 2005, 212 Moore, TR 12-1.30PM

Homework Assignment 5

Due: Thursday, March 17, 2005, by 12 PM (IN CLASS)

Name: __________________________________________________________

Student ID (8 digits): _____________________________________________

Email: __________________________________________________________

Signature: _______________________________________________________

PLEASE, READ THE FOLLOWING INSTRUCTIONS:

• Fill out this form with your name, student ID, email, and signature and return it as the cover page of your homework.

• Turn in your homework at the beginning of your class on the due date described at the top of this page.

• Late assignments will be penalized 25% and will not be accepted after 1:30PM of the day following the due date.

• Late assignments must be turned in to Janean Williams in room 308, 3rd floor, Levine Building.

• All writings must be neat, well-organized, and include sufficient explanations in the delineation of the solutions.

• Full credit will be given only in the case where the correct answer has been properly justified with complete explanations.

• Good luck!

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>15</td>
<td>25</td>
<td>20</td>
<td>25</td>
<td>15</td>
</tr>
</tbody>
</table>

Score: ________________

TOTAL: _____________________________
Problem 1 [15 points]

Let $\Sigma = \{a, b, c\}$. Then,

(a) Give a regular expression that describes the set of all strings $w$ in $\Sigma^*$ such that $|w|$ is odd and $w$ ends with a $b$.

(b) Give a regular expression that describes the set of all strings $w$ in $\Sigma^*$ such that $w$ do not contain two consecutive $c$'s.

Guidelines:

Question (a) counts for 7 points. If you misunderstand 'a $b$' as 'ab' and have a correspondingly correct answer, you are given 3 points. No partial credit is given for other incomplete answers. Question (b) counts for 8 points, and no partial credit is given.
Problem 2 [25 points]

We saw in class that regular languages are closed under complementation, union, concatenation, Kleene closure, and intersection. Use one of these facts and the given fact that the language

\[ L_1 = \{ w \in \{0,1\}^* \mid w = 0^n1^n \text{ for some non-negative integer } n \} \]

is not regular to prove that the language

\[ L_2 = \{ w \in \{0,1\}^* \mid w = 0^j1^k \text{ for some non-negative integers } j, k \text{ such that } j \neq k \} \]

is also not regular.

Guidelines:

You must be completely correct to receive full credit. Partial credit is given as following. 5 points for the correct subtraction, \( L_1 = \{0\}^*\{1\}^* - L_2 \); 10 points for the combination of intersection and complementation, \( L_1 = \{0\}^*\{1\}^* \cap (\{0,1\}^* - L_2) \); and 10 points for the explanation of contradiction.
Problem 3 [20 points]

Let $\Sigma$ be an alphabet, and let $L$ be any language over $\Sigma$. Then, we define the unary operator $E$ as

$$E(L) = \{w \in \Sigma^* \mid w = xy, \text{ where } x \in \Sigma^+ \text{ and } y \in L\}.$$

(a) Given a DFA accepting $L$, describe how to modify this DFA to create another DFA (or an NFA, or an $\epsilon$-NFA) accepting $E(L)$.

(b) Assuming that your construction for (a) is correct, argue that the set of all regular languages closed under the operator $E$.

Guidelines:

(a) 12 points total. 6 points for $F, Q, q_0$, and 6 points for $\delta$.

(b) 8 points total. No partial credit is given.
Problem 4 [25 points]

Let $L$ be a language over an alphabet $\Sigma$, and let $a$ be a symbol of $\Sigma$. We define the quotient $L/a$ of $L$ and $a$ to be the language of all strings $w$ in $\Sigma^*$ such that $wa$ is in $L$. For example, if $\Sigma = \{0, 1\}$ and $L = \{0, 001, 100\}$ then $L/0 = \{\epsilon, 10\}$.

(a) Given a DFA accepting $L$, describe how to modify this DFA to create another DFA (or an NFA, or an $\epsilon$-NFA) accepting $L/a$.

(b) Prove that the finite state machine you provided as an answer for (a) is correct.

Guidelines:

(a) 10 points total. Each of $Q, \Sigma, \delta, q_0, F$ counts for 2 points.
(b) 15 points total. 5 points for a correct claim, and 10 points for the proof.
Problem 5 [15 points]

Let \( N_\Sigma \) be the set of all languages over an alphabet \( \Sigma \) that are not regular. Is \( N_\Sigma \) closed under union? If your answer is yes, prove it. Otherwise, give a counterexample.

Guidelines:

5 points for answering 'No'. 5 points for a correct example, and 5 points for a correct explanation about the example.