Problem 1

(a) Define the term "prefix-condition" in describing a code assigning binary codewords to the letters of a finite-alphabet source.

(b) Consider the following binary code for source alphabet \{X, Y, Z\}:

\[
\begin{align*}
X & \rightarrow 11 \\
Y & \rightarrow 10 \\
Z & \rightarrow 100
\end{align*}
\]

(i) Is this a prefix-condition code?
(ii) For any sequence of source letters encoded using this code, would you be able to decide from the binary coded stream the original sequence of source letters (i.e., is the code uniquely decodable)? Justify your answer briefly.

Problem 2

Source I produces independent symbols from an alphabet of two letters, A and B. For each symbol the probabilities of the letters A and B are 0.6 and 0.4 respectively.

(a) Design a Huffman code operating on blocks of two output symbols from Source I.

(b) What is the average number of bits per symbol for your code? What is the minimum number of bits per symbol that the best code should be able to give you for Source I?

(c) Source II is also a source with a two-letter alphabet \{C, D\} with \(P(C)=0.8\) and \(P(D)=0.2\). A composite source is formed by choosing each time at random with equal probability between Source I and Source II, for its output letters.

Find the Huffman code for single letters for the composite source, and the average bits/letter.

Problem 3

A Go-Back-N ARQ scheme using ACKs and NAKs is implemented on a full-duplex link with the following parameters:

- **Transmit window size** \(K=3\), transmitter (P) re-uses a minimum set of sequence numbers
- **ACK and NAK frames** are of negligible duration
- **I-frames** are of fixed time-duration \(T_{ix}\)
- One-way propagation delay = one I-frame duration
- Processing times for I-frames, ACK and NAK frames = half of I-frame duration

(Note that the I-frame duration is the unit of time measurement in this description).

Draw the frame sequence diagram for the case where the third I-frame from P is lost in transit and the very first ACK frame from S is lost in transit; all other frames are propagated without error. Indicate when frames are accepted and delivered to its higher protocol layer by the receiver (S). (Extend your diagram to 13 I-frame durations from start of transmission. Use the grided sheet to draw the diagram on.)
Problem 4

Consider the following situation in which nodes \( P_1 \) and \( P_2 \) send I-frames to node \( Q \), and \( Q \) forwards I-frames to \( S \). All links are full-duplex.

\[ \begin{align*}
  P_1 & \quad \text{Idle RQ} \\
  800 \text{ Kbps} & \\
  \text{P} & \quad \text{Sel. Rep.} \\
  800 \text{ Kbps} & \\
  Q & \quad 1 \text{ Mbps} \\
  \text{P} & \quad \text{Go-Back-N} \\
  \end{align*} \]

*Frame processing time = 0.5 ms for all frames at all nodes.*

*One-way propagation delay = 3 ms, I-frame size = 1000 bits, ACK/NAK frame size = 50 bits (negligible) for each link.*

(a) What are the maximum possible utilization factors \( U \) for the individual links?

(b) The transmit window at \( P_2 \) is of size \( K \). What is the minimum value of \( K \) to achieve maximum utilization on the \( P_2 \– Q \) link?

(c) For the \( Q- S \) link, let the transmit window at \( Q \) be of size 4, and assume that I-frames are subject to error with probability \( P_f=0.2 \). The other links are error-free. We want to ensure that the buffer at \( Q \) holding frames from \( P_1 \) and \( P_2 \) for transmission to \( S \) does not overflow. What is the maximum allowable window size at node \( P_2 \) for this? What is the average frame arrival rate seen at \( S \) in this case?

\[ \text{Given: For Go-Back-N with transmit window size } K, \text{ frame error probability } P_f, \text{ and considering only propagation delay } T_p \text{ and I-frame duration } T_ix, \text{ the utilization factor is} \]

\[ U_{\text{with error}} = U_{\text{no error}} \frac{1-P_f}{1-P_f + \min\{(1+2a),K\} P_f} \quad \text{where } a = \frac{T_p}{T_ix} \]

For selective repeat with frame error probability \( P_f \),

\[ U_{\text{with error}} = U_{\text{no error}} (1-P_f) \]