Homework 10 (Posted 16th April, Due during or before 1 p.m. 23rd April (Please submit to Ms. Drucilla Spanner. Programming assignment due by 25th April, 11.59 p.m.))

**Policy for Programming Assignment:** Problem 1 has a programming assignment. The design for this one will be posted 23rd April. The **programming assignment is due by 25th April 11.59 p.m.**. If your name is John Smith, then name your program as JohnSmith.c and email it to yjkkim78@gradient.cis.upenn.edu.

**Problem 1: 10+15** Design an algorithm to find the maximum weighted path from a source to all destinations in a DAG. Analyze its complexity. Prove its correctness. Program your algorithm.

**Problem 2: 10 pts** Give an algorithm to detect the existence of a negative weight cycle in a strongly connected digraph. Analyze its complexity and prove its correctness.

**Problem 3: 5 pts** Give a counterexample to show that Dijkstra's algorithm does not generate the shortest paths if edges have negative weights, even if the digraph does not have a nonpositive weight cycle.

**Problem 4: 10 pts** Let $p(u,v)$ be the shortest path weight from $u$ to $v$. Give an $O(VE)$ algorithm to find the $\min_{v \in V} p(v,w)$ for all vertices $w$ in a digraph. Note that the algorithm should find $\min_{v \in V} p(v,w)$ for all vertices $w$ and should run in $O(VE)$. Prove its correctness.