Problem 1 [15 points]: Consider a relation \( R \) with five attributes \( ABCDE \). You are given the following dependencies: \( A \rightarrow D \), \( A \rightarrow E \), \( DE \rightarrow BC \), \( B \rightarrow A \), \( D \rightarrow C \).

1. List all keys for \( R \).
2. Is \( R \) in 3NF?
3. Is \( R \) in BCNF?

Problem 2 [30 points]: You are given the task of designing a DBMS-based email system. The system will be used to store and access email messages and contact lists for a large collection of users. Your client has specified the following requirements:

- Each user will have a unique numeric ID. In addition, you need to store the first name, last name, and email address of each user in the system;
- Each user has a number of contacts. A contact has a first name, a last name, an email address, and one or more phone numbers (where each phone number has a type, e.g., home, cell, etc.);
- An email message belongs to exactly one user and has a “to” field, a “from” field, a date, a subject field, and a body field;
- Each email message is stored in exactly one folder;
- A folder corresponds to a single user and has a name and a parent folder.
- Every user is either a paying customer or a trial user.
- Trial users have expiration dates and remaining quota associated with them.
- Paying customers have no quota but do have a balance associated with them.
- The original users of the system (some still trial users, some paying customers) are further identified as “beta-testers”.

Draw an ER diagram for the email system. Be sure to indicate the various attributes of each entity and relationship set. Also specify the key and participation constraints for each relationship set. Specify any necessary overlap and covering constraints as well.
Problem 3 [25 points]: Consider a relation $R$ with seven attributes $ABCDXYZ$ and the FD set $F = \{AB \rightarrow X, AC \rightarrow D, Y \rightarrow C, YZ \rightarrow X, XB \rightarrow D, BD \rightarrow Z\}$. Let $F^+$ denote the closure set of $F$.

1. For each of the following attribute sets, do the following: (i) write down a minimal cover of the subset of $F^+$ that holds over the set; (ii) name the strongest normal form that is not violated by the relation containing these attributes; (iii) decompose it into a collection of BCNF relations if it is not already in BCNF.
   
   (a) $ABC$
   
   (b) $ABCD$

2. For each of the following decompositions of $R = ABCDXYZ$, with the same set of functional dependencies $F$, say whether the decomposition is (i) dependency preserving, and (ii) lossless join.

   (a) $\{ABXYD, ABCYZ\}$
   
   (b) $\{ABX, ACD, YC, XYZ, BDX, BDZ\}$

Problem 4 [20 points]: Suppose you are given a relation $R(A, B, C, D)$. For each of the following (complete) sets of FDs, (i) identify the candidate key(s) for $R$, and (ii) state whether or not the proposed decomposition of $R$ into smaller relations is a “good” decomposition and briefly explain why or why not.

1. $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$. Decompose into $AB$, $BC$, and $CD$.

2. $C \rightarrow A$, $B \rightarrow D$. Decompose into $AC$ and $BD$.

Problem 5 [15 points]: Let $R$ be a relation on attributes $ABCD$, and let $F = \{B \rightarrow A, B \rightarrow CD\}$ be a set of FDs on $R$. Prove that $S = \{AB, BC, BD\}$ is a lossless join decomposition of $R$ under $F$. (Hint: construct a binary tree for the decomposition, then apply Theorem 3 from Chapter 19 in the text.)