Differential Amplifier
Common & Differential Modes

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Most “Famous” Differential Amplifier

\[ V_o = A (V_1 - V_2) \]

\( V_1 \) & \( V_2 \) are single-ended input voltages defined w.r.t. ground.
Common and Differential Modes

Consider the two-input conceptual circuit shown below.

Define (convention) the voltages:

\[ V_c = \frac{V_1 + V_2}{2} \]
\[ V_d = V_1 - V_2 \]

\( V_c \) = “common mode voltage”
\( V_d \) = “differential mode voltage”

1. Let \( V_1 = V_2 = V \Rightarrow V_c = V \) & \( V_d = 0 \)
2. Let \( V_1 = -V_2 = V \Rightarrow V_c = 0 \) & \( V_d = 2V \)

\( V_c \) “common” to \( V_1 \) & \( V_2 \); \( V_d \) is split between \( V_1 \) & \( V_2 \) s.t. “difference” \( V_1 - V_2 = V_d \)
Replace $V_1$, $V_2$ with DM & CM Sources $V_d$, $V_c$

Definition

$V_c = \frac{V_1 + V_2}{2}$

$V_d = V_1 - V_2$

Solve for $V_1$ and $V_2$

$V_1 = V_c + \frac{V_d}{2}$

$V_2 = V_c - \frac{V_d}{2}$
Our Most “Famous” Differential Amplifier

\[ V_o = A (V_1 - V_2) \]

\[ V_1 \text{ & } V_2 \text{ are single-ended input voltages defined w.r.t. ground.} \]

\[ V_o = A_{v-dm} V_d + A_{v-cm} V_c \]

\[ CMRR = \frac{|A_{v-dm}|}{|A_{v-cm}|} \]
$V_1 = V_c + \frac{V_d}{2}$

$V_2 = V_c - \frac{V_d}{2}$

Typical Op-Amp Input Stage
Common and Differential Mode Currents

Solving for $I_1$ and $I_2$:

$I_1 = \frac{I_c}{2} + I_d$

$I_2 = \frac{I_c}{2} - I_d$

Definition:

$I_d = \frac{I_1 - I_2}{2}$

$I_c = I_1 + I_2$
1. Voltage sources defined as common and differential mode quantities.

2. Differential mode currents take “blue” path.

3. Common mode current takes “red” path.

4. When $Z_1 = Z_2 = Z$, the differential circuit is said to be balanced.

**OBSERVATIONS**

i. No DM $I_d$ flows through $Z_3$.

ii. No CM $I_c$ flows around $Z_1, Z_2$ loop.
Balanced Circuit Differential Mode ($V_C = 0$)

Typical Diff Amp Op-Amp Input Stage

$Z_1 = Z_2 \Rightarrow Q1 = Q2, R_{C1} = R_{C2}, R_{E1} = R_{E2}$ and $R_{B1} = R_{B2}$

balanced circuit $\Rightarrow Z_1 = Z_2 = Z$
Analyze Circuits by Superposition
Balanced Circuit Differential Mode ($V_C = 0$)

$$I_1 = I_d$$
$$V_{d/2}$$
$$I_2 = -I_d$$
$$- V_{d/2}$$

$$I_c = 0$$

balanced circuit => $Z_1 = Z_2 = Z$

$$I_c = I_1 + I_2 = I_d - I_d = 0$$

$$I_c = I_1 + I_2 = \frac{V_d}{2} - V_x + \frac{-V_d}{2} - V_x = -2V_x$$

$$I_c = 0 \Rightarrow V_x = 0$$

$$I_d = I_1 - I_2 = \frac{V_d}{2} - \frac{-V_d}{2} = \frac{V_d}{2Z}$$

$$Z_{in-dm} = \frac{V_d}{I_d} = 2Z$$

NOTE: If $Z_2 = Z_1 + \delta Z$, then $I_c \neq 0 \Rightarrow V_x \neq 0$. 
Analyze Circuits by Superposition

Balanced Circuit Common Mode ($V_d = 0$)

\[ I_c = \frac{V_c}{Z_1 \parallel Z_2 + Z_3} = \frac{V_c}{\frac{Z}{2} + Z_3} \]

\[ I_1 = I_c / 2 \]
\[ I_2 = I_c / 2 \]
\[ \Rightarrow I_d = 0 \]

\[ Z_{in-cm} = \frac{V_c}{I_c} = \frac{Z}{2} + Z_3 \]

balanced circuit $\Rightarrow Z_1 = Z_2 = Z$
Summary

Definitions:

\[ V_c = \frac{V_1 + V_2}{2} \]
\[ I_c = I_1 + I_2 \]
\[ V_d = V_1 - V_2 \]
\[ I_d = \frac{I_1 - I_2}{2} \]

\[ V_1 = V_c + \frac{V_d}{2} \]
\[ V_2 = V_c - \frac{V_d}{2} \]
\[ I_1 = \frac{I_c}{2} + I_d \]
\[ I_2 = \frac{I_c}{2} - I_d \]

All V's & I's have differential & common mode components
Summary cont.

In a **balanced** differential circuit \((Z_1 = Z_2 = Z)\):

1. Differential mode voltages result in differential mode currents.
2. Common mode voltages result in common mode currents.

The **differential mode input impedance** of a balanced circuit is:

\[
Z_{in-dm} = Z_1 + Z_2 = 2Z
\]

The **common mode input impedance** of a balanced circuit is:

\[
Z_{in-cm} = Z_1 \parallel Z_2 + Z_3 = \frac{Z}{2} + Z_3
\]
BJT Differential Amplifier – DC Bias View

Collector bias path inherently common mode.

Choose $I_{cm}$ and $R_C$ for approximate $\frac{1}{2} V_{CC}$ drop across $R_C$.

Recall

\[
\begin{align*}
I_1 &= I_c/2 + I_d \\
I_2 &= I_c/2 - I_d
\end{align*}
\]

0 for dc bias
BJT Differential Amplifier – AC Signal View

\[ v_{od} = v_{o2} - v_{o1} \]

\[ v_{ic} = \frac{v_{i1} + v_{i2}}{2} \]

\[ v_{id} = v_{i1} - v_{i2} \]
BJT Differential Amplifier – AC Diff Mode View

Single-ended DM outputs: $v_{o1d}, v_{o2d}$
Differential DM output: $v_{odm} = v_{o1d} - v_{o2d}$

NOTE: $i_{b1d} = i_{b1}$; $i_{b2d} = -i_{b2}$

DM input impedance: $Z_{in-dm} = \frac{v_{id}}{i_{b1d}}$

$A_{vdm} = \frac{v_{odm}}{v_{id}}$

$s-e A_{dm}$
BJT Differential Amplifier – AC Cmn Mode View

NOTE: \( i_{b1c} = i_{b1} \); \( i_{b2c} = i_{b2} \)

Single-ended CM outputs: \( v_{o1c}, v_{o2c} \)

Differential CM output: \( v_{ocm} = v_{o2c} - v_{o1c} \)

\[
A_{vcm} = \frac{v_{ocm}}{v_{icm}}
\]

\[
A_{vcm1,2} = \frac{v_{o1,2c}}{v_{ic}}
\]

CM input impedance

\[
Z_{in-cm} = \frac{v_{ic}}{i_{b1c}}
\]
BJT Differential Amplifier – Small-signal View

\[ i_{bl} = \frac{i_{ic}}{2} + i_{id} \]

\[ i_{e1} = i_{e1d} + \frac{(1 + \beta) i_{ic}}{2} \]

\[ i_{e2} = -i_{e2d} + \frac{(1 + \beta) i_{ic}}{2} \]

\[ Z_1 = Z_2 = (\beta + 1) r_e \]

\[ Z_3 = (\beta + 1) r_o \]

NOTE:

1. \( r_o \) for amplifier Q1 & Q2 is ignored.
2. \( v_{id}/2 \) and \( v_{ic} \) are ac small signals
Superposition Small-signal Analysis

Differential Mode \((v_{ic} = 0)\)

Balanced circuit \(\Rightarrow i_{e2} = -i_{e1} = i_e, i_{b2} = -i_{b1} = i_b, i_{c2} = -i_{c1} = i_c\)

\[
\begin{align*}
Z_1 &= Z_2 = Z = (\beta + 1)r_e \\
Z_3 &= (\beta + 1)r_o
\end{align*}
\]

DM Path (ignoring both \(R_B\)):

\[
\frac{v_{id}}{2} = i_{ed}r_e + i_{ed}r_e - \frac{v_{id}}{2}
\]

\[
v_{id} = 2r_ei_{ed} = 2(\beta + 1)r_ei_{bd}
\]

\[
Z_{in-dm} = \frac{v_{id}}{i_{bd}} = 2(\beta + 1)r_e
\]

Recall: \(Z_{in-dm} = 2Z\)
Superposition Small-signal Analysis

Differential Mode \((v_{ic} = 0)\) cont.

Balanced circuit

\[ i_{bd} \]

\[ v_{id}/2 \]

\[ i_{ed} \]

\[ i_{bd} \]

\[ R_B \]

\[ - \beta i_{bd} \]

\[ R_C \]

\[ + v_{odm} \]

\[ - v_{odm} \]

\[ - \beta i_{bd} \]

\[ r_e \]

\[ i_e \]

\[ r_o \]

\[ v_x \]

\[ i_i \]

\[ v_{id}/2 \]

\[ i_{ed} \]

\[ i_{bd} \]

\[ v_{o2d} - v_{o1d} \]

\[ v_{odm} = R_C \beta (i_{bd}) - (-R_C \beta i_{bd}) \]

\[ i_{bd} = \frac{v_{id}}{2} \frac{1}{(\beta + 1)r_e} \]

\[ v_{o2d} = \frac{2 R_C \beta v_{id}}{(\beta + 1)r_e} \]

\[ v_{o1d} = \frac{2 R_C \beta v_{id}}{(\beta + 1)r_e} \]

DM Differential voltage gain:

\[ A_{vdm} = \frac{v_{odm}}{v_{id}} = \frac{R_C \beta}{(\beta + 1)r_e} \approx \frac{R_C}{r_e} \]

DM Single-ended voltage gains:

\[ A_{vdm2} = - A_{vdm1} = \frac{v_{o2d}}{v_{id}} = \frac{\beta R_C}{2(\beta + 1)r_e} \approx \frac{R_C}{2r_e} \]
Superposition Small-signal Analysis

Common Mode \( (v_{id} = 0) \)

Balanced circuit

\[ Z_1 = Z_2 = (\beta + 1)r_e \]

\[ Z_3 = (\beta + 1)r_o \]

CM path: both \( r_e \) are in parallel

\[ v_{ic} = (\beta + 1)r_e i_{bc} + 2(\beta + 1)r_o i_{bc} \]

\[ = (\beta + 1)[r_e + 2r_o] i_{bc} \]

Note: \( \frac{i_{ic}}{2} = i_{bc} \)

Recall: \( Z_{in-cm} = Z || Z + Z_3 = \frac{Z}{2} + Z_3 \)
**Superposition Small-signal Analysis**

**Common Mode \((v_{id} = 0)\)**

Balanced circuit

\[
\begin{align*}
\beta i_{bc} & \quad R_C & \quad v_{ocm} & \quad \beta i_{bc} \\
R_B & \quad r_e & \quad i_{cc} & \quad R_C & \quad i_{cc} & \quad R_B
\end{align*}
\]

\[i_{bc} = i_{ic} / 2\]

\[(1 + \beta) i_{ic} \quad i_{bc} = i_{ic} / 2\]

\[v_{ocm} = v_{o2c} - v_{o1c}\]

\[v_{ocm} = -R_C \beta i_{bc} - (-R_C \beta i_{bc}) = 0\]

\[v_{ic} = (\beta + 1)(r_e + 2r_o i_{bc})\]

\[i_{bc} = \frac{v_{ic}}{(\beta + 1)(r_e + 2r_o)} \approx \frac{v_{ic}}{2(\beta + 1)r_o}\]

\[v_{o2c} = v_{o1c} = -R_C \beta i_{bc} = \frac{-R_C \beta}{2(\beta + 1)r_o} v_{ic}\]

**CM Differential voltage gain:**

\[A_{vcm} = \frac{v_{ocm}}{v_{ic}} = 0\]

**CM Single-ended voltage gains:**

\[A_{vcm1} = A_{vcm2} = \frac{v_{o1c}}{v_{ic}} = -\frac{R_C \beta}{2(\beta + 1)r_o} \approx -\frac{R_C}{2r_o}\]
Summary

Differential mode:

\[ Z_{i\text{m-dm}} = 2(\beta + 1) r_e \]

Differential DM Voltage gain:

\[ A_{vdm} = \frac{v_{odm}}{v_{id}} \approx \frac{R_C}{r_e} \]

Single-Ended DM Voltage gain:

\[ A_{vdm1} = -A_{vdm2} = \frac{v_{old}}{v_{id}} \approx -\frac{R_C}{2r_e} \]

Differential-output

\[ CMRR = \frac{A_{vdm}}{A_{cdm}} = \infty \quad \text{i.f.f. Balanced} \]

Common mode:

\[ Z_{i\text{m-cm}} = (\beta + 1) \left( \frac{r_e}{2} + r_o \right) \approx (\beta + 1) r_o \]

Differential CM Voltage gain:

\[ A_{vcm} = \frac{v_{ocm}}{v_{ic}} = 0 \]

Single-Ended CM Voltage gain:

\[ A_{vcm1} = A_{vcm2} = \frac{v_{olc}}{v_{ic}} \approx -\frac{R_C}{2r_o} \]

Single-ended-output (balanced)

\[ CMRR = \frac{A_{vdm1,2}}{A_{vcm1,2}} \approx 20 \log_{10} \left( \frac{r_o}{r_e} \right) \]
Practical Signal Excitation to Test CM
Practical Signal Excitation to Test DM

\[ v_1 = \frac{v_i}{2} + \frac{v_i}{2} = v_i \]

\[ v_2 = \frac{v_i}{2} - \frac{v_i}{2} = 0 \]