TCOM370

Solutions to Homework 99-1

Problem 1

4 hops (point-to-point links) between two terminals nodes; Transmission rate 9600 bps on all links; 24 overhead bits [Header + Trailer] for each packet; 1ms per-hop signal propagation delay. 1 sec. Call set-up time for circuit switched connection across 4 hops.

(a) Message size 5000 bits, packet size 1024 bits, all other parameters the same.

*Circuit switching*

5000 bits at 9600 bps $\Rightarrow$ 0.521 sec. message duration
total propagation delay 0.004 sec.
Total time for message is 0.521 + 0.004 + 1 = 1.921 sec.

*Packet switching*

Number of packets = 5 (1024 - 24 = 1000 bits of message data, 5×1000=5000)
Packet duration = 1024/9600 = 0.107 sec.
Entire 1024-bit packet received by each node from preceding node in 1024/9600 + 0.001 = 0.108 sec.

Total message time is therefore 4×0.108 + 4×0.107 = 0.968 sec. (There are 4 hops, and 4 packets in succession after the first complete packet is received at terminal node)
Transmission delay is 4×0.001 + 3×0.107 = 0.325 sec.

By Transmission Delay we mean the delay between the sending out of a bit in the message and its reception at the other end.

(b) Message size 5000 bits, packet size 512 bits, other parameters the same.

*Circuit switching*

The same as in (a).

*Packet switching*

Number of packets = 11 (512 - 24 = 488 bits of message data for first 10 packets, the 11th packet has 120 bits of message data and 380 dummy bits)
Packet duration = 512/9600 = 0.0535 sec.
Entire 512-bit packet received by each node from preceding node in 512/9600 + 0.001 = 0.0545 sec.
Total message time is therefore $4 \times 0.0545 + 10 \times 0.0535 = 0.753\text{sec.}$ (There are 4 hops, and 10 packets in succession after the first complete packet is received at terminal node) Transmission delay is $4 \times 0.001 + 3 \times 0.0535 = 0.1645\text{ sec.}$

**Following solutions for problems 2.1, 2.4 and 2.5 are from the solutions manual for the textbook.**

**Problem 2 (2.1)**

The main factors that influence the choice of transmission medium to be used in a data communication application are the required bit rate and the distance between the two communicating devices. The various types of transmission media listed in the question each has a defined maximum limit in terms of these factors and it is this that determines the application domain of each type of medium.

(a) Two-wire open lines are used mainly for connecting a data terminal equipment(DTE) - a computer for example - to a local data circuit-terminating equipment(DCE) such as modem. When designed for use with the analog switched telephone network, the maximum bit rate is the order of 100,000 bps. Also, the modem is normally situated less than 50m from the DTE. The actual maximum physical separation of the devices is influenced by the type of driver/receiver electronic circuits being used.

(b) Twisted pair lines have much better immunity to spurious noise signals owing to the two conducting wires being twisted together. This is exploited by the use of differential line driver and receiver circuits since these operate on the difference between the signals on each line rather than their absolute value. By twisting the two conductors together, any noise or spurious signal is picked up in both conductors and hence this does not affect the difference signal. This effect is further reduced by introducing a protective screen or shield around the twisted pair as shown in Fig. 2.2. The two types are then known as unshielded twisted pair (UTP) and shielded twisted pair (STP). UTP is used extensively in the switched telephone network for connecting the telephone to the local exchange. In addition, both types are now used extensively for data communications and, with appropriate differential line driver and receiver circuits, physical separations of up to 100m at 1Mbps are achievable. Also, with additional signal conditioning circuits, data rates as high as 100Mbps are now possible.

(c) Coaxial cable is used extensively for linking computers together in the form of a network. Both thin-wire (0.25 inch diameter) and thick-wire (0.5 inch diameter) versions are used. The operation bit rate is normally 10Mbps and, at this rate, the maximum length of cable is 200m for thin-wire and 500m for thick-wire. Larger diameter coaxial cable - up to 1 inch - is also used in community antenna television (CATV) networks.

(d) Optical fiber cable is the preferred choice for high bit rates - in excess of, say, 100Mbps - and long physical separations in excess of a few kilometers. It is used in computer networks at bit rates of 100Mbps and also in the switched telephone network for linking switching exchanges, either locally, nationally or internationally. In such
applications many voice circuits are multiplexed together and combined bit rates in excess of 600Mbps are frequently used. In addition, because of its superior immunity to electrical interference compared with copper conductors, optical fiber is also used in lower bit rate applications with noisy environments.

(e) Microwave transmission is used in two forms: terrestrial microwave and satellites. Terrestrial microwave is used to transmit data at rates in excess of 100Mbps at distances up to 50km. Normally microwave is used when the laying of cables is either impossible or would be prohibitively expensive.

Satellites are also used for computer-to-computer communications over wide geographical areas. The two alternative configurations are shown in Fig. 2.4. In both applications the satellite is launched to a height of 35,784m above the earth since, at this height, it has the same period of rotation as the Earth. It is then said to be geostationary. The Satellite simply acts as a repeating or relay station, retransmitting the data it receives on one frequency band out on a link between two ground stations while in the second it acts as a point-to-point multipoint (broadcast) link between a central (hub) site and multiple, physically-distributed receivers. In the case of the latter, all data is relayed via the central hub site (computer).

**Problem 3 (2.4)**

![Seven-cell repeat pattern](image)

Seven-cell repeat pattern

Because of its broadcast nature, the use of the radio spectrum has to be strictly controlled otherwise the various services would interfere with each other. Hence for each service, a specific portion of the radio spectrum is allocated. With an application such as television or radio, this is effectively a one-to-many form of communication; that is, the signal transmitted by the transmitter is received by all receivers. Thus a single frequency band is sufficient.

In contrast, data communications normally requires many-to-many communications thus necessitating multiple frequency bands. To optimise the use of the allocated frequency band, the assigned frequency band is normally divided into a number of sub-bands known as cells. The frequency bandwidth associated with each cell is then
selected to provide a wireless (cordless) link between a number of user terminals within the field of coverage of a central (base) station for that cell. The latter is normally connected to the fixed-wire network which is then used to provide a link between cells. The power output from each base station - and hence its field of coverage - is controlled so that it is sufficient to meet the number of active users within that region. Then, by assigning the frequency bands to each base station in a regular repeat pattern, it is possible to obtain complete radio coverage using only a small number of frequency bands.

An example is the 3-cell repeat pattern shown in Fig. 2.5 and an alternative is the 7-cell repeat pattern in which the distance between cells sharing the same frequency channel is larger and hence the risk of interference within a cell from a base station in another cell on the same frequency is reduced. This type of interference is known as channel interference and is discussed further in Section 6.4.1.

**Problem 4 (2.5)**

The effects on a transmitted signal of the different attenuation and distortion sources listed are shown in Fig. 2.6.

(a) **Attenuation**
As a signal propagates along a transmission medium its amplitude decreases, but waveform shape of the signal doesn’t change.

(b) **Limited bandwidth**
As we can see from the Fourier Series of a periodic signal, it is made up of an infinite series of sinusoidal frequency components. But a communications channel has a limited bandwidth, so only those frequency components that are within the channel bandwidth will be received.

(c) **Delay distortion**
The various frequency components of a signal may arrive at the receiver with different amplitudes and different phases, which are called amplitude distortion and phase distortion. An example of amplitude/phase distortion is given in notes 99-4B.

(d) **Line and system noise**
Because the amplitude of the signal is attenuated and different noises are added to the signal, the SNR (signal-to-noise ratio) might be rather small at the receiver. This will cause a problem at the receiver, in that reliable bit decisions cannot be made. The received signal may be unrecoverable.

**Problem 5 (2.6)**

The Fourier series of a periodic function $x(t)$ is:

\[
x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + \sum_{n=1}^{\infty} b_n \sin(2\pi n f_0 t)
\]

where,
\[ a_0 = \frac{1}{T} \int_{-\tau/2}^{\tau/2} x(t) dt, \]

and,

\[ a_n = \frac{1}{T} \int_{-\tau/2}^{\tau/2} x(t) \cos(2\pi nf_0 t) dt, \quad b_n = \frac{1}{T} \int_{-\tau/2}^{\tau/2} x(t) \sin(2\pi nf_0 t) dt, \]

From the class notes 2 and 2B, we have for pulse train of width-\(\tau\) pulses and period \(T\):

\[ a_0 = \frac{A\tau}{T}, \]

and,

\[ a_n = \frac{2A}{n\pi} \sin(\pi nf_0 \tau), \quad b_n = 0 \]

(i) For the unipolar binary signal shown in Fig. 2.7(a), \(\tau = T/2\), \(A = V\).

Therefore,

\[ a_0 = \frac{V\tau}{T} \bigg|_{\tau=T/2} = \frac{V}{2} \]

\[ a_n = \frac{2V}{n\pi} \sin(\pi nf_0 \tau) \bigg|_{\tau=T/2} = \frac{2V}{n\pi} \sin(\pi n/2) \]

\[ a_1 = \frac{2V}{\pi}, \quad a_2 = 0, \quad a_3 = -\frac{2V}{3\pi}, \quad a_4 = 0, \quad a_5 = \frac{2V}{5\pi}, \quad a_6 = 0, \quad a_7 = -\frac{2V}{7\pi} \ldots \]

Hence,

\[ a_n = \frac{2V}{n\pi}, \quad n = 4k + 1, \quad k = 0,1,2\ldots \]

and

\[ a_n = -\frac{2V}{n\pi}, \quad n = 4k + 3, \quad k = 0,1,2\ldots \]

The Fourier series for a unipolar binary signal therefore is:

\[ x(t) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi nf}{T}\right) \text{ with the a's given above.} \]

(ii) The bipolar binary signal shown in Fig. 2.7(a), is simply the unipolar signal of amplitude \(A=2V\) from which a constant level of \(V\) is subtracted. Therefore, the Fourier Series for it is simply that of the unipolar signal with amplitude \(2V\), with \(a_0\) reduced by a constant level of \(V\).

Therefore,

\[ a_0 = 0 \]
\[ a_n = \frac{4V}{n\pi} \sin\left(\frac{n\pi}{2}\right) \]

\[ a_1 = \frac{4V}{\pi}, \quad a_2 = 0, \quad a_3 = -\frac{4V}{3\pi}, \quad a_4 = 0, \quad a_5 = \frac{4V}{5\pi}, \quad a_6 = 0, \quad a_7 = -\frac{4V}{7\pi} \ldots \]

Hence,
\[ a_n = \frac{4A}{n\pi}, \quad n = 4k + 1, \quad k = 0,1,2\ldots \]

and
\[ a_n = -\frac{4A}{n\pi}, \quad n = 4k + 3, \quad k = 0,1,2\ldots \]

\[ b_n = 0 \quad \text{for} \quad n=1,2,3,4,5\ldots \]

**Problem 6 (2.7)**

Assuming one binary digit per signaling element and either unipolar or bipolar encoding, the line signal will change most frequently when the binary sequence 101010... is being transmitted. All other sequences will result in the line signal changing at a lower rate. The sequence 101010..., which has the shortest period, will yield the highest fundamental frequency component. Hence the fundamental and harmonics associated with this sequence will be larger than those of other sequences and hence determine the minimum bandwidth required for the channel.

**Problem 7**

(a) \( \tau = 1/3 \) sec. , \( T=1 \) sec.

\[ x(t) = \begin{cases} 
A(1 - \frac{t}{\tau}) & t \in [0, \frac{\tau}{2}] \\
A(1 + \frac{t}{\tau}) & t \in [-\frac{\tau}{2}, 0] 
\end{cases} \]

is an even function.

\[ a_0 = \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt = \frac{2A}{T} \int_{0}^{\tau/2} (1 - \frac{t}{\tau}) dt = \frac{2A}{T} \left( \left. -\frac{t^2}{2\tau} \right|_{0}^{\tau/2} \right) = \frac{2A}{T} \left( \frac{\tau}{8} \right) = \frac{3A\tau}{4T}, \]

\[ a_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos(2\pi f_0 t) dt = \frac{2A}{T} \int_{0}^{\tau/2} (1 - \frac{t}{\tau}) \cos(2\pi \frac{t}{T}) dt \]
\[
\begin{align*}
&= \frac{4A}{T} \left[ \frac{\sin(2\pi nf_0 t)}{2\pi nf_0} - \frac{1}{2\pi nf_0} \left( t \sin(2\pi nf_0 t) + \frac{1}{2\pi nf_0} \cos(2\pi nf_0 t) \right) \right]_0^{\tau/2} \\
&= \frac{4A}{T} \left[ \frac{\sin(\pi nf_0 \tau)}{2\pi nf_0} - \frac{1}{2\pi nf_0} \left( \frac{\tau}{2} \sin(\pi nf_0 \tau) + \frac{1}{2\pi nf_0} (\cos(\pi nf_0 \tau) - 1) \right) \right] \\
&= \frac{4A}{T} \left[ \frac{\sin(\pi nf_0 \tau)}{4\pi nf_0} - \frac{1}{(2\pi nf_0)^2} (\cos(\pi nf_0 \tau) - 1) \right]
\end{align*}
\]

\[b_n = 0; \text{ (since } x(t) \text{ is an even function)}\]

Amplitude spectrum \( A_n = \sqrt{a_n^2 + b_n^2} = |a_n| \)
(b) $\tau = 1/3$ sec., $T=1$ sec.

(i) $n=3$

(ii) $n=12$
(c) \( \tau = 1/3 \text{ sec.}, T=2 \text{ sec.}, n=6. \)

\[ x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi nf_0 t) \]

Note that for \((n=3,T=1)\) and \((n=6,T=2)\), the plots are reconstructions to within the 3 Hz bandwidth given by the rule: \( \text{bandwidth} = 1/\tau. \) (approx.)