Symbol Error Probabilities for General Cooperative Links

Alejandro Ribeiro, *Student Member, IEEE*, Xiaodong Cai, *Member, IEEE*, and Georgios B. Giannakis, *Fellow, IEEE*

***Abstract*—Cooperative diversity (CD) networks have been receiving a lot of attention recently as a distributed means of improving error performance and capacity. For sufficiently large signal-to-noise ratio (SNR), this paper derives the average symbol error probability (SEP) for analog forwarding CD links. The resulting expressions are general as they hold for an arbitrary number of cooperating branches, arbitrary number of cooperating hops per branch, and various channel fading models. Their simplicity provides valuable insights to the performance of CD networks and suggests means of optimizing them. Besides revealing the diversity, they clearly show from where this advantage comes from and prove that presence of diversity does not depend on the specific (e.g., Rayleigh) fading distribution. Finally, they explain how diversity is improved in multihop CD networks.**

***Index Terms*—Cooperative diversity, diversity, fading, performance analysis.**

I. Introduction

THE USE of diversity to combat the detrimental effects of multiplicative time-selective fading has by now been well documented in wireless communication systems. In particular, the merits of multiantenna links over multiple-input multiple-output (MIMO) channels are well appreciated in terms of boosting capacity and error performance [6], [16]. The application of MIMO technology to mobile networks however, often faces the practical implementation problem of packing many antennas in a small mobile terminal. MIMO gains hinge on the independence of the paths between transmit and receive antennas, for which one must guarantee antenna element separation several times the wavelength, a requirement difficult to meet with small-size terminals. In an effort to overcome this limitation, cooperative diversity (CD) schemes have been introduced in [8]–[10], [13], and [14]. The basic idea is that around a given terminal, there can be other single-antenna terminals which can be used to enhance diversity by forming a virtual (or distributed) multiantenna system (see also Fig. 1) where a source terminal S transmits to a destination D and many cooperating terminals (relays) R. As demonstrated in [1], [3], [5], and [8], CD networks can achieve a diversity order equal to the number of paths between the source and the destination, and thus, they can be used to overcome the antenna array packing limitation.

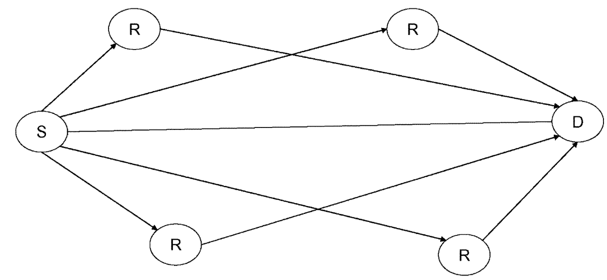

Fig. 1. Cooperative network scheme. The terminals R cooperate with the undergoing transmission between S and D.

Performance analysis of CD networks has yielded many interesting results including outage signal-to-noise ratio (SNR), information theoretic metrics, and average symbol error probability (SEP) expressions over Rayleigh-fading channels [2]–[4]. For sufficiently high SNR, this paper derives general average SEP expressions, for amplify and forward links with multiple cooperating branches, composed of multiple cooperating hops. The general treatment also encompasses various fading models, provided that their probability density functions (pdfs) are nonzero at zero instantaneous SNR (which is true for the widely used Rayleigh and Ricean models). As the resultant SEP expressions are simple and general, they can be used to analyze complex CD scenarios. When restricted to Rayleigh fading, our results coincide with those derived earlier in [2] using a bounding approach which is different from the approach derived here[1].

The rest of the paper is organized as follows. Starting with the simple case of one cooperating terminal, Section II lays out the system model and develops an asymptotic expression for the average SEP, which is subsequently used to optimize relay selection and placement. Section III extends these results to general cooperating setups that include multiple cooperating paths (M) and multiple cooperating hops (N) per cooperating path. Section IV confirms the usefulness of the asymptotic (high SNR) expressions to pragmatic simulated setups, and Section V concludes the paper.

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The authors are with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA (e-mail: aribeiro@ece.umn.edu; caixd@ece.umn.edu; georgios@ece.umn.edu).

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[1]Our approach here is different from that in [3] and [4] and was derived *independently* of [2]. In fact, we became aware of [2] during the review process of this paper.

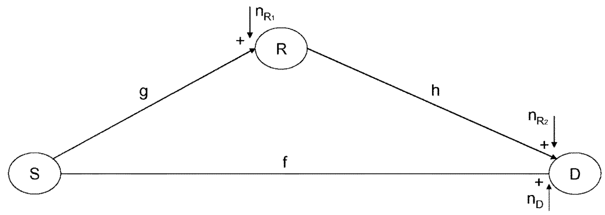

Fig. 2. Single cooperating terminal.

II. SEP With a Single Cooperating Terminal

A. System Model

Consider the simplest CD strategy shown in Fig. 2, where we have an information source S and a destination D communicating over a channel with fading coefficient f. A relay terminal R is willing to participate in this link providing D with a second copy of the original signal through the complex channels $S-R$, and $R-D$ with flat-fading coefficients g and h, respectively. Without loss of generality, we assume that all the additive white Gaussian noise (AWGN) terms n_{R_1}, n_{R_2}, and n_D have equal variance N_0. Similar to [2], [8], and [13], we suppose that the realizations of the random variables f, g, and h have been acquired at the receiver ends, e.g., via training. Note that no particular assumptions are made on channel statistics.

We consider the Amplify and Forward (AF) model where relays simply amplify the signal received from the source [8]. Assuming that S and R transmit through orthogonal channels, the destination D receives two independent copies of the signal x, transmitted by the source

$$\begin{aligned} y_D &= fx + n_D \\ y_R &= hA(gx + n_{R_1}) + n_{R_2} = hAgx + n_R \end{aligned} \tag{1}$$

where $n_R = hAn_{R_1} + n_{R_2}$ and A is the amplification factor which will be discussed later. The receiver collects these copies with a maximum ratio combiner (MRC). We emphasize that the noise terms n_D and n_R do not have identical power because n_R includes a noise contribution at the intermediate stage; for this reason, the MRC should be preceded by a noise normalization step. With this combining rule, we form a decision variable z by weighting the combination with the respective powers. The resulting SNR of the decision variable is

$$\gamma_z = |f|^2 \frac{P_x}{\sigma_D^2} + |Agh|^2 \frac{P_x}{\sigma_R^2} = \gamma_D + \gamma_R \tag{2}$$

where P_x is the transmitted power at $S, \gamma_D := |f|^2 P_x/\sigma_D^2$, and $\gamma_R := |Agh|^2 P_x/\sigma_R^2$. For fixed f, g, and h realizations, z is Gaussian, and the SEP conditioned on the instantaneous SNR γ_z is given by[2] $P_e = Q(\sqrt{k\gamma_z})$, where the constant k depends on the type of modulation [2 for phase shift keying (PSK)], and $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-u^2/2}\, du$.

The term γ_D in (2) is the per-hop SNR associated with the direct channel f; that is, $\gamma_D = \gamma_f = |f|^2 P_x/N_0$, but the term γ_R requires a bit more elaboration. Expanding n_R, the term γ_R takes the form

$$\gamma_R = |Agh|^2 \frac{P_x}{(1+|Ah|^2)N_0}. \tag{3}$$

Here, we have choices over the amplification factor A; a convenient one maintains constant average power output, equal to the original transmitted power

$$A^2 = \frac{P_x}{P_x|g|^2 + N_0}. \tag{4}$$

Substituting (4) into (3) and (2), we obtain

$$\gamma_z = \frac{\gamma_g \gamma_h}{1 + \gamma_g + \gamma_h} + \gamma_f \tag{5}$$

where γ_g and γ_h are the per-hop SNRs associated with the channels g and h, respectively, and are defined similarly to γ_f; that is, $\gamma_g = |g|^2 P_x/N_0$ and $\gamma_h = |h|^2 P_x/N_0$.

At high SNR, the 1 in the denominator of (5) is negligible, and thus, (5) reduces to

$$\gamma_z = \frac{\gamma_g \gamma_h}{\gamma_g + \gamma_h} + \gamma_f. \tag{6}$$

The SNR in (6) is analytically more tractable than that in (5), which will come handy when analyzing the SEP in Sections II-B and III-B. In Section IV, it will be shown that the SEP calculated from the SNR in (6) is very close to the simulated SEP even at moderate SNR values. Furthermore, it is easy to verify that neglecting the 1 in (5) is equivalent to considering an amplification factor $A = 1/g$. Although practically less attractive (because of the noise the transmitter power could be too large), this last choice of A serves as an upper bound for the practically justifiable choice of (4). By comparing with Proposition 1 of the next subsection, we will also prove in Appendix A, that neglecting the one in the denominator of (5) does not affect the SEP, when the SNR is high.

To obtain the average SEP, we have to integrate the Gaussian tail function on the distribution of the variable z, which is often cumbersome. For Rayleigh-fading channels, the distribution of γ_R is known [7], and the distribution of γ_f is exponential. Thus, an immediate approach is to find the pdf of γ_Z by convolving the pdfs of these two independent variables, which at least in principle, is possible. Alternatively we can use the moment generating function (MGF) approach [15, ch. 8]. Whichever approach we select, the calculations get involved, and even if we were able to obtain the SEP in closed form, the resulting expression would provide limited insight.

Instead, we will use a new tool developed in [17], which enables average SEP calculations for sufficiently large SNR by looking at the pdf of the SNR around zero. This approach will allow us to achieve insightful results with relatively simple computations, which we pursue next.

B. SEP Analysis

In fading channels, the probability of error is dominated by the probability of having deep fades, or equivalently, the probability that the channel coefficient is vanishingly small, which

[2]This is valid for many types of modulation including binary phase shift keying (BPSK), quarternary phase shift keying (QPSK), M-pulse amplitude modulation (M-PAM), and rectangular M-quadrature amplitude modulation (M-QAM) [11, Ch. 5].

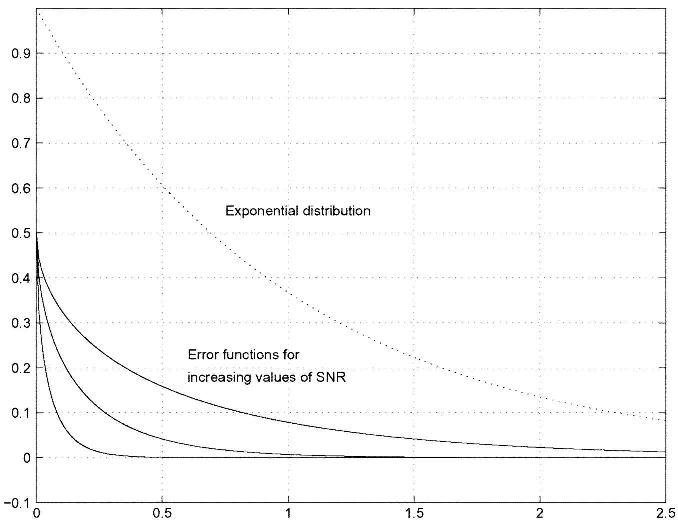

Fig. 3. Behavior of $Q(x)$ and the pdf of the exponential distribution.

in turn pertains to the behavior of the pdf of the SNR $p_\gamma(\gamma)$ around zero. To better explain this assertion, we will normalize γ to isolate the effect of the transmitted power that determines the expected value of γ from the effect of the random variation around this expected value. We define thus, $\hat{\gamma} := \gamma/\bar{\gamma}$, where $\bar{\gamma}$ denotes the average SNR.

With reference to Fig. 3, let us now expand the SEP expression to take into account the averaging over the random SNR

$$\bar{P}_e = \int_0^\infty Q(\sqrt{k\bar{\gamma}\hat{\gamma}})p_{\hat{\gamma}}(\hat{\gamma})\, d\hat{\gamma}. \tag{7}$$

In Fig. 3, we depict the function $Q(\sqrt{\bar{\gamma}x})$ for increasing values of $\bar{\gamma}$ along with an example pdf for the fading effects. It is apparent that for larger values of $\bar{\gamma}$, the behavior of $\hat{\gamma}$ becomes increasingly irrelevant because the Q term in (7) goes to zero so fast that almost throughout all the integration range the integrand is almost null. However, recalling that $Q(0) = 1/2$, regardless of the value of $\bar{\gamma}$, the behavior of $p_{\hat{\gamma}}(\hat{\gamma})$ around zero never loses importance. On the other hand, this behavior can be approximated by a McLaurin series which in general will yield a group of terms easier to handle than the distribution itself. In particular, for an arbitrarily large $\bar{\gamma}$, we consider values of $\hat{\gamma}$ arbitrarily close to zero, and approximate $p_{\hat{\gamma}}(\hat{\gamma})$ arbitrarily well by the first term of the McLaurin series.

All these intuitive arguments have been rigorized in [17], where the pdf of $\hat{\gamma}$ is approximated by

$$p_{\hat{\gamma}}(\hat{\gamma}) = a\hat{\gamma}^t + o(\hat{\gamma}) \tag{8}$$

to deduce that the asymptotic behavior of the average SEP (as $\bar{\gamma} \to \infty$) is given by

$$\bar{P}_e \to \frac{2^t a\Gamma(t+3/2)}{\sqrt{\pi}(t+1)}(k\bar{\gamma})^{-(t+1)} \tag{9}$$

where $\Gamma(x)$ denotes the Gamma function.

The constant t in (9) is not necessarily an integer and a is not necessarily a McLaurin coefficient; but for our purposes, it is convenient to restrict the formula to this case, for which, if the derivatives of $p_{\hat{\gamma}}(\hat{\gamma})$ up to order $t-1$ are null, then a is related to the order t derivative $a = (1/t!)(\partial^t p_{\hat{\gamma}})/(\partial\hat{\gamma}^t)(0)$. Using the relation between distributions of scaled random variables $p_{cx}(cx) = p_x(cx)/|c|$ and making explicit the fact that a is a McLaurin coefficient, we can reduce (9) to the more convenient expression

$$\bar{P}_e \to \frac{\prod_{i=1}^{t+1}(2i-1)}{2(t+1)k^{(t+1)}} \cdot \frac{1}{t!}\frac{\partial^t p_\gamma}{\partial\gamma^t}(0) \tag{10}$$

where we got rid off of the Gamma function by the explicit formula for its half natural values, $\Gamma(n+(1/2)) = (\sqrt{\pi})/(2^n)\prod_{k=1}^n(2k-1)$. Based on (10), we have to study the behavior of $p_\gamma(\gamma)$ around zero, and this is precisely what we are going to do. In particular, we have the following result.

Proposition 1: Let us consider three nonnegative independent random variables X, Y, and Z with pdfs $p_X(x), p_Y(y)$, and $p_Z(z)$, respectively. These pdfs are unknown except for their values at zero that are denoted as x_0, y_0, and z_0 and assumed to be nonzero. If the variable V is defined as

$$V := \frac{XY}{X+Y} + Z \tag{11}$$

then $p_V(0) = 0$, and the first derivative of $p_V(v)$ evaluated at zero is given by

$$\frac{\partial p_V}{\partial v}(0) = (x_0 + y_0)z_0. \tag{12}$$

Before proving this proposition, we note that if X, Y, and Z are the per-hop SNRs in (6), then V is equivalent to γ_z, the SNR of the decision variable. What is more, the derivative in (12) can be plugged in (10) to yield an expression for the average SEP.

Also important to note is that for Ricean fading, γ_g, γ_h, and γ_f are noncentral chi-squared random variables and the values x_0, y_0, and z_0, are proportional to the inverses of the average SNRs: $(K+1)/\bar{\gamma}_g, (K+1)/\bar{\gamma}_h$, and $(K+1)/\bar{\gamma}_f$, with K denoting the so-called specular factor. Besides this particular case, the treatment is also valid for any fading pdf that is nonzero at the origin.

Having motivated Proposition 1, let us proceed with a roadmap of the proof. The variable V is the sum of two independent terms $W := (XY/X+Y)$ and Z. We will begin by demonstrating that the pdf of W at zero satisfies $p_W(0) = x_0 + y_0$, and then, we are going to prove that the sum of the two independent random variables W and Z yields a variable that satisfies (12).

To simplify the notation, we define the function $g(x,y) := xy/(x+y)$, and the auxiliary random variable $W = g(X,Y)$, that represents the first term in (11) and is independent of Z. The pdf of W can be obtained from the following expression for multidimensional functions of random variables [11, Sec. 2.1.2]:

$$p_W(w) = \int\int_{\{(x,y):g(x,y)=w\}} \frac{p_x(x)p_y(y)}{|\nabla g(x,y)|}dxdy \tag{13}$$

where $|\nabla g(x,y)|$ is the modulus of the gradient of $g(x,y)$. Notice that the integral in (13) is over the curves for which $g(x,y)$ has a constant value. In general, it is difficult to evaluate this integral, but since we are interested in $p_W(0)$, and there are only two possibilities for $w=0$, namely $x=0$, or $y=0$, the computations become tractable. Carrying out the integration in (13), we obtain

$$p_W(0) = p_x(0)\int_0^\infty \frac{p_y(y)}{|\nabla g(0,y)|}dy + p_y(0)\int_0^\infty \frac{p_x(x)}{|\nabla g(x,0)|}dx \tag{14}$$

where the first term comes from the curve $x = 0$ and the second from the curve $y=0$. The reader can easily verify that $|\nabla g(x,0)| = |\nabla g(0,y)| = 1$, which yields

$$p_W(0) = x_0\int_0^\infty p_y(y)dy + y_0\int_0^\infty p_x(x)dx. \tag{15}$$

The integrals in (15) are simply equal to one, from where we deduce that

$$w_0 = x_0 + y_0. \tag{16}$$

So far, we have resolved half of the problem, or in fact, more than half. Computing the derivative of $p_V(v)$ at zero is easy if we consider the Laplace transform of $p_V(v)$. Since V is the sum of two independent random variables, we have $L_V(s) = L_W(s)L_Z(s)$, where $L_V(s)$, $L_W(s)$, and $L_Z(s)$ are the Laplace transforms of $p_V(v), p_W(w)$, and $p_Z(z)$, respectively. As we are interested in the value at zero, we can use the initial value theorem of Laplace transforms to arrive at

$$\frac{\partial p_V}{\partial v}(0) = \lim_{s\to\infty} s^2 M_W(s)M_Z(s) \tag{17}$$

where we *assumed* that $p_V(0)=0$, and separated the MGF of V in its two factors. However, this limit can be easily rewritten as a product of two limits

$$\frac{\partial p_V}{\partial v}(0) = \lim_{s\to\infty} sM_W(s) \lim_{s\to\infty} sM_Z(s). \tag{18}$$

However, each of the limits in (18) is precisely the corresponding pdf evaluated at zero, from where we obtain

$$\frac{\partial p_V}{\partial v}(0) = w_0 z_0. \tag{19}$$

The very fact that this limit is not infinite validates the assumption of considering $p_V(0)=0$ (using the initial value theorem, $p_V(0) = \lim_{s\to\infty} sM_W(s)M_Z(s)$, but if the limit with s^2 is finite, the limit with s should be 0) which proves the first statement of Proposition 1 and completes the second part of the proof. Combining the results in (19) and (16), the proof of the second statement of Proposition 1 follows.

Using this proposition, we can calculate the asymptotic average SEP from (10) as

$$\bar{P}_e \to \frac{3}{4k^2}[p_{\gamma_g}(0) + p_{\gamma_h}(0)]p_{\gamma_f}(0). \tag{20}$$

In the special case of Ricean fading, the SNR is noncentral chi-squared distributed according to the expression $p_\gamma(\gamma) = (K+1)/\bar{\gamma}e^{-K-((K+1)\gamma)/(\bar{\gamma})}I_0(2\sqrt{K(K+1)\gamma/\bar{\gamma}})$, and (20) reduces to

$$\bar{P}_e \to \frac{3(K+1)^2}{4k^2}\left(\frac{1}{\bar{\gamma}_g} + \frac{1}{\bar{\gamma}_h}\right)\frac{1}{\bar{\gamma}_f}. \tag{21}$$

This result is strikingly simple and will allow us to draw some interesting conclusions in Section II-C. Furthermore, the treatment in this section can be easily generalized to an arbitrary number of cooperative branches and hops per branch, as we will see in Sections III-A and B, respectively.

By now, we have established that diversity of order two is possible for our simple CD network under various fading channel models. Furthermore, we have seen that the *diversity comes from the product of two independent SNRs*, that of the direct path and the one of the relay path.

C. Relay Selection

Although the main goal of this paper is to develop general and simple expressions for the asymptotic average SEP of CD networks, we wish to illustrate their impact in designing CD relay systems. The average SEP is a good indicator of link performance, and accordingly, having a relatively simple expression for it allows for optimization problems to be efficiently tackled. Even more important, the simplicity of (20), or (21) for the case of Ricean fading, greatly simplifies these problems.

We consider the relay selection problem which is easily tractable with our approach. We focus on Ricean fading but the results can be readily generalized to other models. Suppose that several terminals are available to cooperate with the source, and the source has to decide which one will be the best possible cooperator. The optimization problem under consideration is selecting the relay that minimizes the average SEP. Looking at (21), the SEP is composed of two factors: one that depends solely on the relay path and a second one that depends solely on the direct path. This second factor depends on the source-destination path-loss which is a given quantity. Thus, optimizing relay placement is equivalent to minimizing the function Ω

$$\Omega = \frac{1}{|\bar{g}|^2} + \frac{1}{|\bar{h}|^2} \tag{22}$$

where $|\bar{g}|^2 := E(|\bar{g}|^2)$, and $|\bar{h}|^2 := E(|\bar{h}|^2)$. Note that Ω is twice the inverse of the harmonic mean of $|\bar{g}|^2$ and $|\bar{h}|^2$. So, the solution to the relay selection problem is selecting the pair which maximizes the harmonic mean of the fading coefficients average power from the source and to the destination; i.e.,

$$R_{\rm opt} = R_i : \max\{\mu_H(|\bar{g}_i|^2, |\bar{h}_i|^2)\} \tag{23}$$

where μ_H denotes the harmonic mean function. This is potentially applicable to routing problems in CD networks.

A slightly different problem arises when $|\bar{g}|^2$ and $|\bar{h}|^2$ cannot vary independently due to physical limitations arising from, e.g., the path-loss between two terminals. The latter depends on the distance between them C/d^α, where the constant C depends on propagation parameters such as carrier frequency, and α is the path-loss slope which is greater than two and usually considered to be near four [12, p. 104]. For simplicity, we restrict ourselves to the one dimensional case in which the relay is placed in between S and D. Letting $d_{\rm SR}$ denote the distance between S and R and $d_{\rm SD}$ the distance between S and D, we define the relative distance from source to relay as $\rho = d_{\rm SR}/d_{\rm SD}$, from where we can easily infer that $|\bar{g}|^2$ and $|\bar{h}|^2$ are given by

$$\begin{aligned}|\bar{g}|^2 &= \frac{|\bar{f}|^2}{\rho^\alpha}\\ |\bar{h}|^2 &= \frac{|\bar{f}|^2}{(1-\rho)^\alpha}\end{aligned} \tag{24}$$

where $|\bar{f}|^2 := E(|\bar{f}|^2)$. Given this physical model, (22) takes the form

$$\Omega = \frac{1}{|\bar{f}|^2}\left[\rho^\alpha + (1-\rho)^\alpha\right] \tag{25}$$

which has a maximum for $\rho = 1/2$, regardless of the value of α. This proves the intuitively appealing result that the relay should be placed just in the middle between source and destination. Moreover, this result does not depend on the detailed path-loss parameters (α and C). It is clear from (25) that for exponents $\alpha > 3$, usually encountered in practice, the minimum SEP has also null second derivative. Thus, this minimum is relatively robust to relay displacements from the optimum position, which is always desirable for optimal designs.

While our analysis here focuses on the one-dimensional case, the optimal placement solution for this case also sheds light to more general cooperation scenarios of interest. In this sense, we should focus on CD schemes that work well when h and g have balanced (ideally equal) power profiles, because either placing the relay close to the source or placing it close to the destination offer suboptimal solutions. This result also speaks for the flexibility optimally placed CD systems have to improve the average SEP relative to non-CD multiantenna systems of diversity order two.

III. General Cooperative Links

A. Multibranch CD

In Section II, we worked with the simplest possible scenario of one cooperating terminal. Our closed-form expression for the asymptotic behavior of the average SEP demonstrates that this scheme can achieve diversity of order two. Existing works suggest that the diversity order increases with the number of cooperating terminals [2], [8], [13], and, thus, it would be interesting to obtain a SEP expression for this more complex but better performing scenario.

The ideas of Section II can be generalized to multibranch CD networks, such as those depicted in Fig. 4. In addition to the direct path with fading coefficient f, we consider M cooperating terminals (relays) $\{R_1, \ldots, R_M\}$. The channel coefficient between S and relay R_i is denoted as g_i, while that between R_i and D as h_i. We assume that the relays transmit over mutually orthogonal channels. At each relay, a noise term n_i is present and a second noise term m_i is introduced at reception. At each cooperating terminal, the amplification factor is that defined in (4).

In Section II, we obtained an expression for the first derivative of the pdf of the sum of two independently faded paths evaluated at zero, and proved that this pdf is zero at the origin. Naturally, we expect that for the sum of $M+1$ independent faded paths the first non-null derivative of the pdf at zero is that of order M, and this is precisely what is stated in the following proposition.

Proposition 2: Consider a finite set of nonnegative random variables $\{X\} = \{X_0, X_1, \ldots, X_M\}$ whose pdfs $p_0, p_1, \ldots, p_M$ have nonzero values at zero, and denote these

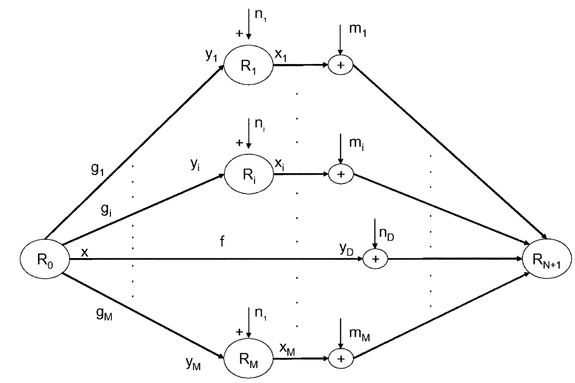

Fig. 4. Multibranch cooperation. M terminals cooperate with S to attain diversity of order $M+1$.

values as $p_0(0), p_1(0), \ldots, p_M(0)$. If the random variable V_M is the sum of the components of the set X

$$V_M = \sum_{i=0}^{M} X_i \tag{26}$$

then all the derivatives of p_{v_M} evaluated at zero up to order $(M-1)$ are zero, while the Mth order derivative is given by

$$\frac{\partial^M p_{V_M}}{\partial v^M}(0) = \prod_{i=0}^{M} p_i(0). \tag{27}$$

The proof is outlined in Appendix B.

Proposition 2 is the tool needed to study the performance of multibranch diversity systems. It is worth mentioning that this result is applicable to various conceivable diversity strategies, even outside the scope of CD networks. This generality will be exploited to analyze the multihop scenario in Section III-C, but for now, we will use the limit in Proposition 2 as an expression for the SNR of the multibranch CD network of Fig. 4. The decision variable z at the MRC output is given by

$$z = \left(\frac{f}{\sigma_D}\right)^* \tilde{y}_D + \sum_{i=1}^{M}\left(\frac{h_i A_i g_i}{\sigma_{R_i}}\right)^* \tilde{y}_{R_i} \tag{28}$$

where we defined the variables $\tilde{y}_D := y_D/\sigma_D$ and $\tilde{y}_{R_i} := y_{R_i}/\sigma_{R_i}$. It follows easily from (28) that the SNR of the decision variable is approximately given by

$$\gamma_z = \gamma_f + \sum_{i=1}^{M} \frac{\gamma_{g_i}\gamma_{h_i}}{\gamma_{g_i} + \gamma_{h_i}} \tag{29}$$

where again we eliminated the one in the denominator of the relay path SNRs, which is equivalent to considering $A_i = 1/g_i$, and has no impact on the asymptotic SEP, as shown in Appendix A. We proceed by analogy to Section II-B defining $W_i := \gamma_{g_i}\gamma_{h_i}/(\gamma_{g_i} + \gamma_{h_i})$. From (16) we know that

$$p_{w_i}(0) = p_{\gamma_{g_i}}(0) + p_{\gamma_{h_i}}(0) \tag{30}$$

and after applying Proposition 2 to the variable defined by (28) we obtain

$$\frac{\partial^M p_{\gamma_z}(\gamma_z)}{\partial \gamma_z^M} = p_{\gamma_f}(0) \prod_{i=1}^{M}\left[p_{\gamma_{g_i}}(0) + p_{\gamma_{h_i}}(0)\right]. \tag{31}$$

Substituting (31) into (10), we obtain the asymptotic expression for the average SEP of the multibranch CD system

$$\bar{P}_e \approx C(M) p_{\gamma_f}(0) \prod_{i=1}^{M} \left(p_{\gamma_{g_i}}(0) + p_{\gamma_{h_i}}(0) \right) \tag{32}$$

where $C(M) = (\prod_{k=1}^{M+1}(2k-1))/(2(M+1)!k^{(M+1)})$ is a constant that depends on the number of cooperating branches M. Its first values are $C(1) = (3/4), C(2) = (5/4)$, and $C(3) = (35/16)$. It is interesting to note that $C(M)$ increases with M, which slightly affects the diversity advantage.

Equation (32) is quite general as it holds under any SNR distribution provided that the underlying pdf at the origin is nonzero. For the particular case of Ricean fading, it takes the form

$$\bar{P}_e \approx \frac{C(M)(K+1)^{M+1}}{k^{M+1}} \cdot \frac{1}{\gamma_f} \prod_{i=1}^{M} \left(\frac{1}{\gamma_{g_i}} + \frac{1}{\gamma_{h_i}} \right) \tag{33}$$

while for Rayleigh faded links, it reduces to that derived in [2].

B. Multihop CD

A second point of interest is to explore what happens when we add multiple hops to each of the diversity branches. In principle, one expects that as the results were generalizable to M branches, they should be generalizable to N hops. Indeed, this is the case. We consider a set of N cooperating relays R_i, as depicted in Fig. 5.

Without loss of generality, the relays will be considered to be an ordered set $\{R\} = \{R_1, \ldots, R_N\}$, and for the sake of uniformity, the source will be named R_0 and the destination R_{N+1}. At node R_i, the received signal will be named y_{i-1} and the transmitted signal x_i. Note that x_0 is the signal transmitted by the source and y_N is the signal received at destination. The fading coefficient between R_i and R_{i+1} will be denoted as h_i, the amplification factor at node R_i as A_i, and the AWGN as n_i. Given this nomenclature, the system equations are

$$\begin{aligned} y_i &= h_{i-1}x_{i-1} + n_i \\ x_i &= A_i y_i. \end{aligned} \tag{34}$$

A detailed treatment of this model for A_i, defined as in (4), seems infeasible. However, as we emphasized in Section II-A, the results for large average SNR are indistinguishable from the model using the simpler definition $A_i = 1/h_{i-1}$. For this reason, we are going to work with the amplification factors $A_i = 1/h_{i-1}$. With this simplification, we obtain the following input–output relationship:

$$y_N = h_N x_0 + h_N \sum_{i=0}^{n-1} A_i n_i + n_N \tag{35}$$

from where we can show that the instantaneous SNR γ of the received variable y_N is[3]

$$\gamma = \frac{\gamma_0\gamma_1, \cdots, \gamma_N}{\sum_{i=0}^{N} \gamma_0\gamma_1, \cdots, \gamma_{i-1}\gamma_{i+1}, \cdots, \gamma_N}, \tag{36}$$

[3]Hasna and Alouini in [7] observe the interesting property that γ is the harmonic mean of γ_i, but as we are interested in the behavior around 0, it is better to avoid inverses.

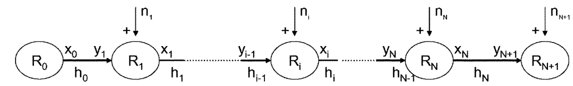

Fig. 5. Multihop system with N intermediate relays ($N+1$ hops).

where γ_i are the per-hop SNRs, defined by $\gamma_i := (P_x/N_0)|h_i|^2$. We note that the denominator in (36) is the sum of all possible products that exclude one and only one of the individual SNRs.

Given this model, the importance of the following proposition (proved in Appendix C) is self evident.

Proposition 3: We consider $N+1$ nonnegative independent random variables $X_0, X_1, \ldots, X_N$, with unknown pdfs $p_0(x_0), p_1(x_1), \ldots, p_N(x_N)$ except for their values at zero that are assumed strictly positive and known. If we define the random variable

$$Z = g(\vec{X}) = \frac{X_0X_1, \ldots, X_N}{\sum_{i=0}^{N} X_0X_1, \ldots, X_{i-1}X_{i+1}, \ldots, X_N} \tag{37}$$

then the pdf of Z at the origin satisfies

$$p_Z(0) = \sum_{i=0}^{N} p_{X_i}(0). \tag{38}$$

Using (38), it is easy to obtain from (10) the following expression for the asymptotic average SEP of a multihop system:

$$\bar{P}_e \approx \frac{1}{2k} \sum_{i=0}^{N} p_{\gamma_i}(0). \tag{39}$$

For the case of Ricean fading, the latter reduces to

$$\bar{P}_e \approx \frac{K+1}{2k} \sum_{i=0}^{N} \frac{1}{\bar{\gamma}_i}. \tag{40}$$

From (40), we see that the multihop system is a diversity one system, and each new hop adds a new term to the probability of error. The advantage of multihop transmissions comes from the path loss gains associated with it. In a practical system, dividing the transmission path will result in a group of average SNRs whose sum of inverses is smaller than the inverse of the original path SNR. In fact, we can obtain from (40) a condition under which a multihop system offers advantages over a single-hop system. If $\bar{\Gamma}$ denotes the average SNR of the single-hop system, the multihop system should be preferred if

$$\sum_{i=0}^{N} \frac{1}{\bar{\gamma}_i} < \frac{1}{\bar{\Gamma}}. \tag{41}$$

Note that the sum in the left-hand side of (41) is $1/N$ times the harmonic mean of the individual hop SNRs. Thus, we have established that for Rayleigh-fading channels multihop should be preferred over single hop if the harmonic mean of the average

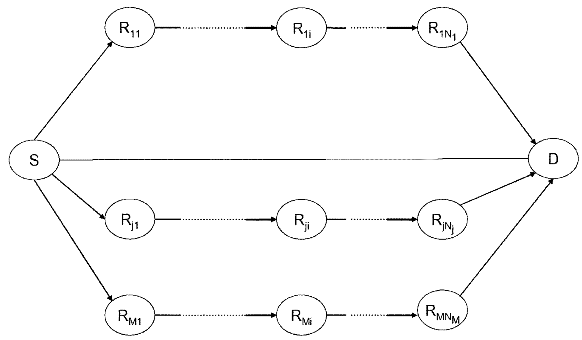

Fig. 6. Multihop, multibranch transmission.

multihop SNRs is larger than the single-hop SNR divided by the number of hops[4].

The gain $\mathcal{G}$ of the multihop system in terms of diminution of the average SEP is precisely the quotient of these two values

$$\mathcal{G} = \frac{\mu_H(\bar{\gamma}_0, \ldots, \bar{\gamma}_N)}{(N+1)\bar{\Gamma}}. \tag{42}$$

Also interesting to note is that from (40), we can easily generalize the optimal relay placement design of Section II-C. Regardless of the underlying path loss model, the result is that equi-spaced relays along the line that connects source with destination are SEP-optimal at sufficiently high SNR. This optimal design enjoys the same properties as that of Section II-C and points to the importance of CD networks having per-hop fading coefficients with balanced average power.

C. Multibranch, Multihop CD

Relying on the results of Sections III-A and B, we are ready to obtain an expression for the average SEP of multibranch, multihop transmissions. The result of Section III-A applies to a sum of random variables regardless of their specific pdfs, provided that their values at the origin are nonzero. In particular, Proposition 2 applies when the pdfs correspond to a multihop transmission as that of Section III-B for which the asymptotic value of the pdf at zero is given by (38).

Based on these two observations, let us consider a cooperative system with $M+1$ diversity branches $\{B_0, B_1, \ldots, B_M\}$ as depicted in Fig. 6, where by convention the diversity branch B_0 corresponds to the direct path. Each of the remaining M branches $\{B_1, \ldots, B_M\}$ is composed of N_i relays $\{R_1, \ldots, R_{N_i}\}$. The channel coefficients between the relays $R_{\rm ij}$ and $R_{i,j+1}$ of branch B_i are denoted by $h_{\rm ij}$, with h_{i0} being the coefficient between the source and the first relay and h_{iN_i} being that between the last relay and the destination.

We define the average per-hop SNRs as usual $\gamma_{\rm ij} = E|h_{\rm ij}|^2 P_x/N_0$, and we also define $p_{\rm ij}(\gamma_{\rm ij})$ to be the pdf of $\gamma_{\rm ij}$. With these definitions and combining the results of (32) and (38), we arrive at

$$\bar{P}_e \approx \frac{C(M)}{k^{M+1}} p_{00}(0) \prod_{i=1}^{M} \left(\sum_{j=0}^{N_i} p_{\rm ij}(0) \right). \tag{43}$$

[4]Although a multihop system uses N times more power than the single hop system, the result can be easily modified to keep the total transmitted power constant.

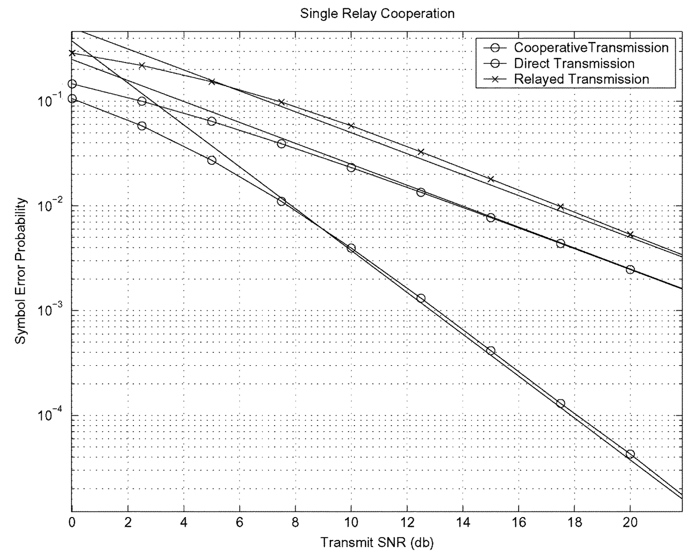

Fig. 7. SEP for single relay cooperation. Rayleigh fading with $|\bar{f}|^2 = |\bar{g}|^2 = |\bar{h}|^2 = 1$ and transmit SNR defined as SNR $= P/N_0$.

Restricting (43) to Ricean fading, we obtain the expression

$$\bar{P}_e \approx \frac{C(M)(K+1)^{M+1}}{k^{M+1}} \frac{1}{\bar{\gamma}_{00}} \prod_{i=1}^{M} \left(\sum_{j=0}^{N_i} \frac{1}{\bar{\gamma}_{\rm ij}} \right) \tag{44}$$

which is neat in its simplicity given its applicability to quite general cooperative networks.

IV. Simulations and Numerical Results

A first concern is how tight the asymptotic results are with respect to pragmatic SNR values. Several simulations ran with this goal confirmed that the asymptotic results provide a very good approximation not only for large but also for moderate SNR values. We tested BPSK modulation, Rayleigh fading (recall that the Rayleigh pdf can be obtained from the Ricean one with unity specular factor; hence, (44) applies to Rayleigh with $K = 1$). The resulting average SEP were plotted against the transmit SNR defined as SNR $= P_x/N_0$.

Fig. 7 presents simulated values for the system of Section II-B. In this case, we consider the channels g and h as having equal expected value and compare the simulated SEP with the analytical line predicted by (21). For SNR values as low as 10 dB, the difference between the observed SEP and the asymptotic SEP is less than 9%.

Similar tightness is observed in Figs. 8, and 9 for the multibranch and multihop case, respectively. The quality of the approximation decreases when the number of cooperating terminals increases, a reasonable result due to the accumulation of approximations. This effect limits the applicability of our expressions to no more than four or five cooperating branches.

In Fig. 9, we show the results of multihop cooperation when the per-hop coefficients h_i satisfy a physical constraint analogous to (24). It is interesting to note that when we use a physical constraint, the approximation is excellent for reasonable target average SEPs even when the number of hops is as large as 10.

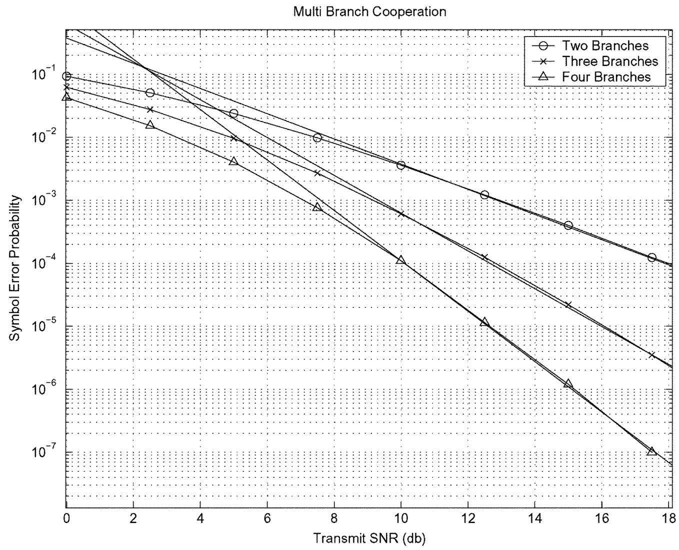

Fig. 8. Multibranch cooperation. One, two, and three cooperating branches are shown ($|\bar{f}|^2 = |\bar{g}_i|^2 = |\bar{h}_i|^2 = 1$, for all branches).

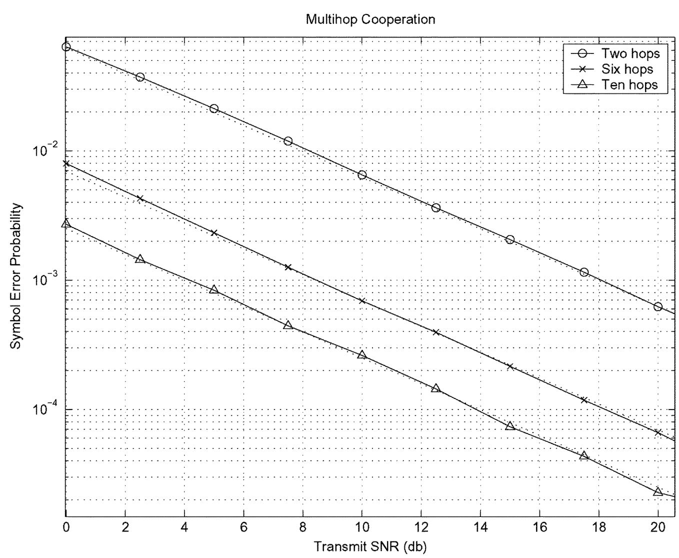

Fig. 9. Multihop cooperation. Two, six, and ten cooperating hops are shown. A physical constraint like (24) with $\alpha = 3$ is used and hops are considered equally spaced.

It is apparent from Figs. 8 and 9, and can be confirmed from (44), that in general, relay power is better used when it adds a cooperative branch than when it adds a hop in an existent branch.

V. Conclusion

We analyzed the average error probability for networks with cooperating terminals amplifying and forwarding their received signals from the source, when the average SNR is sufficiently high. Our performance analysis is applicable to cooperative links with any number of hops and branches; and remains valid for a large class of fading models, whose pdfs have nonzero values at the origin, including Rayleigh and Ricean fading channels. While our error probability formulas were derived for high average SNR, our simulations testified that they match well the simulated error probability even at moderate SNR values, especially when the number of cooperating branches is relatively small.

Our error probability analysis revealed that the error gain of multihop systems stems from the reduced path loss, while that of multibranch systems comes from both the reduced path loss and the diversity. Furthermore, the simplicity of our error probability expressions can be used to design cooperative relays optimally in the sense of minimizing error probability. This may have interesting applications to routing algorithms, relay placement, and power allocation among different terminals in wireless networks, which are directions we have marked in our future research agenda[5].

Appendix A
Asymptotic SEP for (5)

In Proposition 1, we proved that the pdf of the random variable $W = XY/(X+Y)$ at the origin is $p_W(0) = p_X(0) + p_Y(0)$. In this Appendix, we show that the redefined variable $W = XY/(1+X+Y)$ satisfies a similar property when both $E(X)$ and $E(Y)$ are large as well as $p_X(x)$ and $p_Y(y)$ satisfy certain conditions. Hence, the SEP calculated from the expression in (5) is approximately equal to that computed from the SNR in (6).

Proposition 4: Consider two nonnegative independent random variables X and Y, with pdfs p_X and p_Y, respectively. These pdfs are unknown except for their values at zero that are given by $x_0 := p_X(0) = \lambda\tilde{x}_0$ and $y_0 := p_Y(0) = \lambda\tilde{y}_0$ and are assumed to be nonzero. Furthermore, p_X and p_Y are bounded by $p_X(x) < \tilde{g}\lambda e^{-\tilde{g}\lambda x}$ and $p_Y(y) < \tilde{h}\lambda e^{-\tilde{h}\lambda y}$. If $g(x,y) = xy/(1+x+y)$ and $W = g(X,Y)$, then

$$L := \lim_{\epsilon\to 0}\lim_{\lambda\to 0}\frac{P(W<\epsilon)}{\epsilon\lambda} = \tilde{x}_0 + \tilde{y}_0. \tag{45}$$

The variables X and Y correspond to the per-hop SNRs γ_g and γ_h, and the pdf bounds correspond to Rayleigh fading, but are also valid for other channel models. Parameter λ is the inverse of the average SNR; that is, $\tilde{g}\lambda = 1/\bar{\gamma}_g$, and $\tilde{h}\lambda = 1/\bar{\gamma}_h$. Thus, L in (45) describes the behavior of the probability of the SNR of the relay path around zero, which is precisely what we need in order to apply the results of [17]. The analysis becomes more tedious as we are forced to work with limits but once Proposition 4 is established the treatment is analogous to that of Section II-B.

The probability in (45) is the cumulative distribution function (cdf) of W which for a given point ϵ can be calculated from the integral of the joint distribution of X and Y in the $\Re^2$ volume defined by the points in which $g(x,y) < \epsilon$, i.e.,

$$F_W(w) = \int\int_{\{(x,y):g(x,y)<\epsilon\}} p_X(x)p_Y(y)\,dx\,dy. \tag{46}$$

The integration in (46) can be divided over the regions shown in Fig. 10. The integrals over regions C and D are proportional to

[5]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government.

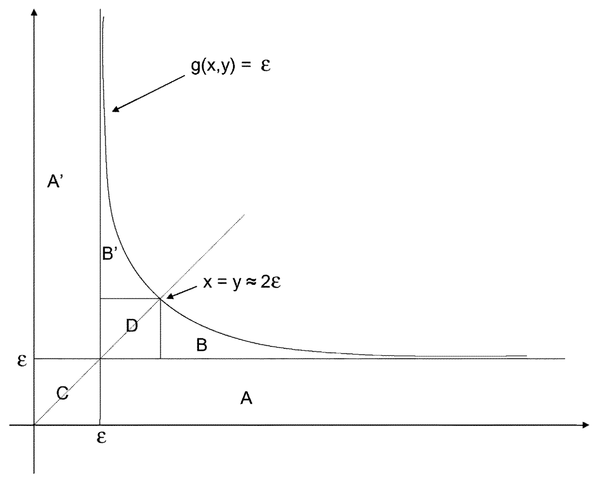

Fig. 10. Decomposition of the integration region used in the Proof of Proposition 4.

ϵ^2 and thus go to zero for all λ. On the other hand, the integrals over A and A' are well approximated by

$$\begin{aligned} I_A + I_{A'} &= \int_A p_x(x)p_y(y)\,dx\,dy + \int_{A'} p_x(x)p_y(y)\,dx\,dy \\ &= \epsilon[p_X(0) + p_Y(0)]. \end{aligned} \tag{47}$$

Since the integrals in all regions are positive, the limit L is lower bounded by (47)

$$L \geq \lim_{\epsilon\to 0}\lim_{\lambda\to 0} \frac{I_A + I_{A'}}{\epsilon\lambda} = x_0 + y_0 \tag{48}$$

regardless of λ. We now show that $L \leq x_0 + y_0$ which will conclude the proof. To show this, we have to prove that the integrals over B and B' go to zero. The integral over B is given by

$$I_B = \int_{2\epsilon}^{\infty} p_x(x)dx \int_{\epsilon}^{\frac{\epsilon(x+1)}{x-\epsilon}} p_y(y)\,dy \tag{49}$$

which can be bounded as follows

$$I_B \leq 2e\tilde{g}\,\tilde{h}\lambda^2\epsilon\ln(1/\epsilon). \tag{50}$$

Since the limit over λ is taken before the limit over ϵ, I_B goes to zero. Mimicking this argument for the integral over B' concludes the proof.

The limit L is similar, but not equal, to the pdf of W evaluated at zero as the SNR grows. So, we have to keep working with the limits and repeat the rest of the proof of Section II-B, including part of the analysis in [17]. However, once we reach the limit L, the rest of the analysis is straightforward.

Appendix B
Proof of Proposition 2

The proof proceeds along the lines of the proof of Proposition 1, where we expressed the Laplace transform of V_M as the product of the Laplace transforms of the individual pdfs, and we used the initial value theorem to compute the value of the pdf at zero. Since the variables X_i are independent, the Laplace transform of V is given by the product of the Laplace transforms of the component variables X_i, and the initial value theorem asserts

$$p_{V_M}(0) = \lim_{s\to\infty} s\prod_{i=0}^{M} L_{X_i}(s). \tag{51}$$

However, this limit can be rewritten as the product of a set of limits

$$p_{V_M}(0) = \lim_{s\to\infty} sL_{X_0}(s)\prod_{i=1}^{M}\lim_{s\to\infty} L_{X_i}(s). \tag{52}$$

The first limit in the product is $p_{X_0}(0)$, while all the other limits are null and, thus, $p_{V_M}(0) = 0$. This allows us to move on to the first derivative

$$\frac{\partial p_{V_M}}{\partial v}(0) = \prod_{i=0}^{1}\lim_{s\to\infty} sL_{X_i}(s)\prod_{i=2}^{M}\lim_{s\to\infty} L_{X_i}(s) \tag{53}$$

where, analogously to (52), the limits of the first product are the individual pdfs at zero, and the limits on the second product are null. Repeating this process until the $(M-1)$st derivative, we conclude that all the lower than M-order derivatives are null, which proves the first claim of proposition 2. For the Mth derivative, we have

$$\frac{\partial^M p_{V_M}}{\partial v^M}(0) = \prod_{i=0}^{M}\lim_{s\to\infty} sM_{X_i}(s). \tag{54}$$

Continuing with our use of the initial value theorem, each of the limits in (54) is precisely the corresponding pdf evaluated at zero, from where we arrive at

$$\frac{\partial^M p_{V_M}}{\partial v^M}(0) = \prod_{i=0}^{M} p_i(0) \tag{55}$$

which is exactly the second claim of Proposition 2.

Appendix C
Proof of Proposition 3

The proof is analogous to that of Section II-B, except that now things are complicated because we work in $N+1$ dimensions. The pdf of multidimensional functions of random variables is

$$p_Z(z) = \int\cdots\int_{\{\vec{x}:g(\vec{x})=z\}} \frac{p_1(x_1)\ldots p_n(x_n)}{|\nabla g(\vec{x})|}dx_0\ldots dx_n. \tag{56}$$

Our interest is at $z = 0$, which is a condition equivalent to nullifying one vector component at a time. This reduces the integration surface to the union of the $N+1$ hyperplanes of dimension N in which any of the coordinates $x_k = 0$. Given this restriction, it is possible to prove that the modulus of the gradient evaluated at $x_k = 0$ is equal to one over all these hyperplanes

$$|(\nabla g)|\,|_{x_k=0} = 1. \tag{57}$$

Since the integration surface is the union of the $N+1, N$-dimensional hyperplanes at which any of the x_k is zero, we obtain

$$p_Z(0) = \sum_{k=0}^{N} p_k(0)\int\cdots\int_{\Re^N}\prod_{j=0,j\neq k}^{N} p_j(x_j)\,dx_j. \tag{58}$$

However, the integrals in (58) are over all possible values of $x_{j,j \neq k}$ which are axiomatically equal to one. After this observation, the proof of Proposition 3 follows.

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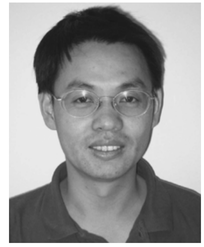
Alejandro Ribeiro (S'05) received the B.Sc. degree in electrical engineering from Universidad de la Republica Oriental del Uruguay, Montevideo, Uruguay in 1998. Since May 2003, he has been working toward the Ph.D. degree in the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis.

From 1998 to 2003, he was a Member of the Technical Staff at BellSouth, Montevideo. His research interests include the areas of communication theory, signal processing, and networking. His current research focuses on wireless cooperative communications, wireless ad hoc, and sensor networks.

Mr. Ribeiro is a Fulbright Scholar.



Xiaodong Cai (M'01) received the B.S. degree from Zhejiang University, Hangzhou, China, the M.Eng. degree from the National University of Singapore, Singapore, and the Ph.D. degree from the New Jersey Institute of Technology, Newark, in 2001, all in electrical engineering.

From February 2001 to June 2001, he was a Member of Technical Staff at Lucent Technologies, Murray Hill, NJ, working on the WCDMA project. From July 2001 to October 2001, he was a Senior System Engineer with the Sony Technology Center, San Diego, CA, where he was involved in developing high data rate wireless modem. Since November 2001, he has been a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis. His research interests include in the areas of communication theory, signal processing, and networking. His current research focuses on cooperative communications and collaborative signal processing for wireless ad hoc and sensor networks.

Georgios B. Giannakis (S'84–M'86–SM'91–F'97) received the Diploma in electrical engineering from the National Technical University of Athens, Athens, Greece, in 1981, and the M.Sc. degree in electrical engineering, the M.Sc. degree in mathematics, and the Ph.D. degree in electrical engineering, all from the University of Southern California (USC), Los Angeles, in 1983 and 1986, respectively.

After lecturing for one year at USC, he joined the University of Virginia, Charlottesville, in 1987, where he became a Professor of electrical engineering in 1997. Since 1999, he has been a Professor with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, where he now holds an ADC Chair in Wireless Telecommunications. His general interests span the areas of communications and signal processing, estimation and detection theory, time-series analysis, and system identification—subjects on which he has published more than 200 journal papers, 350 conference papers, and two edited books. His current research interests include transmitter and receiver diversity techniques for single- and multiuser fading communication channels, complex-field and space-time coding, multicarrier, ultrawide-band wireless communication systems, cross-layer designs, and distributed sensor networks.

Dr. Giannakis is the (co-)recipient of six paper awards from the IEEE Signal Processing (SP) and Communications Societies (1992, 1998, 2000, 2001, 2003, and 2004). He also received the SP Society's Technical Achievement Award in 2000. He served as Editor-in-Chief for the IEEE SIGNAL PROCESSING LETTERS, as Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING and the IEEE SIGNAL PROCESSING LETTERS, as Secretary of the SP Conference Board, as member of the SP Publications Board, as member and Vice Chair of the Statistical Signal and Array Processing Technical Committee, as Chair of the SP for Communications Technical Committee, and as a member of the IEEE Fellows Election Committee. He has also served as a member of the the IEEE-SP Society's Board of Governors, the Editorial Board for the PROCEEDINGS OF THE IEEE, and the steering committee of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS.