Solutions to Homework #7

Problem 1

Stop-and-wait

\[ U_{\text{noerror}} = \frac{T_{ix}}{T_{ix} + 2T_p} \]

\[ T_{ix} = \frac{N}{5 \times 10^3} \], where \( N = \) I-frame length in bits. We want:

\[ U = \frac{N}{N + 2T_p \times 5 \times 10^3} = \frac{N}{N + 2 \times 20 \times 10^{-3} \times 5 \times 10^3} = \frac{N}{N + 200} \geq 0.5 \]

\[ \frac{N}{2} \geq 100 \]

\[ N \geq 200 \]

Problem 2

\[ T_p = 270ms \]

\[ T_{ix} = \frac{5000}{10^6} = 5ms \]

\[ a = \frac{T_p}{T_{ix}} = 54 \]

a) \( U = \frac{1}{1 + 2a} = 0.0092 \)

b) \( U = \frac{K}{1 + 2a} = 0.064, K = 7 \)

c) \( U = 1, K = 255 \) since \( K > 1 + 2a \)

Problem 3 (Exercise 4.17, Halsall)

The minimum number of frame identifiers - sequence numbers - can be determined by considering the case when the full send window limit of information frames is transmitted, all are received correctly, but all the acknowledgements relating to them are corrupted. It is then necessary for the secondary to determine whether the next frame received is the next in-sequence frame (as it expects), or a duplicate of the first frame in the sequence retransmitted by P as a result of the timeout (as is the case).

a) With an idle RQ scheme, the send window is 1, and hence the range of sequence numbers need only be 2 to detect the above condition.

b) With a selective repeat scheme, since frames can be received out of order, it is necessary for the primary to use a completely different set of identifiers for the next set of K frames it sends. The maximum number of identifiers is thus 2K.

c) With a go-back-N scheme, the secondary will only accept frames in the correct sequence and hence the maximum number of identifiers is K+1. Note that the idle RQ scheme is a special case of go-back-N with a send window of 1.
Problem 4 (Exercise 4.18, Halsall)

\[ T_p = \frac{4 \times 10^6}{2 \times 10^6} = 2 \times 10^{-2} \]
\[ T_{ix} = \frac{10^3}{2 \times 10^6} = 0.5 \times 10^{-3} \]
\[ a = \frac{T_p}{T_{ix}} = \frac{2 \times 10^{-2}}{0.5 \times 10^{-3}} = 40 \]
\[ P_f = N_f P = 10^3 \times 10^{-4} = 0.1 \]

a) Idle RQ

Without errors: \( U = \frac{1}{1+2a} = \frac{1}{81} = 0.012 \)
With errors: \( U = \frac{1-P_f}{1+2a} = \frac{0.9}{81} = 0.011 \)

b) Selective repeat \( (K = 7 < 1+2a) \)

Without errors: \( U = \frac{K}{1+2a} = \frac{7}{81} = 0.086 \)
With errors: \( U = \frac{K(1-P_f)}{1+2a} = \frac{7 \times 0.9}{81} = 0.078 \)

c) Go-back-N \( (K = 127 > 1+2a) \)

Without errors: \( U = 1 \)
With errors: \( U = U_{\text{no error}} \frac{1-P_f}{1-P_f + \min\{1+2a, K\}P_f} = \frac{0.9}{1-0.1+81 \times 0.1} = 0.1 \)

Problem 5

Link A-B

Calculate the utilization efficiency (U) to determine the effective bit rate of the link.
\[ T_p = 8 \times 10^{-6} \times 2000 = 1.6 \times 10^{-2} \text{ sec} \]
\[ T_{ix} = \frac{800}{100 \times 10^3} = 8 \times 10^{-3} \text{ sec} \]

Thus, \( a = \frac{T_p}{T_{ix}} = \frac{1.6 \times 10^{-2}}{8 \times 10^{-3}} = 2 \). The time for ACK to return to A from B is \( a+1+a = 1+2a = 5 \) frame times. Since \( K = 3 < 5 \), we can calculate \( U = \frac{K}{1+2a} = \frac{3}{5} = 60\% \). Thus the effective average bit rate on this link is 100kbps \( \times 60\% = 60\text{ kbps} \).

Link B-C

\[ T_p = 8 \times 10^{-6} \times 500 = 4 \times 10^{-3} \text{ sec} \]
\[ T_{ix} = \frac{800}{r} \text{ sec}, \text{ where } r \text{ is the transmission rate of the link in bps.} \]
Thus, $a = \frac{T_p}{T_{ix}} = \frac{4 \times 10^{-3}}{800/r} = 5 \times 10^{-6} \times r$. Since this link uses stop-and-wait, the utilization efficiency is

$$U = \frac{1}{1 + 2a} = \frac{1}{1 + r \times 10^{-5}}$$

and the effective average bit rate is $r \times U = \frac{r}{1 + r \times 10^{-5}}$.

In order to prevent buffer overflow, we need to have $R_{B-C} \geq R_{A-B}$, where $R$ is the effective bit rate on each link.

$$R_{B-C} \geq R_{A-B}$$

$$\frac{r}{1 + r \times 10^{-5}} \geq 60 \times 10^3$$

$$0.4r \geq 60 \times 10^3$$

$$r \geq 150 \text{ kbps}$$

Thus, $R_{B-C}$ must be greater than or equal to 150 kbps.

**Problem 6**

Link 1

$T_{p1} = 40\mu s$

$T_{ix1} = \frac{1000}{10 \times 10^3} = 0.1 \text{ sec}$

$T_{ax1} = \frac{100}{10 \times 10^3} = 0.01 \text{ sec}$

Link 2

$T_{p2} = 20\mu s$

$T_{ix2} = \frac{1000}{5 \times 10^3} = 0.2 \text{ sec}$

$T_{ax2} = \frac{100}{5 \times 10^3} = 0.02 \text{ sec}$

a) Minimum time-out periods

$$\text{min} T_{out} = 2T_p + T_{ix} + T_{ax}$$

Link 1: $\text{min} T_{out} \approx 0.11 \text{ sec}$

Link 2: $\text{min} T_{out} \approx 0.22 \text{ sec}$

b) Retransmission probability

$$\Pr\{\text{frame retransmission}\} = 1 - \Pr\{\text{no frame retransmission}\}$$

$$\Pr\{\text{no frame retransmission}\} = \Pr\{1\text{- frame correct}\} \times \Pr\{\text{ACK frame correct}\}$$

Link 1: $\Pr\{\text{frame retransmission}\} = 1 - \Pr\{\text{no frame retransmission}\} \approx 1 - 0.9 \times 0.9 = 0.19$

Link 2: $\Pr\{\text{frame correct}\} = (1 - p)^N = 1 - Np$, where $p=\text{BER}$

$$\Pr\{\text{frame retransmission}\} = 1 - \Pr\{\text{no frame retransmission}\}$$

$$\approx 1 - (1 - 1000 \times 10^{-5})(1 - 100 \times 10^{-5})$$

$$\approx 0.011$$

c) Utilization efficiency

$$U = \frac{(1 - p)T_{ix}}{2T_p + T_{ax} + T_{ix}}$$
\[
U_1 = \frac{0.81 T_{ix1}}{2T_p + T_{ax1} + T_{ix1}} \approx 0.736 \\
U_2 = \frac{0.989 T_{ix2}}{2T_p + T_{ax2} + T_{ix2}} = 0.899
\]

d) Achievable transmission rate

Link 1: effective rate = \(10 \text{ kbps} \times U_1 = 7.36 \text{ kbps}\)

Link 2: effective rate = \(5 \text{ kbps} \times U_2 = 4.495 \text{ kbps}\)

For transmission through both links, the achievable transmission rate is the smaller of the above 2 values, which equals 4.495 kbps.

**Problem 7**

\[
T_p = \frac{90000}{3 \times 10^5} = 0.3s
\]

\[
T_{ix} = T_{ax} = \frac{1000}{64 \times 10^3} = \frac{1000}{64} \text{ ms}
\]

Total time for ACK to be received and processed at the original sender =

\[
2T_p + T_{ix} + T_{ax} + 2 \times 5 \text{ ms} = 600 + \frac{2000}{64} + 10 = 641.25 \text{ ms}
\]

For efficient continuous RQ utilization, the send window size must be at least

\[
\frac{641.25}{1000/64} = 41.04, \text{ so } K=42
\]

is the minimum window size for 100% utilization in absence of errors.

If frame error probability equals 1%, then \(U = 1 - P_f = 0.99\), so the average packet delivery rate equals

\[
64 \times 0.99 = 63.36 \text{ packets/s.}
\]

If Go-back-N protocol is used, a different formula for the utilization efficiency must be used.

\[
U_{\text{with error}} = U_{\text{no error}} \frac{1 - P_f}{1 - P_f + \min \left( \frac{2T_p + T_{ix} + T_{ax} + 2 \times 5 \text{ ms}}{T_{ix}}, K \right) P_f} = \frac{1 - 0.01}{1 - 0.01 + \min\{41.04, 42\} \times 0.01} = 0.707
\]

Thus, the average packet delivery rate equals \(64 \times 0.707 = 45.2\) packets/sec. **Note that you cannot** use the "exact" formula derived in class. You must account for the ACK time and the processing time (as they are not negligible).
Problem 8

Sequence numbers for K=2 are 0, 1, 2.

Problem 9

a) Idle RQ utilization:

\[ U = \frac{T_{ix}}{T_t} = \frac{T_{ix}}{2T_p + T_{ix} + T_{ax}} = \frac{1}{1 + \frac{2T_p + T_{ax}}{T_{ix}}} = \frac{1}{1 + \frac{2 \times 0.1 + 0.005}{T_{ix}}} \]

To maximize U, we need to maximize T_{ix}. T_{ix} is maximized when I-frame length is maximized. Thus, T_{ix,max} = 20 ms, and

\[ U = \frac{1}{1 + \frac{0.205}{0.02}} = 0.089 \]

b) 4 bits → 2^4 = 16 sequence numbers. Thus, for SR, K_{max} = 8 for both transmit and receive.

c) For go-back-N, the maximum transmit window size equals 15 (K=15), and the receive window size equals 1(K=1).

d) (i) SR: \( U_{max} = \frac{KT_{ix}}{T_t} = \frac{8}{11.25} = 0.71 \) (if \( U_{max} < 1 \) with \( T_{ix} = 20 \) ms)

(ii) Go-back-N: K=15, so U=1.

e) \( P_t = 0.1 \)

SR: \( U_{max} = \frac{8}{11.25} \times 0.9 = \frac{7.2}{11.25} = 0.64 \)

Go-back-N: Note that the formula given in the problem is for \( T_{ax} = 0 \). Suppose we make this approximation ignoring \( T_{ax} \). We will get a more optimistic result:

\[ U_{max,witherror} = 1 \times \frac{0.9}{0.9 + 1.1} = \frac{0.9}{2} = 0.45 \] since K=15 and 1+2a=11.

Therefore, SR is better with \( P_t = 0.1 \).