High Frequency BJT Model
Gain of 10 Amplifier – Non-ideal Transistor

Gain starts dropping at about 1MHz.

Why!
Because of internal transistor capacitances that we have ignored in our models.
Sketch of Typical Voltage Gain Response for a CE Amplifier

\[ |A_v|(dB) \]

Low Frequency Band

Due to external blocking and bypass capacitors

Midband

ALL capacitances are neglected

High Frequency Band

Due to BJT parasitic capacitors \( C_\pi \) and \( C_\mu \)

\[ 20 \log_{10} |A_v|(dB) \]

\[ BW = f_H - f_L \approx f_H \]

GBP = \[|A_v|BW\]

2008 Kenneth R. Laker, update 12Oct10 KRL
High Frequency Small-signal Model

Two capacitors and a resistor added.

A base to emitter capacitor, $C_\pi$

A base to collector capacitor, $C_\mu$

A resistor, $r_x$, representing the base terminal resistance ($r_x \ll r_\pi$)

$$C_\mu = \frac{C_\mu 0}{m} \left( 1 + \frac{V_{CB}}{V_{0c}} \right)$$

$$C_\pi = C_{de} \frac{+C_{je0}}{m} \left( 1 - \frac{V_{BE}}{V_{0e}} \right) \approx C_{de} + 2C_{je0}$$

$C_{de} = \tau_F g_m$

$\tau_F$ = forward-base transit time

SPICE

$C_{JC} = C_{\mu 0}$

$C_{JE} = C_{je0}$

$TF = \tau_F$

$RB = r_x$
High Frequency Small-signal Model

The internal capacitors on the transistor have a strong effect on circuit high frequency performance! They attenuate base signals, decreasing $v_{be}$ since their reactance approaches zero (short circuit) as frequency increases.

As we will see later $C_{\mu}$ is the principal cause of this gain loss at high frequencies. At the base $C_{\mu}$ looks like a capacitor of value $k C_{\mu}$ connected between base and emitter, where $k > 1$ and may be $>> 1$.

This phenomenon is called the Miller Effect.
The relationship \( i_c = \beta i_b \) does not apply at high frequencies \( f > f_H \)!

Using the relationship \( -i_c = f(V_{\pi}) \) – find the new relationship between \( i_b \) and \( i_c \). For \( i_b \) (using phasor notation \((I_x & V_x)\) for frequency domain analysis):

\[
I_b = \left( \frac{1}{r_{\pi}} + sC_{\pi} + sC_{\mu} \right) V_{\pi} \quad \text{where} \quad r_{\pi} \approx 0 \quad (\text{ignore} \ r_{\pi})
\]

\[NOTE: \ s = \sigma + j\omega, \ \text{in sinusoidal steady-state} \ s = j\omega.\]
Frequency-dependent $h_{fe}$ or “beta”

\[ I_b = \left( \frac{1}{r_{\pi}} + sC_\pi + sC_\mu \right) V_\pi \]

@ node C: \[ I_c = (g_m - sC_\mu)V_\pi \] (ignore \( r_0 \))

Leads to a new relationship between the $I_b$ and $I_c$:

\[ h_{fe} = \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{\frac{1}{r_{\pi}} + sC_\pi + sC_\mu} \]
Frequency Response of $h_{fe}$

\[ h_{fe} = \frac{g_m - s C_\mu}{1 + s C_\pi + s C_\mu} \]

multiply N&D by $r_\pi$ and set $s = j\omega$

\[ h_{fe} = \frac{(g_m - j\omega C_\mu) r_\pi}{1 + j\omega(C_\pi + C_\mu) r_\pi} \]

factor N to isolate $g_m$

\[ h_{fe} = \frac{(1 - j\omega \frac{C_\mu}{g_m}) g_m r_\pi}{1 + j\omega(C_\pi + C_\mu) r_\pi} \]

\[ g_m = \frac{I_C}{V_T}, \quad r_\pi = \beta \frac{V_T}{I_C} \]

For small $\omega = \omega_{low}$: $\omega_{low} \frac{C_\mu}{g_m} \ll 1 < \frac{1}{10}$

and:

$\omega_{low} (C_\pi + C_\mu) r_\pi \ll 1 < \frac{1}{10}$

Note:

$\omega_{low} (C_\pi + C_\mu) r_\pi = \omega_{low} (C_\pi + C_\mu) \frac{\beta}{g_m} \gg \omega_{low} \frac{C_\mu}{g_m}$

We have:

\[ h_{fe} = g_m r_\pi = \beta \]
Frequency Response of $h_{fe}$ cont.

\[
\begin{align*}
   h_{fe} &= \frac{(1 - j \omega \frac{C_\mu}{g_m}) g_m r_\pi}{1 + j \omega (C_\pi + C_\mu) r_\pi} = \frac{1 - j \frac{\omega}{\omega_z}}{1 + j \frac{\omega}{\omega_\beta}} g_m r_\pi = \frac{1 - j \frac{f}{f_z}}{1 + j \frac{f}{f_\beta}} \beta \\
   \omega_\beta &= \frac{(C_\pi + C_\mu) r_\pi = (C_\pi + C_\mu) \frac{\beta}{g_m} \gg \frac{C_\mu}{g_m} \implies f_z \gg f_\beta}

\text{Hence, the lower break frequency or } -3\text{dB frequency is } f_\beta
\end{align*}
\]

\[
\begin{align*}
   f_\beta &= \frac{1}{2\pi (C_\pi + C_\mu) r_\pi} = \frac{g_m}{2\pi (C_\pi + C_\mu) \beta} \quad \text{the upper: } f_z &= \frac{1}{2\pi C_\mu / g_m} = \frac{g_m}{2\pi C_\mu} \\
   \text{where } f_z &> 10 f_\beta
\end{align*}
\]
Frequency Response of $h_{fe}$ cont.

Using Bode plot concepts, for the range where: $f < f_\beta$

$$h_{fe} = g_m r_\pi = \beta$$

For the range where: $f_\beta < f < f_z$ s.t. $|1 - j f / f_z| \approx 1$

We consider the frequency-dependent numerator term to be 1 and focus on the response of the denominator:

$$h_{fe} = \frac{g_m r_\pi}{\left(1 + j \frac{f}{f_\beta}\right)} = \frac{\beta}{\left(1 + j \frac{f}{f_\beta}\right)}$$
Frequency Response of $h_{fe}$ cont.

Neglecting numerator term:

$$h_{fe} = \frac{g_m r_\pi}{\left(1 + j \frac{f}{f_\beta}\right)} = \frac{\beta}{\left(1 + j \frac{f}{f_\beta}\right)}$$

And for $f / f_\beta >> 1$ (but $< f / f_z$):

$$|h_{fe}| \approx \frac{\beta}{f / f_\beta} = \beta \frac{f_\beta}{f}$$

Unity gain bandwidth:

$$|h_{fe}| = 1 \Rightarrow \beta \frac{f_\beta}{f} \quad |f = f_T = 1 \Rightarrow f_T = \beta f_\beta$$

BJT unity-gain frequency or GBP

$$f_T = \frac{\omega_T}{2 \pi} = \beta f_\beta$$
Frequency Response of $h_{fe}$ cont.

$\beta = 100 \quad r_\pi = 2500 \Omega \quad C_\pi = 12 \ pF \quad C_\mu = 2 \ pF \quad g_m = 40 \cdot 10^{-3} \ S$

$$\omega_\beta = \frac{1}{(C_\pi + C_\mu)r_\pi} = \frac{10^{12} \cdot 10^{-3}}{(12 + 2) \cdot 2.5} = 28.57 \cdot 10^{6} \ rps$$

$$f_\beta = \frac{\omega_\beta}{2\pi} = \frac{28.57}{6.28} \cdot 10^6 \ Hz = 4.55 \ MHz \quad f_T = \beta f_\beta = 455 \ MHz$$

$$\omega_z = \frac{g_m}{C_\mu} = \frac{40 \cdot 10^{-3} \cdot 10^{12}}{2} \ Hz = 20 \cdot 10^9 \ rps$$

$$f_z = \frac{\omega_z}{2\pi} = 3.18 \cdot 10^9 \ Hz = 3180 \ MHz$$
Scilab $f_T$ Plot

```scilab
//fT Bode Plot
Beta=100;
KdB = 20*log10(Beta);
fz=3180;
fp=4.55;
f=1:1:10000;
term1=KdB*sign(f); //Constant array of len(f)
term2=max(0,20*log10(f/fz)); //Zero for f < fz;
term3=min(0,-20*log10(f/fp)); //Zero for f < fp;
BodePlot=term1+term2+term3;
plot(f,BodePlot);
```
$h_{fe}$ Bode Plot

$h_{fe}$ vs. Frequency

Freq. (MHz.)

$T_f$
Multisim Simulation

Insert 1 ohm resistors – we want to measure a current ratio.

\[ h_{fe} = \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{\frac{1}{r_{\pi}} + s(C_\pi + C_\mu)} \]
Simulation Results

Low frequency $|h_{fe}|$

Unity Gain frequency about 440 MHz

Theory: $f_T = \beta f_\beta = 455 \text{ MHz}$