Homework Assignment 6

Due: Tuesday, March 22, 2005, by 12 PM (IN CLASS)

Name: ________________________________________________________________

Student ID (8 digits): __________________________________________________

Email: _______________________________________________________________

Signature: _____________________________________________________________

PLEASE, READ THE FOLLOWING INSTRUCTIONS:

• Fill out this form with your name, student ID, email, and signature and return it as the cover page of your homework.

• Turn in your homework at the beginning of your class on the due date described at the top of this page.

• Late assignments will be penalized 25% and will not be accepted after 1:30PM of the day following the due date.

• Late assignments must be turned in to Janean Williams in room 308, 3rd floor, Levine Building.

• All writings must be neat, well-organized, and include sufficient explanations in the delineation of the solutions.

• Full credit will be given only in the case where the correct answer has been properly justified with complete explanations.

• Good luck!

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TOTAL: ________________________________________________________________
Problem 1 [25 points]

Show that the language $L$ such that

$$L = \{ x \in \{a, b\}^* \mid x = ww, \text{ for some } w \in \{a, b\}^* = \{ \epsilon, aa, bb, aaaa, abab, baba, bbbb, \ldots \} \}$$

is not a regular language.

Guidelines:

You must be completely right for full credit. Partial credit is given as following: 5 points for refering to the Pumping Lemma; 10 points for giving a correct pumping string $w$ and its correct decomposition $xyz$; and 10 points for finding an $i$ such that $xyz^iz$ not $\in L$, and its reason.
**Problem 2 [25 points]**

Show that the language \( L \) such that

\[
L = \{ w \in \{a\}^* \mid |w| = j^2, \text{ for some integer } j \} = \{ \epsilon, a, aaaa, aaaaaaaaaa, \ldots \}
\]

is not a regular language.

**Guidelines:**

You must be completely right for full credit. Partial credit is given as following: 5 points for refering to the Pumping Lemma; 10 points for giving a correct pumping string \( w \) and its correct decomposition \( xyz \); and 10 points for finding an \( i \) such that \( xy^iz \) not \( \in L \), and its reason.
Problem 3 [25 points]

Show that the language $L$ such that

$$L = \{ w \in \{a, b\}^* \mid |w|_a \geq |w|_b \} = \{ \epsilon, a, aab, aba, baa, \ldots \}$$

is not a regular language.

Guidelines:

You must be completely right for full credit. Partial credit is given as following: 5 points for refering to the Pumping Lemma; 10 points for giving a correct pumping string $w$ and its correct decomposition $xyz$; and 10 points for finding an $i$ such that $xy^iz$ not $\in L$, and its reason.
Problem 4 [25 points]

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a deterministic finite automaton. Then, show that $L(D)$ is infinite if and only if there exists a string $w$ in $L(D)$ such that $|Q| \leq |w| < 2|Q|$, where $|w|$ is the length of $w$ and $|Q|$ is the number of states of $Q$.

Guidelines:

10 points: $w$ exists such that $|Q| \leq |w| < 2|Q| \Rightarrow L(D)$ is infinite. 5 points for referring to the Pumping Lemma; and 5 points for infinite number strings like $xy^i z \in L(D)$.

15 points: $L(D)$ is infinite $\Rightarrow w$ exists such that $|Q| \leq |w| < 2|Q|$. 5 points for that there exists a string $w \in L(D)$ such that $|w| \geq |Q|; 5$ points for using the shortest string $w$ with $|w| \geq 2|Q|$ and finding a contradiction afterwards; and 5 points for proving $|xy^0 z| \geq |Q|$ and finding the other contradiction.