Homework 3 (Posted 12th February, Due during or before class 19th February)

General Comment: Unless otherwise mentioned we consider binary search trees with keys as real numbers. Also, nodes and their keys will be used interchangeably.

Problem 1: $5+3+2$ Analyze the Preorder Traversal. You have a list of $n$ real numbers, and you want to form a binary search tree with them. What is the tree formation complexity? Would your answer change, if I tell you that when your tree has $k$ nodes then its depth varies from that of a complete binary tree of $k$ nodes by at most a constant?

Problem 2: 5 Sort a list of real numbers using a binary search tree. Analyze the complexity of your algorithm. Your grade depends on the complexity of your algorithm.

Problem 3: 7 You have a binary search tree. Consider a leaf $l$. $B$ is the set of keys in the path of $l$ including $l$ and the root. $A$ is the set of keys to the left of this path. $C$ is the set of keys to the right of the path. Is the following statement true or false? Given any element $a$ in $A$, $b$ in $B$, $c$ in $C$, $a \leq b \leq c$. Justify your answer.

Problem 4: 8 Node $A$ is a leaf and node $B$ is its parent in a binary search tree. Show that either $B$ is the smallest element larger than $A$ in the tree, or the largest element smaller than $A$.

Problem 5: 5 A strict binary tree is one where every node has 0 or 2 children. Prove that if there are $n$ leaf nodes in a strict binary tree then the total number of nodes is $2n - 1$. 