Differential Amplifier Cont.

- Differential amplifier with emitter resistors
- Single-ended output
- Common mode rejection ratio (CMRR)
Quick Review

CM DC

DC Bias is ALWAYS COMMON MODE

\[ R_{C1} = R_{C2} = R_C = \frac{V_{CC} - V_C}{\alpha \frac{I_{CM}}{2}} \]

\[ R_{B1} = R_{B2} = R_B = \text{large} \]
Quick Review

**Differential mode:**

\[ Z_{in\text{-}dm} = 2(\beta+1)r_e \]

Differential DM Voltage gain:

\[ A_{vdm} = \frac{v_{odm}}{v_{id}} \approx \frac{R_C}{r_e} \]

Single-Ended DM Voltage gain:

\[ A_{vdm1} = -A_{vdm2} = \frac{v_{old}}{v_{id}} \approx -\frac{R_C}{2r_e} \]

**Common mode:**

\[ Z_{in\text{-}cm} = (\beta+1)\left(\frac{r_e}{2} + r_o\right) \approx (\beta+1)r_o \]

Differential CM Voltage gain:

\[ A_{vcm} = \frac{v_{ocm}}{v_{ic}} = 0 \]

Single-Ended CM Voltage gain:

\[ A_{vcm1} = A_{vcm2} = \frac{v_{ocl}}{v_{ic}} \approx -\frac{R_C}{2r_o} \]

\[ CMRR_{dif} = \frac{A_{vdm}}{A_{vcm}} = \infty \]

\[ CMRR_{s\text{-}e} = \frac{A_{vdm1}}{A_{vcm1}} = \frac{r_o}{r_e} \]
Differential Amplifier With Emitter Resistors

Balance assumed:

\[ R_{C1} = R_{C2} = R_C \]
\[ R_{E1} = R_{E2} = R_E \]
\[ R_{B1} = R_{B2} = R_B \]

\[ Q1 = Q2 \text{ (Matched Pair)} \]

What is the purpose for \( R_{E1} \) and \( R_{E2} \)?
**Differential Mode Analysis (v_{ic} = 0)**

Balanced circuit

NOTE:
1. $r_o$ for Q1 and Q2 are ignored.
2. Both $R_B$'s are ignored.

\[
Z_{in-dm} = 2(\beta + 1)(r_e + R_E)
\]

Differential DM Voltage gain:

\[
A_{vdm} = \frac{v_{odm}}{v_{id}} \approx \frac{R_C}{R_E + r_e}
\]

Single-Ended DM Voltage gains:

\[
A_{vdm1} = \frac{v_{old}}{v_{id}} = \frac{-\beta R_C}{2(\beta + 1)R_E} \approx \frac{-R_C}{2(R_E + r_e)}
\]

\[
A_{vdm2} = \frac{v_{o2d}}{v_{id}} = \frac{\beta R_C}{2(\beta + 1)R_E} \approx \frac{R_C}{2(R_E + r_e)}
\]
Common Mode Analysis ($v_{id} = 0$)

Balanced circuit

\[ Z_{in-cm} = (\beta + 1) \left( \frac{r_e + R_E}{2} + r_o \right) \approx (\beta + 1) r_o \]

Differential CM Voltage gain:

\[ A_{vcm} = \frac{v_{ocm}}{v_{ic}} = 0 \]

Single-Ended CM Voltage gains:

\[ A_{vcm1} = \frac{v_{o1c}}{v_{ic}} = -\frac{\beta R_C}{(\beta + 1)(r_e + R_E + 2r_o)} \approx -\frac{R_C}{2r_o} \]

\[ A_{vcm2} = \frac{v_{o2c}}{v_{ic}} = A_{vcm1} \]

NOTE:
1. $r_o$ for Q1 and Q2 are ignored.
2. Both $R_B$'s are ignored.
Common Mode Rejection Ratio

Common mode rejection ratio is the magnitude ratio of the differential mode gain over the common mode gain expressed in dB:

Differential:
\[ A_{vcm} = 0 \quad \text{(iff balanced)} \]

Single-Ended:
\[ A_{vcm2} = A_{vcm1} \approx -\frac{R_C}{2 r_o} \]

Differential:
\[ A_{vdm} \approx \frac{R_C}{R_E + r_e} \]

Single-Ended:
\[ A_{vdm2} = -A_{vdm1} \approx \frac{R_C}{2 (R_E + r_e)} \]

Differential:
\[ CMRR_{db} = 20 \log_{10} \left( \frac{A_{vcm}}{A_{vdm}} \right) = \infty \]

(Iff balanced)

Single-Ended:
\[ CMRR = 20 \log_{10} \left( \frac{A_{vcm1,2}}{A_{vdm1,2}} \right) \]

\[ CMRR \approx 20 \log_{10} \left( \frac{R_C}{2 (R_E + r_e)} \ast \frac{2 r_o}{R_C} \right) \]

\[ CMRR_{db} \approx 20 \log_{10} \left( \frac{r_o}{R_E + r_e} \right) \]
Let $R_{C2} > R_{Cl}$, s.t. $R_{Cl} = R_C$ & $R_{C2} = R_C + \Delta R_C$

where $\Delta R_C << R_C$

### Differential Output CM Gain

$$A_{vcm1} = \frac{v_{ocl}}{v_{ic}} \approx - \frac{R_C}{(r_e + R_E + 2r_0)} \approx - \frac{R_C}{2r_o}$$

$$A_{vcm2} = \frac{v_{ocl}}{v_{ic}} \approx - \frac{R_C + \Delta R_C}{2r_0}$$

$$= - \frac{R_C}{2r_o} \left(1 + \frac{\Delta R_C}{R_C}\right)$$

Hence!

$$A_{vcm1} \approx - \frac{R_C}{2r_o} \neq A_{vcm2}$$

$$A_{vcm} = A_{vcm2} - A_{vcm1} = - \frac{R_C}{2r_o} \frac{\Delta R_C}{R_C} \neq 0$$
What if the Diff Amp isn't Perfectly Balanced?

Let $R_{C1} = R_C$ and $R_{C2} = R_C + \Delta R_C$

From previous slide

$$A_{vcm}(unbal) = A_{vcm2} - A_{vcm1} = -\frac{R_C}{2} - \frac{\Delta R_C}{R_C}$$

$$A_{vdm1} \approx \frac{-R_C}{2(R_E + r_e)} \quad A_{vdm2} \approx \frac{-(R_C + \Delta R_C)}{2(R_E + r_e)}$$

$$A_{vdm}(unbal) = A_{vdm2} - A_{vdm1} \approx \frac{R_C}{R_E} \approx \frac{R_C}{R_E}$$

$$A_{vdm}(unbal) \approx A_{vdm}(bal) = A_{vdm}$$

$$CMRR_{db} = 20 \log_{10}\left(\frac{|A_{vdm}|}{|A_{vcm}(unbal)|}\right) = 20 \log_{10}\left(\frac{2r_o}{R_E + r_e} \frac{1}{\Delta R_C / R_C}\right) \neq \infty$$
What if the Diff Amp isn't Perfectly Balanced?

Single-ended outputs

\[ A_{vdm2}(\text{bal}) = -A_{vdm1}(\text{bal}) \approx \frac{R_C}{2(R_E + r_e)} \]

\[ A_{vdm1}(\text{unbal}) = -A_{vdm1}(\text{bal}) \quad A_{vdm2}(\text{unbal}) \neq -A_{vdm2}(\text{bal}) \approx \frac{R_C + \Delta R_C}{2(R_E + r_e)} \approx \frac{R_C}{2(R_E + r_e)} \]

\[ A_{vcm2}(\text{bal}) = A_{vcm1}(\text{bal}) \approx -\frac{R_C}{2r_o} \]

\[ A_{vcm1}(\text{unbal}) = -A_{vcm1}(\text{bal}) \quad A_{vcm2}(\text{unbal}) \neq A_{vcm2}(\text{bal}) \approx -\frac{R_C + \Delta R_C}{2r_o} \approx -\frac{R_C}{2r_o} \]

\[ \text{CMRR}_{db}(\text{unbal}) \approx \text{CMRR}_{db}(\text{bal}) = 20 \log_{10} \left( \frac{r_o}{R_E + r_e} \right) \]
Differential mode (balanced)

\[ Z_{in-dm} = \frac{v_{id}}{i_{bd}} = (\beta + 1) \frac{r_e}{2 \left( R_e + r_e \right)} \]

\[ A_{vdm2} = \frac{v_{o2d}}{v_{id}} = - A_{vdm1} \approx \frac{R_C}{2 \left( R_e + r_e \right)} \]

Common mode (balanced)

\[ Z_{in-cm} = \frac{v_{ic}}{2 i_{bc}} = (\beta + 1) \left( \frac{r_e + R_E}{2} + r_0 \right) \]

\[ A_{vcm1} = \frac{v_{oci}}{v_{ic}} = A_{vcm2} \approx \frac{R_C}{2 r_o} \]

\[ A_{vcm} = \frac{v_{ocm}}{v_{ic}} = 0 \quad \text{i.f.f. Balanced} \]

Unbalanced

\[ A_{vdm\,(unbal)} \approx A_{vdm\,(bal)} = A_{vdm} \]

\[ A_{vcm\,(unbal)} = \frac{v_{ocm}}{v_{ic}} \approx \frac{R_C}{2 r_o} \frac{\Delta R_C}{R_C} \]

\[ CMRR_{db} = \frac{A_{vdm,1,2}}{A_{vcm,1,2}} \approx 20 \log_{10} \left( \frac{r_o}{r_e + R_e} \right) \]

Differential-output

\[ CMRR = \infty \quad \text{i.f.f. Balanced} \]

\[ CMRR_{db} = 20 \log_{10} \left( \frac{2}{r_e + R_e} \frac{r_o}{\Delta R_C / R_C} \right) \]
Differential Amplifier Design

- Design with ideal emitter current source bias.
- Differential and common mode gain results
- Add finite output resistance $r_o$ to ideal current source.
- Replace ideal current source & $r_o$ with current mirror.
By inspection DC bias ($v_{i-dm} = v_{i-cm} = 0$) for Q1 & Q2 is common mode:

$$I_E = \frac{I}{2} = 5 \text{mA} \quad V_C = 7 \text{V}$$
$$V_{BE} = 0.7 \text{V}$$

Neglecting $I_B$: let $I_B = 0$

$$V_{Re} = 5 \text{V} \Rightarrow V_C = V_{CC} - V_{Re} = 7 \text{V}$$
$$I_C = I_E = 5 \text{mA} \Rightarrow V_{Re} = \frac{I_C}{R_C} = 5 \text{V}$$

Single-ended voltage gains w.r.t. $v_{i\\text{d}}/2$ and $v_{ic}$ (for Q2 side):

$$A_{vdm1} = \frac{v_{o1d}}{v_{id}} \approx \frac{-R_C}{2(R_E + r_e)} = -4.76$$
$$r_e = \frac{V_T}{I_E} \approx 5 \Omega$$
$$A_{vcm1} = \frac{v_{o1c}}{v_{ic}} \approx -\frac{R_C}{2r_o} = 0$$

ideal current source, i.e. $r_o = \infty$
Differential-Mode AC Gain Results

“Scope” output B at collector of Q1, i.e. $v_B = v_{old}$. 

Input voltage $v_A = v_{id}$

$0.14 \, V_{\text{peak}} \, \arg (0^\circ)$ at $f = 1 \, \text{kHz}$. 

Output voltage $v_B = v_{old}$

$1.33 \, V_{\text{peak}} \, \arg (180^\circ)$. 

Measured gain:

$$A_{vdm} = \frac{v_B}{v_A} = \frac{v_{old}}{v_{id}} = \frac{-0.664 \, V}{0.141 \, V} = -4.7 \approx \frac{-R_C}{2(R_E + r_e)}$$
Common Mode AC Results

"Scope" output B at collector of Q1, i.e. $v_B = v_{olc} = 0 \text{ V}$. Input voltage $v_A = v_{ic}$

$0.14 \text{ V}_{\text{peak}} \arg (0^\circ) \text{ at } f = 1 \text{ kHz}$.

Since $I$ is an ideal current source $r_o = \infty \implies A_{vcm1} = 0$.

$$A_{vcm1} = \frac{v_B}{v_A} = \frac{v_{olc}}{v_{ic}} = \frac{-0 \text{ V}}{0.14 \text{ V}} = 0$$

$$CMRR_{db} = 20 \log_{10} \left| \frac{A_{vdm1}}{A_{vcm1}} \right| = 20 \log_{10} \left| \frac{4.7}{0} \right| = \infty$$
Common Mode Results - Add $r_o$ to Model

\[ \begin{align*}
    v_{vcm1} &= v_{o1c} \approx -\frac{R_C}{2} = -0.005 \\
    v_{vdm1} &= v_{o1d} \approx -\frac{R_C}{2 \left( R_E + r_e \right)} = -4.76 \\
    A_{vcm1} &= \frac{v_{olc}}{v_{ic}} \approx -\frac{1}{2 r_o} = -0.005 \\
    CMRR_{db} &= 20 \log_{10} \left| \frac{A_{vdm1}}{A_{vcm1}} \right| = 59.6 \text{ dB}
\end{align*} \]

insert finite $r_o$ to model non-ideal current source

Kenneth R. Laker, update KRL 18Oct13
Common Mode Results - Add Finite $r_o$

$r_o = 100 \; \text{k}\Omega$

“Scope” output B at collector of Q1, i.e. $v_B = v_{olcm}$.

Input voltage $v_A = v_{ic}$

1.4 $V_{peak}$ $\arg (0^\circ)$ at $f = 1 \; \text{kHz}$.

Output voltage $v_B = v_{olcm}$

0.007 $V_{peak}$ $\arg (180^\circ)$.

$A_{vcm1} \approx \frac{-0.007}{1.4} = -0.005$

$A_{vdm1} = -4.7$ (unchanged)

$CMRR_{db} \approx 20 \log_{10} \left( \frac{4.7}{0.005} \right) \approx 59.5 \; \text{dB}$
**Simulation with 10 mA Current Mirror**

Matched 2N2222 BJTs (Q1, Q2, Q3 and Q4).

\[ I_{C3} \approx 10 \text{ mA} \]

\[ \Rightarrow I_{C1} = I_{C2} \approx 5 \text{ mA} \]

\[ R_{ref} = 2.33 \text{ k} \Omega \]

\[ I_{REF} = \frac{V_{CC} - V_{BE4} - V_{EE}}{R_{ref}} \approx 10 \text{ mA} \]
Simulation with 10 mA Current Mirror cont.

Input voltage \( v_A = v_{ic} \)

1.4 \( V_{peak} \) \( \arg (0^\circ) \) at \( f = 1 \ kHz \).

Output voltage \( v_B = v_{o1c} \)

60 \( mV_{peak} \) \( \arg (180^\circ) \).

\[
A_{vcm1} \approx -0.043
\]

Infer \( r_o \) with \( R_C = 1 \ k\Omega \)

\[
r_o = \frac{-R_C}{2 A_{vcm1}} = \frac{1 \ k\Omega}{0.086} = 11.67 \ k\Omega
\]

Infer \( V_A \) with \( r_o = 11.67 \ k\Omega \)

\[
r_o = \frac{V_A}{I_C} \Rightarrow V_A = r_o I_C = 58.3 \ V
\]
Simulation with 10 mA CM - Bode Plots

10 mA current: $A_{vcm1}(dB) + 3 \text{ dB frequency}$

$$A_{vcm1}(db)(f = 1 \text{ kHz}) \approx 20 \log_{10} \frac{R_C}{2r_o} \approx 20 \log_{10} \frac{0.06}{1.4} = -27.3 \text{ dB}$$

$$A_{vcm1}(db)(f = 9.7 \text{ MHz}) = -24.2 \text{ dB}$$

$$f_B = -3 \text{ dB } A_{vcm1} \text{ corner frequency}$$
Simulate the 10 mA Design with 2 pF Parasitics

-3dB common mode bandwidth with 2 pF base-emitter and base collector caps.

Parasitic caps reduces $A_{v-cm(dB)}$ break frequency $f_B$ from 9.7 MHz to 2.5 MHz!

RECALL: 2 pF is about the capacitance between 2 rows of Protoboard pins!
Observations

1). $r_o$ can be inferred from $A_{vcm}$.

2). Minimize parasitic capacitance around mirror transistor to increase common mode gain -3dB bandwidth.

3). Since no differential mode current flows through the mirror transistor ($Q3$, i.e. $r_o$), it should have no effect on differential mode performance.