Problem 1.2

Here we need to calculate the maximum packing fraction, treating the atoms as hard spheres.

Atom at the origin \((0,0,0)\) a

Atom at the center \(\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\) a

Nearest neighbor distance \(= \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 5 = 4.330\) angstrom

Radius of hard spheres \(= \frac{1}{2} \times 4.330 = 2.165\) angstrom

Volume of each atom \(= \frac{4}{3} \pi (2.165)^3 = 42.5\) angstrom\(^3\)

Number of atoms per cube \(= 1 \times \frac{1}{8} + 1 = 2\)

Packing fraction \(= \frac{42.5 x^2}{(5)^3} = 68\%\)

Problem 1.4

Si has diamond structure (fcc lattice with basis of 2 atoms)

Number of atoms per cube \(= (8 \times 1/8 + 1/2 \times 6) \times 2 = 8\)

Density \(= \frac{8}{(5.43 \times 10^{-8})^3} = 5.00 \times 10^{22} \text{ atoms/cm}^3\)

To calculate density on the (110) plane let us look at the figure
The number of atoms in the (110) plane indicated in the figure is:

\[
\text{# of atoms} = 4 \times \frac{1}{4} + 2 \times \frac{1}{2} + 2 \times 1 = 4
\]

\[
\text{area} = a \times \sqrt{2}a = \sqrt{2}a^2 = \sqrt{2}(5.143 \times 10^{-8})^2 = 0.417 \times 10^{14} \text{ cm}^2
\]

Area density = \[
\frac{\text{# of atoms}}{\text{area}} = \frac{4}{0.417 \times 10^{14}} = 9.59 \times 10^{14} \text{ cm}^{-2}
\]

[111] direction is perpendicular to (111) plane. [111] crosses the two adjacent plane at points (0,0,0) and \((\frac{9}{4}, \frac{9}{4}, \frac{9}{4})\).

The distance between these two points is the separation of these two planes

\[
d = \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{a}{4}\right)^2 + \left(\frac{a}{4}\right)^2} = \frac{\sqrt{3}}{4}a = 2.39 \text{ angstrom}
\]

**Problem 1.6**

The figure shows the two dimensional lattice that we see in the NaCl lattice looking down the <100> direction.

Two possible unit cells are shown in the figure.
**Problem 1.12**

For sc

- atoms/ cell = $1/8 \times 8 = 1$
- nearest neighbor distance = $a$

For bcc

- atoms / cell = $1/8 \times 8 + 1 = 2$
- nearest neighbor distance = $\frac{\sqrt{3}}{2} a$

For fcc

- atoms / cell = $8 \times 1/8 + 1/2 \times 6 = 4$
- nearest neighbor distance = $\frac{\sqrt{2}}{2} a$

**Problem 1.17**

From the Table in Appendix III
a (AlSb) = 6.14 angstrom  
a (AlAs) = 5.66 angstrom  
a (InP) = 5.87 angstrom

If the lattice constant varies linearly with x:

\[ A(\text{AlSb}_x\text{As}_{1-x}) = (1-x)a(\text{AlAs}) + x a(\text{AlSb}) \]

We want this to be equal to a (InP), therefore:

\[ (1-x) 5.66 + x (6.14) = 5.87 \]
\[ x = 0.21/0.48 = 0.44 \]

From figure 1-15 we see that the curve representing AlSb\(_x\)As\(_{1-x}\) crosses the InP lattice constant at x = 0.44 where E\(_g\) = 1.9eV

For In\(_x\)Ga\(_{1-x}\)P:

\[ a (\text{InP}) = 5.87 \text{ angstrom} \]
\[ a (\text{GaP}) = 5.45 \text{ angstrom} \]
\[ a (\text{GaAs}) = 5.65 \text{ angstrom} \]

\[ x(5.87) + (1-x)(5.45) = 5.65 \]
\[ x = \frac{0.20}{0.42} = 0.48 \]

From Figure 1-15 we see that the curve representing In\(_x\)Ga\(_{1-x}\)P crosses GaAs lattice constant at x = 0.48 where E\(_g\) = 2eV

**Problem 2.4**

In order to show Eq. (2-17) and (2-3) are equivalent, it is sufficient to show that:

\[
cR = \frac{mq^4}{2K^2 \hbar^2 h} \]

\[ cR = 3 \times 10^{10} \text{ cm/s} \times 109.678 \text{ cm}^{-1} = 3.29 \times 10^{15} \text{ s}^{-1} \]

\[
\frac{mq^4}{2K^2 \hbar^2 h} = \frac{mq^4}{2(2\pi\varepsilon_0)^2 h^2 h} = \frac{mq^4}{8\varepsilon_0^2 h^3} = \frac{9.11 \times 10^{-31} (1.6 \times 10^{-19})^4}{8(8.85 \times 10^{-12})^2 (6.63 \times 10^{-34})^3} \]
\[ = 3.27 \times 10^{15} \text{ s}^{-1} \]
ESE 218 Assignment 1 Solutions
Spring 2005

We can see that the two equations are equivalent.

**Problem 2.6**

\[ \lambda = \frac{h}{mv} \] de Broglie wavelength of a particle

For an electron:

\[ E = \frac{mv^2}{2} \rightarrow v = \sqrt{\frac{2E}{m}} \]

Therefore:

\[
\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2} \times 9.11 \times 10^{-31}} E^{-\frac{1}{2}} = 4.9 \times 10^{-19} E^{-\frac{1}{2}} \text{ (E in J)}
\]

\[
= \frac{4.9 \times 10^{-19}}{(1.6 \times 10^{-19})^\frac{1}{2}} E^{-\frac{1}{2}} \text{ (E in eV)}
\]

\[
= 1.23 \times 10^{-9} E^{-\frac{1}{2}} \text{ (E in eV)}
\]

Electron with \( E = 100 \text{ eV} \)

\[
\lambda = 1.23 \times 10^{-9} (100)^{-\frac{1}{2}} = 1.23 \times 10^{-10} \text{ m} = 1.23 \text{ angstrom}
\]

Electron with \( E = 12 \text{ KeV} \)

\[
\lambda = 1.23 \times 10^{-9} (12000)^{-\frac{1}{2}} = 1.12 \times 10^{-11} \text{ m} = 0.112 \text{ angstrom}
\]

Visible light us about \( 0.5 \mu m = 5000 \text{ angstrom} \), so the resolution of electron microscope is much better.