Differential Amplifier Offset

- Causes of dc voltage and current offset
- Modeling dc offset
  - $R_C$ mismatch
  - $I_S$ mismatch
  - $\beta$ mismatch
  - dc offsets in differential amplifiers due to component mismatch can be modeled as differential phenomena
1. Let the ac sources be zero, i.e. the bases of matched Q1 & Q2 are connected to ground.

2. To further simplify, let there be no external base $R_B$ and emitter resistors $R_E$.

3. Let the mismatch be in $R_C$ where $R_{C1} \neq R_{C2}$.

Since $Q1=Q2$ and $I$ is fixed, currents are matched i.e.

$$I_{C1} = I_{C2} = \frac{\beta}{\beta + 1} \frac{I}{2} = \alpha \frac{I}{2}$$

$$V_{C1} = V_{CC} - R_{C1} I_{C1}$$

$$V_{C2} = V_{CC} - R_{C2} I_{C2}$$

$$V_{Odm} = V_{C2} - V_{C1} = R_{C1} I_{C1} - R_{C2} I_{C2}$$

If $V_{Odm} = 0 \Rightarrow R_{C1} I_{C1} = R_{C2} I_{C2}$

$$I_{C1} = I_{C2} = \alpha \frac{I}{2} \Rightarrow R_{C1} = R_{C2}$$
Assume matched currents:

\[ I_{C1} = I_{C2} = \alpha \frac{I}{2} \]

Let: \( R_{C1} \neq R_{C2} \) s.t. \( R_{C2} > R_{C1} \)

Split the mismatch between both collector resistors, and decompose into common and differential components:

\[ R_C = \frac{R_{C2} + R_{C1}}{2} \quad (\text{common}) \]

\[ \Delta R_C = R_{C2} - R_{C1} \quad (\text{differential - mismatch}) \]

Solving for \( R_{C1} \) & \( R_{C2} \):

\[ R_{C2} = R_C + \frac{\Delta R_C}{2} = R_C \left( 1 + \frac{1}{2} \frac{\Delta R_C}{R_C} \right) \]

\[ R_{C1} = R_C - \frac{\Delta R_C}{2} = R_C \left( 1 - \frac{1}{2} \frac{\Delta R_C}{R_C} \right) \]
Recall:

\[ V_{Odm} = R_{C2} I_{C2} - R_{C1} I_{C1} \]

\[ I_{C1} = I_{C2} = \alpha \frac{I}{2} \quad R_{C1} \neq R_{C2} \]

Hence:

\[ V_{Odm} = \left( R_C + \frac{\Delta R_C}{2} \right) I_{C2} - \left( R_C - \frac{\Delta R_C}{2} \right) I_{C1} \]

or:

\[ V_{Odm} = \left( R_C + \frac{\Delta R_C}{2} \right) \alpha \frac{I}{2} - \left( R_C - \frac{\Delta R_C}{2} \right) \alpha \frac{I}{2} \]

iff balanced, \( R_{C1} = R_{C2} \) : \( V_{Odm} = 0 \) V

random variable (rv)

output offset voltage
Collector-collector voltage due to resistor mismatch:

\[ V_{Odm} = \alpha \frac{I}{2} \Delta R_C \]

Define the **input offset voltage** as that input voltage that will cancel \( V_{Odm} \). If the amplifier differential gain is \( A_{vdm} \):

\[ V_{OS} \equiv \frac{V_{Odm}}{A_{vdm}} \]

\( V_{OS} \) is highly variable, \( rv \) & can be + or -

Input offset voltage is the output offset voltage referred to the input due to mismatch \( \Delta R_C \).
\[ V_{OS} = \frac{V_{Odm}}{A_{vdm}} \]

where

\[ V_{Odm} = \alpha \frac{I}{2} \Delta R_C \]

\[ A_{vdm} = g_m R_C \]

\[ V_{OS} = \frac{\alpha I}{2} \Delta R_C \]

\[ g_m = \frac{I}{V_T} = \frac{I}{2 \cdot \frac{1}{V_T}} \]

Input referred offset due to \( \Delta R_C \) mismatch:

\[ V_{OS(\Delta R_C)} = V_T \frac{\Delta R_C}{R_C} \]
Offset Voltage From Transistor Mismatch

Perfect balance requires:

\[ V_{Odm} = 0 \Rightarrow R_{C1} I_{C1} = R_{C2} I_{C2} \]

Previous case considered \( R_{C1} \neq R_{C2} \)

& \( Q_1 = Q_2 \Rightarrow I_{C1} = I_{C2} \).

Consider now \( Q_1 \neq Q_2 \Rightarrow I_{C1} \neq I_{C2} \).

ASSUME:  
1. \( R_{C1} = R_{C2} = R_C \)
2. \( V_{BE1} = V_{BE2} \)
3. \( V_{T1} = V_{T2} \)

Only difference is in the saturation currents of the transistors, i.e.

\[ I_{S1} \neq I_{S2} \]
Again using common and differential mode concepts:

\[
I_S = \frac{I_{S2} + I_{S1}}{2}
\]

\[
\Delta I_S = I_{S2} - I_{S1}
\]

The two transistor saturation currents are:

\[
I_{S2} = I_S + \frac{\Delta I_S}{2} + \frac{v_{id}}{2}
\]

\[
I_{S1} = I_S - \frac{\Delta I_S}{2} - \frac{v_{id}}{2}
\]
\[ I_{C1} \approx I_S e^{\frac{V_{BE}}{V_T}} = \left( I_S - \frac{\Delta I_S}{2} \right) e^{\frac{V_{BE}}{V_T}} \]

\[ I_{C2} \approx I_S e^{\frac{V_{BE}}{V_T}} = \left( I_S + \frac{\Delta I_S}{2} \right) e^{\frac{V_{BE}}{V_T}} \]
The parallel current sources are illustrated in the schematic.

Large signal Model:

\[ I_{C1} \approx IS \left( \frac{\Delta IS}{2} \right) e^{V_{BE}/VT} \]

\[ I_{C2} \approx IS e^{V_{BE}/VT} - \frac{\Delta IS}{2} e^{V_{BE}/VT} \]

\[ I_{C1} \approx IS e^{V_{BE}/VT} \]

\[ I_{C2} \approx IS e^{V_{BE}/VT} + \frac{\Delta IS}{2} e^{V_{BE}/VT} \]
Note: differential $I_C$ components cause current flow in opposite directions through the $R_C$'s resulting in an offset voltage. The common $I_C$ components cause no offset voltage.

$$I_{C2} \approx \frac{I}{2} \left( 1 + \frac{\Delta I_S}{2 I_S} \right)$$

$$I_{C1} \approx \frac{I}{2} \left( 1 - \frac{\Delta I_S}{2 I_S} \right)$$

$$V_{Odm} = R_C I_{C2} - R_C I_{C1} \approx \frac{I}{2} \frac{\Delta I_S}{I_S} R_C$$
Recall:

\[ g_m = \frac{I_C}{V_T} \approx \frac{I}{2} \frac{1}{V_T} \]

\[ V_{Odm} \approx \frac{I}{2} \frac{\Delta I_S}{I_S} R_C \]

\[ V_{Odm} \approx \frac{I}{2} \frac{V_T}{V_T} \frac{\Delta I_S}{I_S} R_C \]

\[ V_{Odm} \approx g_m V_T \frac{\Delta I_S}{I_S} R_C \]

\[ V_{OS(\Delta I_S)} = \frac{V_{Odm}}{A_{vdm}} = \frac{V_{Odm}}{g_m R_C} \approx V_T \frac{\Delta I_S}{I_S} \]

\[ V_{OS(\Delta I_S)} \approx V_T \frac{\Delta I_S}{I_S} \]

random variable
Offset Voltage Summary

1. We considered two sources of offset voltage:
   a. Unbalanced collector resistors
   b. Unbalanced saturation currents
      i. Due to mismatched transistor geometries

2. We ignored base or emitter circuit unbalance

3. Since the relationship between resistor and current unbalances are random and assumed independent, we combine their effect as an rms quantity:

$$V_{OS(rms)} = \sqrt{(V_{OS-\Delta R_c})^2 + (V_{OS-\Delta I_s})^2} = V_T \sqrt{\left(\frac{\Delta R_c}{R_c}\right)^2 + \left(\frac{\Delta I_s}{I_s}\right)^2}$$
Average & Offset Base (input) Bias Currents

Consider the case where the base currents differ $I_{B1} \neq I_{B2}$.

Since $I_{B1}$ & $I_{B2}$ are related to bias current $I$, their mismatch may be due to $\beta_1 \neq \beta_2$.

Let's represent $\beta_1$ & $\beta_2$ in terms of differential & common components:

$$
\beta_1 = \beta + \frac{\Delta \beta}{2} + \frac{v_{id}}{2}
$$

$$
\beta_2 = \beta - \frac{\Delta \beta}{2} - \frac{v_{id}}{2}
$$
Using this notation, the two base currents are:

\[
I_{B1} \approx \frac{I}{2} \frac{1}{\beta_1} = \frac{I}{2} \frac{1}{\beta} + \frac{\Delta \beta}{2} \frac{1}{\beta} (1 + \frac{\Delta \beta}{2 \beta})
\]

\[
I_{B2} \approx \frac{I}{2} \frac{1}{\beta_2} = \frac{I}{2} \frac{1}{\beta} - \frac{\Delta \beta}{2} \frac{1}{\beta} (1 - \frac{\Delta \beta}{2 \beta})
\]

For \( x \ll 1 \) we can expand the fraction as the series:

\[
\frac{1}{1+x} = 1 - x + x^2 - x^3 + ... \approx 1 - x
\]

where \( x \approx \pm \frac{\Delta \beta}{2 \beta} \)
Using the expansion and approximating $\beta + 1 \approx \beta$:

$$I_{B1} \approx \frac{I}{2} \frac{1}{\beta} \left(1 - \frac{1}{2} \frac{\Delta \beta}{\beta}\right)$$

$$I_{B2} \approx \frac{I}{2} \frac{1}{\beta} \left(1 + \frac{1}{2} \frac{\Delta \beta}{\beta}\right)$$

$$I_{OS(\Delta \beta)} = \left|I_{B1} - I_{B2}\right| = \frac{I}{2 \beta} \left|\Delta \beta\right|$$

The base or input offset current can also be written as:

$$I_{OS(\Delta \beta)} = I_B \left|\Delta \beta\right|$$

Since the input bias current $I_B$ is defined as:

$$I_B = \frac{I_{B1} + I_{B2}}{2} = \frac{I}{2 \beta}$$
**DC Offset Voltage & Current Summary**

1. DC offset voltage and current occur due to mismatches in the BJT differential amplifier external resistors and transistor parameters ($I_s$ and $\beta$).

2. These mismatches are imperfections that are inherent to all differential amplifiers and their applications (e.g. op amps).

3. DC offset voltage and current are statistical quantities, i.e. no two differential amplifiers will have the same offset voltage and current.

4. MOS differential amplifiers have zero input offset current.