Differential Amplifier Offset

- Causes of DC voltage and current offset
- Modeling DC offset
  - $R_C$ mismatch
  - $I_S$ mismatch
  - $\beta$ mismatch
To understand imbalance effects, consider the simple case where the differential amplifier bases are grounded and there are no external emitter or base resistors:

\[
V_{c1} = V_{cc} - R_{c1} I_{c1}
\]

\[
V_{c2} = V_{cc} - R_{c2} I_{c2}
\]

\[
V_{O_{dm}} = -R_{c2} I_{c2} - (-R_{c1} I_{c1}) = R_{c1} I_{c1} - R_{c2} I_{c2}
\]

Hence, if \( V_{O_{dm}} = 0 \) then \( R_{c1} I_{c1} = R_{c2} I_{c2} \)
First, assume that:

\[ I_{C1} = I_{C2} = \alpha \frac{I}{2} \]

Let:

\[ R_{C1} \neq R_{C2} \quad \text{and} \quad R_{C2} > R_{C1} \]

Split the mismatch between both collector resistors:

\[ R_C = \frac{R_{C2} + R_{C1}}{2} \quad \text{(common or mean or nominal)} \]

\[ \Delta R_C = R_{C2} - R_{C1} \quad \text{(differential)} \]

Then, it can be shown that:

\[ R_{C2} = R_C + \frac{\Delta R_C}{2} = R_C \left( 1 + \frac{1}{2} \frac{\Delta R_C}{R_C} \right) \]

\[ R_{C1} = R_C - \frac{\Delta R_C}{2} = R_C \left( 1 - \frac{1}{2} \frac{\Delta R_C}{R_C} \right) \]
Recall:

\[ V_{O-\text{dm}} = R_{C2} I_{C2} - R_{C1} I_{C1} \]
\[ I_{C1} = I_{C2} = \alpha \frac{I}{2} \]

Hence:

\[ V_{O-\text{dm}} = \left( R_C - \frac{\Delta R_C}{2} \right) I_{C1} - \left( R_C + \frac{\Delta R_C}{2} \right) I_{C2} \]

or:

\[ V_{O-\text{dm}} = \left( R_C - \frac{\Delta R_C}{2} \right) \alpha \frac{I}{2} - \left( R_C + \frac{\Delta R_C}{2} \right) \alpha \frac{I}{2} \]

random variable (rv)

output offset voltage

\[ V_{O-\text{dm}} = -\alpha \frac{I}{2} \Delta R_C \]
Collector-collector voltage due to resistor mismatch:

\[ V_{O-\text{dm}} = -\alpha \frac{I}{2} \Delta R_C \]

Define the **input offset voltage** as that input voltage that will cancel \( V_{O-\text{dm}} \). If the amplifier differential gain is \( G_{\text{dm}} \):

\[ V_{OS} \equiv \frac{V_{O-\text{dm}}}{G_{\text{dm}}} \quad V_{OS} \text{ is highly variable, rv & can be + or -} \]

Input offset voltage is the output offset voltage referred to the input due to mismatch \( \Delta R_C \).
**Differential Mode Small-signal Analysis**

Collector-collector voltage:

\[ V_{o-\text{dm}} = V_{c2g-\text{dm}} - V_{c1g-\text{dm}} \]

\[ V_{o-\text{dm}} = R_{C2} \beta i_{b-\text{dm}} - (-R_{C1} \beta (-i_{b-\text{dm}})) \]

\[ \frac{V_{o-\text{dm}}}{V_{dm}} = \frac{2 \beta R_C}{2(\beta+1) r_e} \]

\[ r_e = \frac{V_T}{I_E} = \frac{\beta}{\beta+1} \]

\[ I_C = \frac{\beta}{\beta+1} g_m \]

\[ G_{\text{dm}} = \frac{I_C}{V_T} \]

\[ R_C = g_m R_C \]
\[ V_{OS} = \left| \frac{V_{O-\text{dm}}}{G_{\text{dm}}} \right| \quad \text{where} \quad G_{\text{dm}} = g_m R_C \]

\[ V_{OS} = \frac{\alpha \frac{I}{2} \Delta R_C}{g_m R_C} \]

\[ g_m = \frac{I_C}{V_T} = \alpha \frac{I}{2} \frac{1}{V_T} \]

Input referred offset due to \( \Delta R_C \) mismatch:

\[ V_{OS(\Delta R_C)} = V_T \frac{\Delta R_C}{R_C} \]
Offset Voltage From Transistor Mismatch

Perfect balance requires:

\[ V_{O_{-\text{dm}}} = 0 \Rightarrow R_{C1} I_{C1} = R_{C2} I_{C2} \]

Previous case considered unequal resistors \( R_{C1} \neq R_{C2} \). Here consider unequal collector currents.

ASSUME: 1. \( V_{BE1} = V_{BE2} \)

2. \( V_{T1} = V_{T2} \)

Only difference is in the saturation currents of the transistors, i.e.

\[ I_{S1} \neq I_{S2} \]
Again using common and differential mode concepts:

\[ I_S = \frac{I_{S2} + I_{S1}}{2} \]

\[ \Delta I_S = I_{S2} - I_{S1} \]

The two transistor saturation currents are:

\[ I_{S2} = I_S + \frac{\Delta I_S}{2} \]

\[ I_{S1} = I_S - \frac{\Delta I_S}{2} \]
Large signal Model:

\[ I_{C1} = \left( I_S - \frac{\Delta I_S}{2} \right) e^{\frac{V_{BE}}{V_T}} \]

\[ I_{C1} = I_S e^{\frac{V_{BE}}{V_T}} - \frac{\Delta I_S}{2} e^{\frac{V_{BE}}{V_T}} \]

also

\[ I_{C2} = I_S e^{\frac{V_{BE}}{V_T}} + \frac{\Delta I_S}{2} e^{\frac{V_{BE}}{V_T}} \]

The parallel current sources are illustrated in the schematic.
Note: differential $I_C$ components cause current flow in opposite directions through the $R_C$'s resulting in an offset voltage. The common mode $I_C$ components cause no offset voltage.

$$I_{C2} = I_S e^{\frac{V_{BE}}{V_T}} + \frac{\Delta I_S}{2} e^{\frac{V_{BE}}{V_T}} = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{\Delta I_S}{2 I_S}\right)$$

$$= \alpha \frac{I}{2} \left(1 + \frac{\Delta I_S}{2 I_S}\right)$$

$$I_{C1} = I_S e^{\frac{V_{BE}}{V_T}} - \frac{\Delta I_S}{2} e^{\frac{V_{BE}}{V_T}} = I_S e^{\frac{V_{BE}}{V_T}} \left(1 - \frac{\Delta I_S}{2 I_S}\right)$$

$$= \alpha \frac{I}{2} \left(1 - \frac{\Delta I_S}{2 I_S}\right)$$

$$V_{O_{-dm}} = R_C I_{C1} - R_C I_{C2} = -2 \alpha \frac{I}{2} \frac{\Delta I_S}{2 I_S} R_C = -\frac{\alpha I}{2} \frac{\Delta I_S}{I_S} R_C$$
Recall:

\[
g_m = \frac{I_C}{V_T} = \frac{\alpha I}{2} \frac{1}{V_T}
\]

\[
V_{O-\text{dm}} = -\alpha I \frac{I_s \Delta I_s}{I_s^2} R_C
\]

\[
V_{O-\text{dm}} = -\alpha I \frac{V_T \Delta I_s}{2 V_T} I_s R_C
\]

\[
V_{O-\text{dm}} = -g_m V_T \frac{\Delta I_s}{I_s} R_C
\]

\[
V_{OS(\Delta I_s)} = \frac{V_{O-\text{dm}}}{G_{dm}} = \frac{V_{O-\text{dm}}}{g_m R_C} = V_T \frac{\Delta I_s}{I_s}
\]

\[
V_{OS(\Delta I_s)} = V_T \frac{\Delta I_s}{I_s}
\]
Offset Voltage Summary

We considered two sources of offset voltage:
- Unbalanced collector resistors
- Unbalanced saturation currents
- Mismatched transistor geometries

We ignored base or emitter circuit unbalance.

Since the relationship between resistor and current unbalances are statistically random and assumed independent, we combine their effect as an rms quantity:

\[ V_{OS(rms)} = \sqrt{\left( V_{OS-\Delta R_C} \right)^2 + \left( V_{OS-\Delta I_S} \right)^2} = V_T \sqrt{\left( \frac{\Delta R_C}{R_C} \right)^2 + \left( \frac{\Delta I_S}{I_S} \right)^2} \]
Average & Offset Base (input) Bias Currents

Consider the case where the base currents differ $I_{B1} \neq I_{B2}$.

Since $I_{B1}$ & $I_{B2}$ are related to bias current $I$, their mismatch is due to $\beta_1 \neq \beta_2$.

Let's use differential-common mode models for beta mismatch:

$$\beta_1 = \beta + \frac{\Delta \beta}{2}$$

$$\beta_2 = \beta - \frac{\Delta \beta}{2}$$
Using this notation, the two base currents are:

\[ I_{B1} = \frac{I}{2} \left( \frac{1}{\beta_1 + 1} \right) = \frac{I}{2} \left( \frac{1}{\beta + \frac{\Delta \beta}{2} + 1} \right) = \frac{I}{2} \left( \frac{1}{\beta + 1} \right) \left( 1 + \frac{\Delta \beta}{2(\beta + 1)} \right) \]

\[ I_{B2} = \frac{I}{2} \left( \frac{1}{\beta_2 + 1} \right) = \frac{I}{2} \left( \frac{1}{\beta - \frac{\Delta \beta}{2} + 1} \right) = \frac{I}{2} \left( \frac{1}{\beta + 1} \right) \left( 1 - \frac{\Delta \beta}{2(\beta + 1)} \right) \]

For \( x < 1 \) we can expand the fraction as the series:

\[ \frac{1}{1 + x} = 1 - x + x^2 - x^3 + \ldots \]

where 

\[ x = \pm \frac{\Delta \beta}{2(\beta + 1)} \approx \frac{1}{2} \frac{\Delta \beta}{\beta} \]

nonlinear f(\( \Delta \beta \))
Using the expansion and approximating $\beta + 1 \approx \beta$:

$$I_{B1} \approx \frac{I}{2} \frac{1}{\beta + 1} \left(1 - \frac{1}{2} \frac{\Delta \beta}{\beta}\right)$$

$$I_{B2} \approx \frac{I}{2} \frac{1}{\beta + 1} \left(1 + \frac{1}{2} \frac{\Delta \beta}{\beta}\right)$$

$$I_{OS} \equiv |I_{B1} - I_{B2}| = \frac{I}{\beta + 1} \frac{\Delta \beta}{\beta}$$

(input offset current)

Since the input bias current $I_B$ is defined as:

$$I_B = \frac{I_{B1} + I_{B2}}{2} = \frac{I}{2(\beta + 1)}$$

The base or input offset current can also be written as:

$$I_{OS(\Delta \beta)} = I_B \left(\frac{\Delta \beta}{\beta}\right)$$