Demand Response Management in Smart Grids with Heterogeneous Consumer Preferences

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***Abstract*—Consumer demand profiles and fluctuating renewable power generation are two main sources of uncertainty in matching demand and supply. This paper proposes a model of the electricity market that captures the uncertainties on both, the operator and the user side. The system operator (SO) implements a temporal linear pricing strategy that depends on real-time demand and renewable generation in the considered period combining Real-Time Pricing with Time-of-Use Pricing. The announced pricing strategy sets up a noncooperative game of incomplete information among the users with heterogeneous but correlated consumption preferences. An explicit characterization of the optimal user behavior using the Bayesian Nash equilibrium solution concept is derived. This explicit characterization allows the SO to derive pricing policies that influence demand to serve practical objectives such as minimizing peak-to-average ratio or attaining a desired rate of return while at the same time hedging renewable generation uncertainty.**

I. Introduction

Matching power production to power consumption is a complex problem in conventional energy grids, exacerbated by the introduction of renewable sources, which, by their very nature, exhibit significant output fluctuations. This problem can be mitigated with a system of smart meters that control the power consumption of customers by managing the energy cycles of various devices while also enabling information exchange between customers and the system operator (SO) [1], [2]. The flow of information between meters and the SO can be combined with sophisticated pricing strategies so as to encourage a better match between power production and consumption [3]–[5]. The effort of operators to guides the consumption of end users through suitable pricing policies is referred to as demand response management (DR) [6].

To implement DR we can consider pricing mechanisms that combine *Real-Time Pricing (RTP)* with *Time-of-Use Pricing (TOU)*. That is, the price depends on total consumption at each time slot (RTP) and, in addition, the SO divides the operation cycle into periods (TOU). The use of TOU allows the SO to apply temporal policies based on its anticipation of consumption and renewable source generation in each period. The use of RTP transfers part of the risks and benefits to consumers and encourages their adaptation to power production. When producers use RTP customers agree to a pricing function but actual prices are unknown a priori because they depend in the realized aggregate demand. In this context, customers must reason strategically about the consumption of others that will ultimately determine the realized price. Game-theoretic models of user behavior then arise naturally and various mechanisms and analyses have been proposed [3], [4], [6]–[9] – see also [10], [11] for more comprehensive expositions. A common feature of these schemes is that the SO and its customers run an iterative algorithm to solve a distributed optimization problem prior to the start of an operating cycle. The outcome of this optimization results in individual power targets that the customers agree to consume once the operating cycle starts.

This paper proposes a RTP mechanism for DR in which customers agree to a linear price function that depends on the total consumption and a parameter to incentivize the use of energy produced from renewable sources. Both, total consumption and the amount of energy produced by renewable sources are unknown a priori and customers must decide their consumption based on uncertain estimates made public by the SO. Instead of running an iterative optimization problem prior to the start of the operating cycle, we assume that this is all the information exchange that occurs between customers and the SO. To determine their consumption levels customers use this information to anticipate the behavior of others, be aware of their influence on price, and mind renewable resource generation forecasts. We provide an analysis of this pricing policy in which customers' anticipatory behavior is formally modeled as the actions of rational consumers with heterogeneous preferences repeatedly taking actions in a game with *incomplete* information. We provide explicit expressions for the Bayesian Nash equilibria (BNE) of these games and use the resulting characterizations to show desirable properties of the proposed RTP mechanism – e.g., we can adapt pricing policies to achieve a desired rate of return or to minimize the power peak-to-average ratio (PAR) without using any further information exchange with customers other than the unidirectional broadcasting of the pricing policy parameters.

We begin with an introduction of the mathematical model for operator and consumer behavior (Sections II-A and II-B). The SO hedges its uncertainty in renewable sources on the price where predicted abundance of renewables creates consumption incentives through fixed discounts and the predicted scarcity discourages consumption at each period. The pricing policy and renewable generation prediction is broadcasted to the consumers at the beginning of the period. We model consumption behavior by designing payoffs for each customer that depends on self-preferences and price. The self-preference is private to the customer itself and is not known by the SO and other customers. The consumers act to maximize their myopic expected payoffs with respect to their belief on others' preferences and renewable power generation estimate (Section II-B). We explicitly characterize consumption behavior at each time with respect to self-preference by using BNE as the solution concept when the preferences come from a jointly normal distribution (Section III). We compare the selfish behavior to the case when customers are altruistic; that is, they care for each other and act to maximize aggregate utility given the uncertainties. Our results show selfish users overweigh their self-preferences compared to the aggregate utility

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maximizing user making demand less predictable for the operator. We provide numerical experiments exploring the behavior of aggregate utility, consumption, price, and operator's realized profit with respect to the population's preference distribution, renewable prediction errors and price policy parameters (Section IV). Finally, we show that the explicit characterization of consumer behavior allows the SO to execute pricing policies that attain desired rate of return or minimize PAR consumption without having access to users' preferences (Section V).

II. Smart Grid Model

A system operator oversees a DR model with N customers. Customers, each equipped with a power consumption scheduler, are characterized by their individual load consumption l_{ih} defined as the power consumed by customer $i \in \mathcal{N}$ at time slot $h \in \mathcal{H} := \{1, \ldots, H\}$. Accordingly, we represent the total consumption of the population at time slot h with $L_h := \sum_{i \in \mathcal{N}} l_{ih}$. In order to be able to respond to changing conditions in the environment, e.g. resource prices or consumption preferences, the SO divides the day into K time zones $t_1, t_2, \ldots, t_K$. The time zone k is a batch of time slots starting at $h_k^s \in \mathcal{H}$ and ending at $h_k^e \in \mathcal{H}$, i.e., $t_k := [h_k^s, h_k^e)$. The time zones do not overlap and span the operation cycle $\mathcal{H}$, that is, $h_{k-1}^e = h_k^s$ for $k \in \{2, \ldots, K\}$ and $h_1^s = 1$ and $h_K^e = H$.

A. Power provider model

For a time slot $h \in t_k$ the total power consumption L_h results in the SO incurring a production cost of $C_k(L_h)$ units. Observe that the production cost function $C_k(L_h)$ depends on the time zone k and the total power produced L_h. When the generation cost per unit is constant, $C_k(L_h)$ is a linear function of L_h. More often, increasing the load L_h results in increasing unit costs as more expensive energy sources are brought online. This results in superlinear cost functions $C_k(L_h)$ with a customary model being the quadratic form[1]

$$C_k(L_h) = \frac{1}{2} \kappa_k L_h^2, \tag{1}$$

for given constants $\kappa_k > 0$ that depend on the day's time zone k. The cost in (1) has been experimentally validated for thermal generators [12] and is otherwise widely accepted as a reasonable approximation [4], [6], [7].

The SO utilizes an adaptive pricing strategy whereby customers are charged a slot-dependent price p_h that varies linearly with the total power consumption L_h. The SO owns renewable source plants such as wind farms and solar arrays, and incorporates renewable source generation into the pricing strategy by introducing a random variable $\omega_k \in \mathbb{R}$ that depends on the amount of renewable power produced. The per-unit power price at time slot $h \in t_k$ is set as

$$p_k(L_h; \omega_k) = \gamma_k (L_h + \omega_k), \tag{2}$$

where $\gamma_k > 0$ is a policy parameter to be determined by the SO based on its objectives. We present how the operator can pick its policy parameter $\gamma_k > 0$ to minimize PAR or achieve a desired rate of return in Section V after modeling and analyzing consumption behavior. The random variable ω_k is such that $\omega_k = 0$ when renewable sources operate at their nominal benchmark capacity $\bar{W}_k$; that is, the generation at time zone k W_k equals $\bar{W}_k$. If the realized production exceeds this benchmark, $W_k > \bar{W}_k$, the SO agrees to set $\omega_k < 0$ to discount the energy price and share the windfall brought about by favorable weather conditions. If the realized production is below benchmark, i.e., $W_k < \bar{W}_k$, the SO sets $\omega_k > 0$ to reflect the additional charge on the customers. The specific dependence of ω_k with the realized energy production and the policy parameter γ_k, are part of the supply contract between the SO and its customers.

The operator's price function maps the amount of energy demanded to the market price. This is a standard model in pricing – see [13] for a similar formulation. A fundamental observation here is that the prices $p_k(L_h; \omega_k)$ in (2) become known *after* the end of the time zone t_k. This is because prices depend on the total demand L_h and the value of ω_k, which is determined by the amount of renewable power produced in time zone t_k. Both of these quantities are unknown a priori as shown in Fig. 1.

We assume that the SO uses a model on the renewable power generation – see, e.g., [3] for the prediction of wind generation – to estimate the value of ω_k at the beginning of the time zone k. The corresponding probability distribution P_{ω_k} is made available to all customers at the beginning of the time zone. Henceforth, we use E_{ω_k} to denote expectation with respect to the belief P_{ω_k} and $\bar{\omega}_k := E_{\omega_k}[\omega_k]$ to denote the mean of the distribution P_{ω_k}. By including a term that depends on renewable generation in the price function, the SO aims to use the flexibility of consumption behavior to compensate for the uncertainties in renewables in real-time [3], [14].

Remark 1 *The time zone-based adjustability of the pricing function in* (2) *allows the SO to be responsive to not only changes in predictions of renewable power but also to changes in predicted consumption behavior. Recent studies have shown that consumption behavior is constant majority of the time with few significant changes [15]. Hence, few time zones suffice in order to be responsive to changes in consumption preferences within the operation cycle. In this paper, we assume that the SO is able to correctly predict changes in consumption preferences within the operation cycle-based on past consumer data and determines time zones* $t_1, \ldots, t_K$ *accordingly. This pricing scheme is in line with the argument that pricing schemes that consider hour-by-hour consumption behavior are* fairer *compared to the ones that consider the total load across the horizon as they enable charging customers according to their load profile [16].*

B. Power consumer model

The consumption preferences of users are determined by random variables $g_{ik} > 0$ that are possibly different across customers and time zones. When user i consumes l_{ih} units of power at time slot h we assume that it receives the linear marginal utility $g_{ik} l_{ih}$. The user has a diminishing marginal utility from consumption which is captured by the introduction of a quadratic penalty $\alpha_k l_{ih}^2$. This quadratic penalty implies that even when the price charged by the SO is zero, e.g., when $\gamma_k = 0$, it is not in users' interest to consume infinite amounts of energy. Note that the constant α_k may change across time zones k but is the same for all consumers. For each unit of power consumed, the SO charges the price $p_k(L_h; \omega_k)$, which results in user i incurring

[1]It is possible to add linear and constant cost terms to $C_k(L_h)$ and have all the results in this paper still hold. We exclude these terms to simplify notation.

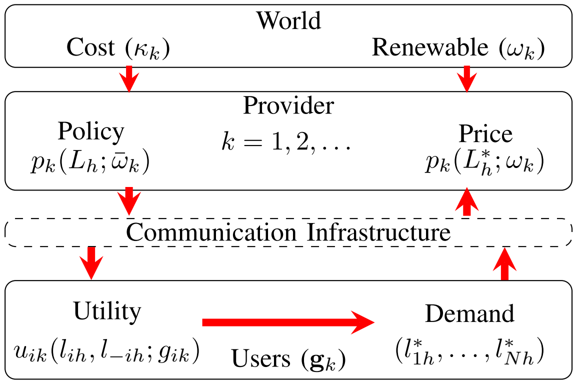


Fig. 1. Illustration of information flow between the power provider and the consumers. The provider determines the pricing policy (2) and broadcasts it to the users along with its prediction of renewable energy term P_{ω_k}. Selfish (4) or altruistic (6) users respond optimally to realize demand $L_h^* = \sum_{i\in\mathcal{N}} l_{ih}^*$. The realized total demand L_h^* together with realized renewable generation term ω_k determines the price of time zone k.

the total cost $l_{ih}p_k(L_h;\omega_k)$. The utility of user i is then given by the difference between the consumption return $g_{ik}l_{ih}$, the power cost $l_{ih}p_k(L_h;\omega_k)$ and the overconsumption penalty $\alpha_k l_{ih}^2$,

$$u_{ik}(l_{ih}, L_h; g_{ik}, \omega_k) = -l_{ih}p_k(L_h;\omega_k) + g_{ik}l_{ih} - \alpha_k l_{ih}^2. \quad (3)$$

Using the expressions for prices in (2) and L_h we express (3) as

$$u_{ik}(l_{ih}, l_{-ih}; g_{ik}, \omega_k) \quad (4)$$
$$= -l_{ih}\left[\gamma_k\left(\sum_{j\in\mathcal{N}} l_{jh} + \omega_k\right)\right] + g_{ik}l_{ih} - \alpha_k l_{ih}^2,$$

where we have also rewritten the utility of user i as $u_{ik}(l_{ih}, L_h; g_{ik}, \omega_k) = u_{ik}(l_{ih}, l_{-ih}; g_{ik}, \omega_k)$ to emphasize the fact that it depends on the consumption $l_{-ih} := \{l_{jh}\}_{j\neq i}$ of other users. Note that if the provider's policy parameter is set to $\gamma_k = 0$, the utility of user i is maximized by $l_{ih} = g_{ik}/2\alpha_k$ – see [7] for a similar formulation.

The utility of user i depends on the powers l_{-ih} that are consumed by other users in the current slot. These l_{-ih} power consumptions depend partly on their respective preferences, i.e., marginal utilities $g_{-ik} := \{g_{jk}\}_{j\neq i}$, which are, in general, *unknown* to user i. We assume, however, that there is a probability distribution $P_{\mathbf{g}_k}(\mathbf{g}_k)$ on the vector of marginal utilities $\mathbf{g}_k := [g_{1k}, \ldots, g_{Nk}]^T$ from where these marginal utilities are drawn and this probability distribution is *known* to all users. We further assume that $P_{\mathbf{g}_k}$ is normal with mean $\bar{g}_k\mathbf{1}$ where $\bar{g}_k > 0$ and $\mathbf{1}$ is an $N\times 1$ vector with one in every element, and covariance matrix $\mathbf{\Sigma}_k$,

$$P_{\mathbf{g}_k}(\mathbf{g}_k) = \mathcal{N}\Big(\mathbf{g}_k;\ \bar{g}_k\mathbf{1}, \mathbf{\Sigma}_k\Big). \quad (5)$$

We use the operator $E_{\mathbf{g}_k}$ to signify expectation with respect to the distribution $P_{\mathbf{g}_k}$ and $\sigma_{ij}^k := ((\mathbf{\Sigma}_k))_{ij}$ to denote the (i,j)th entry of the covariance matrix $\mathbf{\Sigma}_k$. Having mean $\bar{g}_k\mathbf{1}$ implies that all customers have equal average preferences in that $E_{\mathbf{g}_k}(g_{ik}) = \bar{g}_k$ for all i. If $\sigma_{ij}^k = 0$ for some pair $i \neq j$, it means that the marginal utilities of these customers are uncorrelated. In general, $\sigma_{ij}^k \neq 0$ to account for correlated preferences due to, e.g., common weather. It is assumed that marginal utilities $\mathbf{g}_k$ and $\mathbf{g}_l$ for different time zones $k \neq l$ are independent, e.g., the jump in consumption preference from off-peak to peak zone is independent.

The probability distributions P_{ω_k} and $P_{\mathbf{g}_k}$ and the parameters α_k and γ_k are common knowledge among the operator and its customers. That is, the probability distribution $P_{\mathbf{g}_k}$ in (5) is correctly predicted by the SO based on past data by assumption and is announced to the customers. The pricing parameter γ_k and the operator's belief on the renewable energy parameter ω_k, P_{ω_k} is also announced. In addition, customer i knows its private value of consumption preference g_{ik}.

A selfish customers' goal is to maximize the utility $u_{ik}(l_{ih}, l_{-ih}; g_{ik}, \omega_k)$ in (4) given its partial knowledge of the others' consumptions l_{-ih}. An altruistic customers' goal is to maximize the aggregate utility defined as the sum of consumers' utility functions,

$$U_k(\{l_{jh}\}_{j\in\mathcal{N}}; \mathbf{g}_k, \omega_k) := \sum_{i\in\mathcal{N}} u_{ik}(l_{ih}, l_{-ih}; g_{ik}, \omega_k). \quad (6)$$

Both of these maximizations require a model of behavior for other users that comes in the form of a BNE that we introduce next.

III. Customers' Bayesian Game

User i's load consumption at time $h \in t_k$ is determined by its *belief* q_{ih} and *strategy* s_{ih}. The belief of i is a conditional probability distribution on $\mathbf{g}_k$ given g_{ik}, $q_{ih}(\cdot) := P_{\mathbf{g}_k}(\cdot|g_{ik})$. We use $E_{ih}[\cdot] := E_{\mathbf{g}_k}[\cdot|g_{ik}]$ to indicate conditional expectation with respect to belief of user i at time h. In order to second-guess the consumption of other customers, user i forms beliefs on their preferences given the common prior $P_{\mathbf{g}_k}$ and self-preferences up to time zone k $\{g_{im}\}_{m=1,\ldots,k}$. Observe that self-preferences of previous time zones $\{g_{im}\}_{m<k}$ are not relevant to belief at time zone k as they are independent from the preferences at time zone k. Note further that user i's belief is static over the time horizon as it receives no other information about the preferences of others. User i's load consumption at time $h \in t_k$ is determined by its strategy which is a complete contingency plan that maps any possible local observation that it may have to its consumption, that is, $s_{ih} : g_{ik} \mapsto \mathbb{R}$ for any g_{ik}. In particular, for user i, its best response strategy is to maximize expected utility with respect to its belief q_{ih} given the strategies of other customers $\mathbf{s}_{-ih} := \{s_{jh}\}_{j\neq i}$,

$$BR(g_{ik}; \mathbf{s}_{-ih}) = \arg\max_{l_{ih}} E_{\omega_k}\Big[E_{ih}\big[u_{ik}(l_{ih}, \mathbf{s}_{-ih}; g_{ik}, \omega_k)\big]\Big]. \quad (7)$$

A BNE strategy profile $\mathbf{s}^* := \{s_{ih}\}_{i\in\mathcal{N}, h\in\mathcal{H}}$ at time $h \in t_k$ is a strategy in which each customer maximizes expected utility with respect to its own belief given that other customers play with respect to BNE strategy.

Definition 1 *A Bayesian Nash equilibrium (BNE) strategy* $\mathbf{s}^*$ *is such that for all* $i \in \mathcal{N}$, $k = 1, \ldots, K$, $h \in t_k$, *and* $\{q_{ih}\}_{i\in\mathcal{N}, h\in\mathcal{H}}$,

$$E_{\omega_k}\Big[E_{ih}\big[u_{ik}(s_{ih}^*, \mathbf{s}_{-ih}^*; g_{ik}, \omega_k)\big]\Big] \geq$$
$$E_{\omega_k}\Big[E_{ih}\big[u_{ik}(s_{ih}, \mathbf{s}_{-ih}^*; g_{ik}, \omega_k)\big]\Big]. \quad (8)$$

A BNE strategy (8) is computed using beliefs formed according to Bayes' rule. Note that the BNE strategy profile is defined for all time slots, that is, no user at any given point in time has a profitable deviation to another strategy. Equivalently, a BNE strategy is one in which users play best response strategy given their individual beliefs as per (7) to best response strategies of other users – see [13], [17], [18] for a detailed explanation. As a result, the BNE strategy is defined with the following fixed point equations

$$s_{ih}^*(g_{ik}) = BR(g_{ik}; \mathbf{s}_{-ih}^*) \tag{9}$$

for all $i \in \mathcal{N}$, $h \in t_k$, and g_{ik}. Using the definition in (9), the following result characterizes the unique linear BNE strategy.

Proposition 1 *Consider the game defined by the payoff in* (4) *at time $h \in t_k$ for $k = 1, \ldots, K$. Let the information given to customer i be its preference g_{ik}, the common normal prior on preferences $P_{\mathbf{g}_k}$ as per* (5) *and the prior on renewable generation P_{ω_k}. Then, the unique BNE strategy of customer i is linear in $\bar{\omega}_k, \bar{g}_k, g_{ik}$ for all $k = 1, \ldots, K$ such that*

$$s_{ih}^*(g_{ik}) = a_{ih}(\bar{g}_k - \bar{\omega}_k\gamma_k) + b_{ih}(g_{ik} - \bar{g}_k) \tag{10}$$

where the constants a_{ih} and b_{ih} are entries of the vectors $\mathbf{a}_h = [a_{1h}, \ldots, a_{Nh}]^T$ and $\mathbf{b}_h = [b_{1h}, \ldots, b_{Nh}]^T$ which are given by

$$\mathbf{a}_h = ((N+1)\gamma_k + 2\alpha_k)^{-1}\mathbf{1}, \quad \mathbf{b}_h = \rho_k \mathbf{d}(\mathbf{\Sigma}_k), \tag{11}$$

with constant $\rho_k = (2(\gamma_k + \alpha_k))^{-1}$ and inference vector

$$\mathbf{d}(\mathbf{\Sigma}_k) = (\mathbf{I} + \rho_k\gamma_k\mathbf{S}(\mathbf{\Sigma}_k))^{-1}\mathbf{1}. \tag{12}$$

obtained from the pairwise inference matrix $\mathbf{S}(\mathbf{\Sigma}_k)$ defined as

$$[[\mathbf{S}(\mathbf{\Sigma}_k)]]_{ii} = 0, [[\mathbf{S}(\mathbf{\Sigma}_k)]]_{ij} = \sigma_{ij}^k/\sigma_{ii}^k \quad \forall i \in \mathcal{N}, j \in \mathcal{N} \setminus i. \tag{13}$$

Proof: See Appendix. ■

Proposition 1 shows that there exists a unique BNE strategy that is linear in self-preference g_{ik} at each time slot. This is a direct consequence of the fact that the utility in (4) has quadratic form and the prior on preferences is normal (5). From the linear strategy in (10), we observe that increase in mean preference $\bar{g}_k$ causes an increase in consumption when $a_{ih} > b_{ih}$. From the first set of strategy coefficients in (11), $\mathbf{a}_h$, we observe that the estimated effect of renewable power $\bar{\omega}_k$ has a decreasing effect on individual consumption. This is expected since increasing $\bar{\omega}_k$ implies an expected increase in the price which lowers the incentive to consume. Observe that the BNE strategy (10) does not contain any time slot h dependent parameter hence the consumption level of an individual is fixed for all $h \in t_k$. This is due to the fact that users do not receive any new information within a time zone. This is supported by the finding that significant changes to consumption behavior are few within an operation cycle [15].

Further observe that the strategy coefficients a_{ih} and b_{ih} do not depend on information specific to customer i. A consequence of this observation is that the SO knows the strategy functions of all the users via the action coefficient equations in (11). On the other hand, the realized load consumption l_{ih} is a function of realized preference g_{ik}, i.e., $l_{ih}^* = s_{ih}^*(g_{ik})$, which is private. Hence, knowing the strategy function does not imply that the SO knows the consumption level of the users. Nevertheless, the SO can use the BNE strategies of users to estimate the expected total consumption in order to achieve its policy design objectives defined in Section V.

The strategy coefficients $\mathbf{a}_h$ and $\mathbf{b}_h$ in (11) depend on the inference vector $\mathbf{d}(\mathbf{\Sigma}_k)$ which is driven by the covariance matrix $\mathbf{\Sigma}_k$. In order to identify the effect of correlation among preferences on user behavior, we define the notion of σ-correlated preferences.

Definition 2 *The preferences of users are σ-correlated at time zone k if $\sigma_{ij}^k = \sigma$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus i$ and $\sigma_{ii}^k = 1$ for all $i \in \mathcal{N}$ where $0 \le \sigma \le 1$.*

In σ-correlated preferences, the correlation among all users vary according to the parameter σ. Hence, the definition does not allow heterogeneous correlation among pairs. When the parameter σ is varied, the preference correlation change is ubiquitous. The inference vector $\mathbf{d}(\mathbf{\Sigma}_k)$ is well-defined for σ-correlated preferences where $0 \le \sigma \le 1$. We interpret the effect of correlation on the BNE strategies of users with respect to varying σ in the next result.

Proposition 2 *Denote the BNE strategy weights by $\mathbf{a}_h^\sigma$, $\mathbf{b}_h^\sigma$ when preferences are σ-correlated at time zone k. Then, when $\sigma'' > \sigma'$, we have the following relationship in time zone k,*

$$a_{ih}^{\sigma'} = a_{ih}^{\sigma''} \text{ and } b_{ih}^{\sigma'} > b_{ih}^{\sigma''} \quad \forall i \in \mathcal{N}. \tag{14}$$

Proof: When the preferences are σ-correlated, the off-diagonal elements of the inference matrix $\mathbf{S}(\mathbf{\Sigma}_k)$ is equal to σ. As a result, it can be expressed as $\mathbf{S}(\mathbf{\Sigma}_k) = \sigma(\mathbf{1}\mathbf{1}^T - \mathbf{I})$ which allows us to express the inference vector as $\mathbf{d}(\mathbf{\Sigma}_k) = (\mathbf{I} + \rho_k\gamma_k\sigma(\mathbf{1}\mathbf{1}^T - \mathbf{I}))^{-1}\mathbf{1}$. Use the relationship that $(\mathbf{I} + c(\mathbf{1}\mathbf{1}^T - \mathbf{I}))^{-1}\mathbf{1} = ((N-1)c+1)^{-1}$ for a constant c to obtain the following weights for $\mathbf{a}_h^\sigma$ and $\mathbf{b}_h^\sigma$ in (11),

$$\begin{aligned} \mathbf{a}_h^\sigma &= ((N+1)\gamma_k + 2\alpha_k)^{-1}\mathbf{1}, \\ \mathbf{b}_h^\sigma &= \rho_k((N-1)\gamma_k\rho_k\sigma + 1)^{-1}\mathbf{1}. \end{aligned} \tag{15}$$

The result is obtained by comparing individual entries of (15). ■

Proposition 2 shows that user i's strategy is to place less weight on self-preference g_{ik} when the correlation between the users increases. If the user i's preference is higher than the mean, $g_{ik} > \bar{g}_k$, then increasing correlation coefficient decreases consumption of user i. When $g_{ik} < \bar{g}_k$, user i's consumption increases as σ is increased. The intuition is as follows. Consider the case where $g_{ik} > \bar{g}_k$. As the correlation coefficient increases, it is more likely that others' preferences are also above the mean. For instance, others' preferences are certainly above the mean when $\sigma = 1$, given $g_{ik} > \bar{g}_k$. This implies that consumption willingness of others is similar to i, which then means the price will be higher than what is expected when the population's preference is at the mean. As a result, user i decreases its consumption. An identical reasoning follows when $g_{ik} < \bar{g}_k$.

The increase in correlation coefficient enhances the ability of individuals to predict each others' preferences. Alternatively, this increase in prediction ability can be achieved via communication among individuals. Hence, Proposition 2 states that if communication is such that the predictive ability of all the individuals increase, then users place less weight on self-preferences and more on the mean estimate $\bar{g}_k$. In [18], a similar result is shown to hold for the beauty contest game where in contrast to the game considered here, individuals have the incentive to increase their action when others increase theirs.

We note that the strategy coefficients of all users are the same when the preferences are σ-correlated; that is, $a_{ih}^\sigma = a_{jh}^\sigma$ and $b_{ih}^\sigma = b_{jh}^\sigma$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus i$. Furthermore, the effect of γ_k on strategy coefficients is readily identified from (15). BNE strategy coefficients $\mathbf{a}_h^\sigma$ and $\mathbf{b}_h^\sigma$ decrease with respect to increasing γ_k – see equations in (15). The downward trend on consumption is conceivable since increasing γ_k means increasing the elasticity of price with respect to total consumption.

We remark that similar analysis as in Proposition 2 follows when σ_{ii} is equal to some constant $c > \sigma$ for all $i \in \mathcal{N}$, that is, it suffices that the diagonals of $\mathbf{\Sigma}_k$ are equal.

Finally, we compare the strategies of altruistic users to the strategies of selfish users. Next result characterizes user behavior when users regard aggregate utility in (6).

Proposition 3 *Consider the game where each customer maximizes the aggregate utility in* (6) *given self-preference* g_{ik} *and priors on* $\mathbf{g}_k$ *and* ω_k. *Then the unique optimal strategy of customer* i *for* $h \in t_k$ *is given by*

$$s_{ih}^U(g_{ik}) = a_{ih}^U(\bar{g}_k - \bar{\omega}_k\gamma_k) + b_{ih}^U(g_{ik} - \bar{g}_k) \tag{16}$$

where $\mathbf{a}_h^U = [a_{1h}^U, \ldots, a_{Nh}^U]^T$ *and* $\mathbf{b}_h^U = [b_{1h}^U, \ldots, b_{Nh}^U]^T$ *are*

$$\mathbf{a}_h^U = (2(N\gamma_k + \alpha_k))^{-1}\mathbf{1}, \mathbf{b}_h^U = \rho_k(\mathbf{I} + 2\rho_k\gamma_k\mathbf{S}(\mathbf{\Sigma}_k))^{-1}\mathbf{1}, \tag{17}$$

with constant ρ_k *and inference matrix* $\mathbf{S}(\Sigma_k)$ *as defined in Proposition 1.*

Proof: See Appendix. ■

We observe that the users place less weight on self-preferences compared to when they are selfish. Note that $[[(\mathbf{I} + 2\rho_k\gamma_k\mathbf{S}(\mathbf{\Sigma}_k))^{-1}\mathbf{1}]]_i \le [[(\mathbf{I} + \rho_k\gamma_k\mathbf{S}(\mathbf{\Sigma}_k))^{-1}\mathbf{1}]]_i$ for $i = 1, \ldots, N$. Consequently, when the individuals are selfish, they overweigh their preferences, that is, $b_{ih} > b_{ih}^U$. The first coefficient of selfish user i is larger than altruistic user, that is, $a_{ih} > a_{ih}^U$. Consequently, when $\bar{g}_k - \gamma_k\omega_k > 0$ and $g_{ik} > \bar{g}_k$, selfish user i consumes more than that of an altruistic user i, that is, $s_{ih}^*(g_{ik}) > s_{ih}^U(g_{ik})$. When preferences are σ-correlated, the strategy coefficients of the altruistic user i is given by

$$a_{ih}^{U\sigma} = (2(N\gamma_k + \alpha))^{-1}, b_{ih}^{U\sigma} = \rho_k(2(N-1)\gamma_k\rho_k\sigma + 1)^{-1}. \tag{18}$$

The effects of the correlation coefficient σ and the policy parameter γ_k on the consumption of altruistic users are identical to the discussion following Proposition 2.

IV. Numerical Examples

We numerically evaluate the effects of the preference distribution $P_{\mathbf{g}_k}$ (Section IV-A), policy parameter γ_k (Section IV-B) and prediction errors of renewable power term ω_k (Section IV-C) on aggregate utility (6), total consumption L_h, price (2), operators' costs, $C_k(L_h)$, realized rate of return defined as revenue divided by the cost. In our simulations, there are three time zones $K = 3$ in a $H = 24$ hour day where each time slot is an hour. The start and end times of the time zones are given by $t_1 = [1, 8]$, $t_2 = [9, 17]$ and $t_3 = [18, 24]$. While the first and last time zones are off-peak time zones with mean marginal utilities equal to $\bar{g}_1 = 30$ and $\bar{g}_3 = 35$, the second time zone is the peak zone with mean preference $\bar{g}_2 = 50$. We choose the variance of the preferences to be identical for all time zones, that is, $\sigma_{ii} = 4$. The covariances σ_{ij} are set to 2 for all agents at all time zones unless otherwise stated. That is, we consider σ-correlated preferences but use the variable σ_{ij} to refer to off-diagonal elements of $\mathbf{\Sigma}_k$ for $k = 1, 2, 3$. There are $N = 10$ users. We consider selfish users (4) with the decay parameter chosen as $\alpha_k = 1.5$. The cost function of the SO is as given in (1) with the parameter values

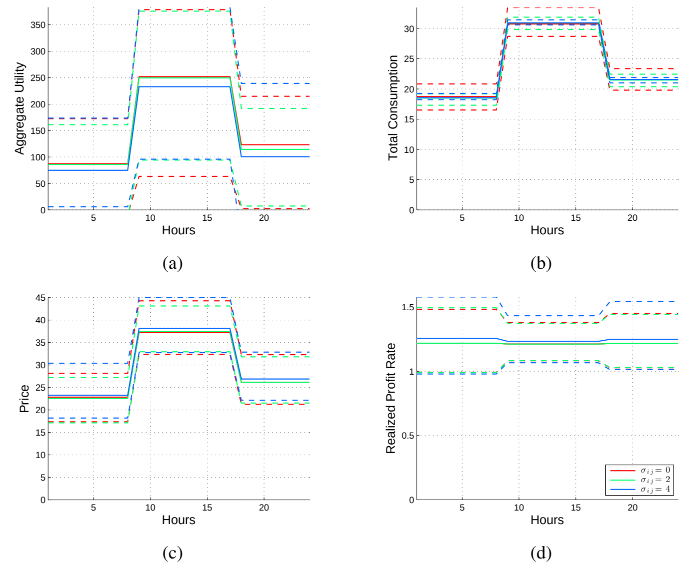


Fig. 2. Effect of preference distribution on performance metrics Aggregate Utility U_h (a), total consumption L_h (b), price $p_h(L_h; \beta_k, \omega_k)$ (c), and realized profit $R_k(L_h)/C_k(L_h)$ (d). Each line represents the value of the performance metric with respect to three values of $\sigma_{ij} \in \{0, 2, 4\}$ as color coded in the legend of (d). Solid lines represent the average value over 20 instantiations. Dashed lines indicate the maximum and minimum values of 20 instantiations. Changes in user preferences do not affect realized profit rate of the operator.

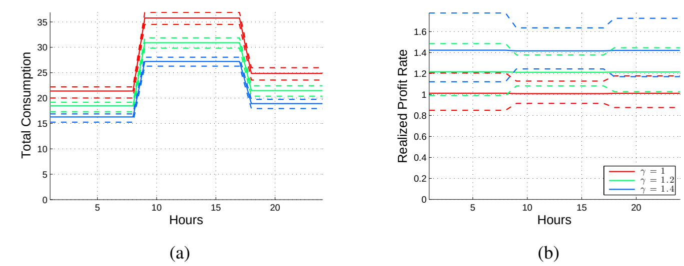
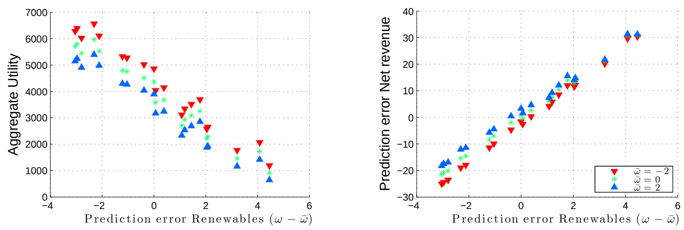


Fig. 3. Effect of policy parameter on performance metrics total consumption L_h (a), and realized profit $R_k(L_h)/C_k(L_h)$ (b). Each solid line represents the average value (over 20 realizations) of the performance metric with respect to three values of $\gamma \in \{1, 1.2, 1.4\}$ where $\gamma_k = \gamma$ for $k = 1, 2, 3$ as color coded in the legend of (d). Dashed lines mark minimum and maximum values over all scenarios. Total consumption decreases with increasing γ.

$\kappa_k = 1$. For the baseline results, the policy parameter is set to $\gamma_k = 1.2$ for $k = 1, 2, 3$. Unless stated otherwise, we let the renewable power term ω_k come from normal distribution with mean $\bar{\omega}_k = 0$ and variance $\sigma_{\omega_k} = 2$ for $k = 1, 2, 3$.

Our findings can be summarized as follows. The mean of marginal utility $\bar{g}_k$ is a significant factor shaping consumption behavior while the adaptive pricing scheme protects the realized profit to be affected by the changes in $\bar{g}_k$. Furthermore, increased correlation among users decreases the uncertainty in total consumption. Based on the decreasing effect of increasing price policy parameter γ_k on total consumption, we observe that increasing γ_k during peak time zones can reduce PAR. Prediction error of renewable generation $\omega_k - \bar{\omega}_k$ is beneficial to the SO if it is positive; otherwise, it is beneficial to the customer. Furthermore, we observe that given the same amount of prediction error in renewable generation a predicted discount $\bar{\omega}_k < 0$ is always preferable by the consumers.



(a) (b)

Fig. 4. Effect of prediction error of renewable power uncertainty ω_k on aggregate utility (a) and net revenue (b). For both figures horizontal axis shows the prediction error for the renewable term in price, that is, $\omega_k = \omega$ and $\bar{\omega}_k = \bar{\omega}$ and it shows $\omega - \bar{\omega}$. Each point in (a) or (b) corresponds to the value of the aggregate utility or the difference between net revenue and estimated net revenue at a single run. When the realized renewable term ω is larger, net revenue is increases while the aggregate utility decreases.

A. Effect of marginal utility distribution

In Figs. 2(a)-(d), we plot aggregate utility, total consumption, price and realized profit ratio at each hour of the day, respectively. Each solid line is the mean value from 20 realizations of the $\mathbf{g}_k$ and ω_k. Each dashed plot refers to the maximum and minimum values among the scenarios considered. Plots with different covariance values are color coded in Fig. 2(d).

Mean marginal utility $\bar{g}_k$ has a significant effect on all of the performance metrics except realized profit. We observe that in the peak time zone total consumption and price is higher in Fig. 2(b) and Fig. 2(c), respectively. The increase in price is expected in peak hours with a jump in total consumption – see (2). Increase in price does not automatically translate to an increase in realized profit ratio in Fig. 2(d) since the cost also grows quadratically with total consumption. The correlation value σ_{ij} affects the band that total consumption moves in as shown by Fig. 2(b). Specifically, the uncertainty in consumption is higher when individuals are less correlated. This indicates that the SO can predict consumption behavior with higher accuracy when the preferences are highly correlated.

B. Effect of policy parameter

Figs. 3(a)-(b) illustrate the effect of policy parameter γ_k on total consumption and realized profit, respectively. We fix the policy parameter across time zones $k = 1, 2, 3$, that is, $\gamma_k = \gamma \in \{1, 1.2, 1.4\}$ for all $k = 1, 2, 3$. As before, solid lines indicate average value over 20 instantiations and dashed lines indicate minimum and maximum values over the 20 runs. The legend in Fig. 3(b) color codes each line according to the policy parameter.

Total consumption decreases as γ increases in Fig. 3(a) as noted in the discussion following Proposition 2. We observe that the mean realized profit ratio overlap with the policy parameter γ in Fig. 3(b). Furthermore, PAR in total consumption is not altered when γ is fixed over the time horizon in Fig. 3(b). As a policy to reduce PAR, the SO might choose high γ in the peak time zone and low γ when demand is low. Based on this we propose a formal PAR minimizing policy in Section V and compare it with other commonly used pricing schemes.

C. Effect of uncertainty in renewable power

From the BNE strategy of customers in (10), we observe that the announced expectation of $\omega_k = \omega$ for $k = 1, 2, 3$, $\bar{\omega}_k = \bar{\omega}$ affects the load of the customers linearly. Hence, the SO can use the response of its customers to mitigate the effects of fluctuations in renewable source generation. However, the contract between the operator and the customers is such that the latter are charged based on the realization of the random variable ω. We analyze the effect of prediction errors of ω on aggregate utility and the operator's net revenue in Figs. 4(a)-(b) where we plot the two metrics with respect to prediction error $\omega - \bar{\omega}$. Fig. 4(a) shows that aggregate utility decreases as the realized value grows. Fig. 4(b) shows that the realized net revenue is most likely larger than the estimated net revenue when the realized value of $\omega - \bar{\omega}$ is positive. Furthermore, observe that a decrease in the announced estimate, $\bar{\omega}$ is always beneficial to the aggregate utility of users when the amount of prediction error is fixed. On the other hand, an expected discount decreases the net revenue of the SO.

V. Comparison among pricing policy mechanisms

We propose desired rate of return and PAR minimization as the two objectives that the provider may determine the pricing policy parameter with respect to. Below we first explain these two objectives and then compare them with flat and TOU pricing schemes in numerical experiments.

Desired Rate of Return RTP. The rate of return is defined as the revenue divided by the cost. The SO's revenue at time slot $h \in t_k$ is obtained by multiplying the total consumption L_h by the price in (2), $R_k(L_h) := L_h p_k(L_h; \omega_k)$. The operator's rate of return for the time slot is given by the ratio $R_k(L_h)/C_k(L_h)$. Given its uncertainties in user marginal utilities $\mathbf{g}_k$, the SO relies on the consumer behavior determined by the BNE (10) to obtain a target expected rate of return $r_k^* = E[R_k(L_h(\gamma_k))/C_k(L_h(\gamma_k)]$ at time zone k by adjusting its policy parameter γ_k. The term $L_h(\gamma_k)$ makes the operator's possible influence on consumption behavior through the adjustment of γ_k explicit. In a budget balancing scheme, the SO would set desired rate of return to $r_k^* = 1$. Otherwise, it is customary that the desired profit rate is larger than $r_k^* > 1$ – see [6], [8] for similar pricing policies. Solving the desired rate of return $r_k^* = E[R_k(L_h(\gamma_k))/C_k(L_h(\gamma_k))]$ with respect to price yields that the policy parameter is equal to $\gamma_k = r_k^* \kappa_k$ when we neglect the renewable generation term $\omega_k = 0$. This explains the overlap between γ_k and mean rate of return observed in Fig. 3(b). In our comparisons, we choose the desired rate of return to be $r_k^* = 1.2$ which yields $\gamma_k = 1.2$.

PAR Minimizing Price (PAR). The PAR of load profile $\{L_h\}_{h=1,\ldots,H}$ is defined as the ratio of the maximum load over the operation cycle to the average load profile. The SO can pick the policy parameter $\{\gamma_k\}_{k=1,\ldots,K}$ to minimize the expected PAR of consumption behavior which is formulated as follows

$$\min_{\{\gamma_k\}_{k=1,\ldots,K}} E\Big[\frac{H \max_{h=1,\ldots,H} L_h(\gamma_k)}{\sum_{h=1}^{H} L_h(\gamma_k)}\Big]. \tag{19}$$

In computing its expected PAR, the SO relies on the model of user optimal behavior as defined by the BNE in (10). The closed form solution to the above optimization problem does not exist. For this reason, we use an evolutionary algorithm to determine the minimizing set of policy parameters $\{\gamma_k^*\}_{k=1,\ldots,K}$ within the range $[1, 1.5]$ and compute the expected PAR using Monte Carlo sampling. The optimal policy parameters are at the boundaries of the parameter range $[1, 1.5]$, that is, $\gamma_1^* = 1$, $\gamma_2^* = 1.5$, and

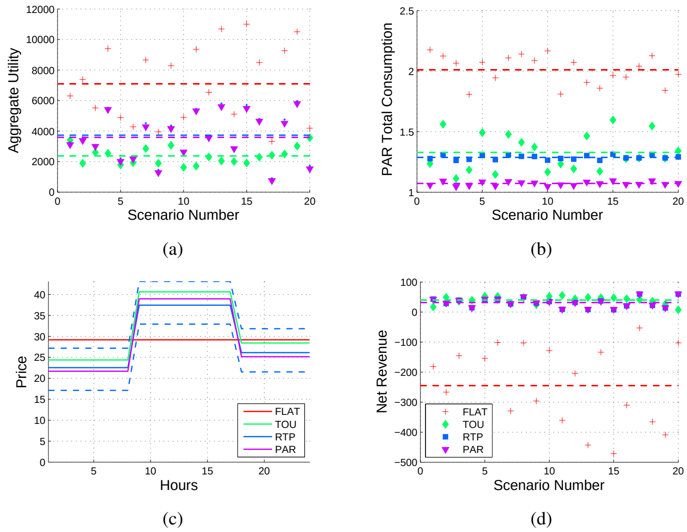


Fig. 5. Comparison of different pricing schemes with respect to Aggregate Utility $\sum_h U_h$ (a), PAR of Total Consumption (b), Price (c) and Net Revenue $\sum_{h\in\mathcal{H}} pL_h - C_k(L_h)$ (d). In (c), mean Price over 20 runs is depicted for each hour with solid lines and dashed lines mark minimum and maximum values over 20 scenarios. In (a), (b) and (d), each point corresponds to the value of the metric for that scenario and dashed lines correspond to the average value of these points over all scenarios. The PAR-minimizing policy performs better than others in minimizing PAR of consumption while at the same time being comparable to the best pricing mechanism in other performance metrics.

$\gamma_3^* = 1$. The PAR minimizing choices of high policy parameter in the peak time zone $k = 2$ and low γ_k during off-peak $k = 1, 3$ supports the intuition developed from Fig. 3(b).

We compare the above pricing schemes with commonly used flat and TOU pricing schemes which we explain below.

Flat Price (FLAT). Customers are charged with a flat price p across the horizon that is determined as the average of realized RTP prices in (2), that is, $p = \sum_{h\in\mathcal{H}} p_k(L_h^*, \omega_k)/H$. Customers respond by optimizing their utility in (3) with price replaced by flat price p, that is, they are price-takers. The user response is obtained by solving the first order conditions $l_{ih}^* = (-p + g_{ik})/2\alpha_k$.

TOU Price. Customers are charged with hourly prices p_h that are determined by maximizing hourly expected net revenue, that is, $p_h = \max_p E[pL_h - C_k(L_h)]$. Customers optimally respond to hourly prices by $l_{ih}^* = (-p_h + g_{ik})/2\alpha_k$.

Figs. 5(a)-(d) compare the aforementioned pricing schemes with respect to their influence on customer utility, load behavior, price and the operator's net revenue, respectively. We observe that flat pricing scheme results in high PAR in consumption Fig. 5(b), negative revenue Fig. 5(d) and high variation in all performance metrics across scenarios. We note that while it is possible to increase the net revenue by raising the flat price, this lowers aggregate utility below the levels observed in RTP and PAR pricing causing customer dissatisfaction. The TOU scheme performs comparable to RTP in terms of net revenue and also has a comparable mean PAR of total consumption. However, in TOU, the aggregate utility is considerably lower than other pricing schemes and there is higher variation in PAR of total consumption when compared with RTP scheme. The RTP and PAR schemes achieve identical net revenues as shown in Fig. 5(d). This is due to the fact that the policy parameter in RTP $\gamma_k^* = 1.2$ is equal to the time weighted average of the policy parameters in PAR pricing. The dashed lines in Fig. 5(c) illustrating the minimum and maximum prices observed in 20 runs of RTP show that the variation in price for different scenarios is low. We note that some of the variation observed in metrics for RTP and PAR are due to the uncertainty introduced by the renewable energy term ω_k in (2). In comparison to RTP, the PAR scheme improves on total consumption PAR. When we compare the total consumption over the whole horizon for the two cases we observe no difference; that is, the average aggregate consumption over 20 runs is equal to 576 kWh for both RTP and PAR. This implies that users are shifting their consumption from peak time zone to off-peak time zones based on the policy parameter in the PAR pricing. Furthermore, the mean of aggregate utility of PAR is close to the mean in RTP as observed from Fig. 5(a). This means customer satisfaction is not significantly hurt by the PAR pricing.

VI. Conclusion

We considered a DR model where customers with unknown and heterogeneous marginal utilities respond to RTP announced by the SO ahead of each time zone in the operation cycle. The pricing mechanism incorporated a renewable energy term that allows the provider to incentivize consumption when there is estimated abundance of renewable source within a time zone. Given the pricing mechanism, we characterized selfish and altruistic customer behavior using the BNE and discuss the effects of changes in preference distribution, and policy parameters on the customer satisfaction, total consumption and net revenue of the provider. Based on the characterized user behavior and pricing strategy, we proposed a consumption PAR-minimizing pricing scheme which can be implemented without any prior communication with the users. Numerical comparisons proved that the proposed PAR minimizing scheme is the most effective in minimizing PAR while performing as good in comparison to other pricing schemes in customer satisfaction and net revenue.

Appendix

Proof: Our plan is to propose a linear strategy as in (10) and use the fixed point equations of BNE in (9) to solve for the linear strategy coefficients. First, we obtain a general form for the best response function that incorporates the best response for both the selfish users with utility (4) and the altruistic users with the utility (6). In order to compute the best response in (7) we take the derivative of conditional expected utility with respect to l_{ih}, equate the resultant to zero and solve for l_{ih},

$$BR(g_{ik}; \mathbf{s}_{-ih}) = \frac{g_{ik} - \gamma_k\left(\bar{\omega}_k + \lambda \sum_{j\neq i} E_{\omega_k}[E_{ih}[s_{jh}]]\right)}{2(\gamma_k + \alpha_k)} \quad (20)$$

where $\lambda = 1$ if the users are selfish, i.e., maximize (4) or $\lambda = 2$ if they are altruistic, i.e., maximize (6).

Next, we use the best response expression in (20) in the BNE definition (9) and substitute the proposed linear strategy in (10) for the corresponding terms to get the following fixed point equation,

$$a_{ih}(\bar{g}_k - \bar{\omega}_k\gamma_k) + b_{ih}(g_{ik} - \bar{g}_k) = \rho_k\left(g_{ik} - \gamma_k(\bar{\omega}_k + \lambda \sum_{j\neq i} E_{ih}[a_{jh} + b_{jh} g_{jk}])\right) \quad (21)$$

for all $i \in \mathcal{N}$ where we use the definition of ρ_k. Furthermore, given g_{ik} and the normal prior on $\mathbf{g}_k$, we have $E_{ih}[g_{jk}] = (1 - \sigma_{ij}^k/\sigma_{ii}^k)\bar{g}_k + (\sigma_{ij}^k/\sigma_{ii}^k)g_{ik}$. Substituting the term for the expectation,

$$a_{ih}(\bar{g}_k - \bar{\omega}_k\gamma_k) + b_{ih}(g_{ik} - \bar{g}_k) = \rho_k g_{ik} - \rho_k\gamma_k\Big(\bar{\omega}_k + \lambda\sum_{j\neq i} a_{jh} + b_{jh}\big((1 - \frac{\sigma_{ij}^k}{\sigma_{ii}^k})\bar{g}_k + \frac{\sigma_{ij}^k}{\sigma_{ii}^k} g_{ik}\big)\Big). \tag{22}$$

Next, we add and subtract $\rho_k\bar{g}_k$ to the right hand side and group the terms that multiply $g_{ik} - \bar{g}_k$ and $\bar{g}_k - \bar{\omega}_k\gamma_k$,

$$\begin{aligned} a_{ih}(\bar{g}_k - \bar{\omega}_k\gamma_k) + b_{ih}(g_{ik} - \bar{g}_k) = \\ \rho_k\big(1 - \gamma_k\lambda\sum_{j\neq i} a_{jh}\big)\big(\bar{g}_k - \bar{\omega}_k\gamma_k\big) \\ + \rho_k\big(1 - \gamma_k\lambda\sum_{j\neq i}\frac{\sigma_{ij}^k}{\sigma_{ii}^k} b_{jh}\big)\big(g_{ik} - \bar{g}_k\big) \end{aligned} \tag{23}$$

Equating the terms that multiply $(\bar{g}_k - \bar{\omega}_k\gamma_k)$ and $(g_{ik} - \bar{g}_k)$ for all $i \in \mathcal{N}$, we get the following set of equations for $\mathbf{a}_h$ and $\mathbf{b}_h$,

$$a_{ih} = \rho_k\big(1 - \gamma_k\lambda\sum_{j\neq i} a_{jh}\big) \tag{24}$$

$$b_{ih} = \rho_k\big(1 - \gamma_k\lambda\sum_{j\neq i}\frac{\sigma_{ij}^k}{\sigma_{ii}^k} b_{jh}\big) \tag{25}$$

for all $i \in \mathcal{N}$. Next, we stack the equations above and write them in vector form

$$\big(\mathbf{I} + \rho_k\gamma_k\lambda(\mathbf{1}\mathbf{1}^T - \mathbf{I})\big)\,\mathbf{a}_h = \rho_k\mathbf{1} \tag{26}$$

$$\left(\mathbf{I} + \rho_k\gamma_k\lambda\mathbf{S}(\mathbf{\Sigma}_k)\right)\mathbf{b}_h = \rho_k\mathbf{1}. \tag{27}$$

where in (27) we used the definition of the inference matrix (13). Action coefficient a_{ih} is obtained from (26) by multiplying both sides by $(\mathbf{I} + \rho_k\gamma_k\lambda(\mathbf{1}\mathbf{1}^T - \mathbf{I}))^{-1}$ and using the following identity

$$(\mathbf{I} + \rho_k\gamma_k\lambda(\mathbf{1}\mathbf{1}^T - \mathbf{I}))^{-1}\mathbf{1} = ((N-1)\rho_k\gamma_k\lambda + 1)^{-1}\mathbf{1}. \tag{28}$$

The action coefficient b_{ih} is obtained from (27) by multiplying both sides of the equation by $(\mathbf{I} + \rho_k\gamma_k\lambda\mathbf{S}(\mathbf{\Sigma}_k))^{-1}$. Hence, we have shown that there exists a BNE strategy that is of linear form as given in Proposition 1 when users are selfish $\lambda = 1$ and as given in Proposition 3 when they are altruistic $\lambda = 2$.

To prove uniqueness, we first show that the games defined by payoffs in (4) and (6) are both Bayesian potential games with Bayesian potential function $v(\{l_{ih}\}_{i\in\mathcal{N}}; \mathbf{g}_k, \omega_k, \lambda)$ and then argue that the potential function $v(\{l_{ih}\}_{i\in\mathcal{N}}; \mathbf{g}_k, \omega_k, \lambda)$ is strictly concave which implies unique solution for the original games with payoffs in (4) or (6). Define the symmetric matrix $\mathbf{Q}_k \in \mathbb{R}^{N\times N}$ where $((\mathbf{Q}_k))_{ii} = 1$ for all $i = 1, \dots, N$ and $((\mathbf{Q}_k))_{ij} = \lambda\rho_k$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus i$. Let $\mathbf{l}_h := \{l_{ih}\}_{i\in\mathcal{N}}$. For the stage game $h \in t_k$ with quadratic payoffs u_i in (4) and information on $\mathbf{g}_k$ and ω_k, there exists a Bayesian potential function

$$v(\mathbf{l}_h; \mathbf{g}_k, \omega_k, \lambda) = -(\gamma_k + \alpha_k)\mathbf{l}_h^T\mathbf{Q}_k\mathbf{l}_h + \mathbf{l}_h^T(\mathbf{g}_k - \gamma_k\omega_k\mathbf{1}^T). \tag{29}$$

Note that when $\lambda = 1$, $\partial v(\mathbf{l}_h)/\partial l_{ih} = \partial u_{ik}(\mathbf{l}_h)/\partial l_{ih}$ and when $\lambda = 2$, $\partial v(\mathbf{l}_h)/\partial l_{ih} = \partial U_k(\mathbf{l}_h)/\partial l_{ih}$ for all $i \in \mathcal{N}$. Hence, both stage games $h \in t_k$ with selfish and altruistic users and information on $\mathbf{g}_k$ and ω_k are Bayesian potential games with potential functions as in (29) by Lemma 6 in [19].

This result implies that the equilibrium of the Bayesian potential game with function $v(\mathbf{l}_h; \mathbf{g}_k, \omega_k, \lambda)$ is the same as the equilibrium of the stage game at time $h \in t_k$ with payoffs u_{ik} in (4) when $\lambda = 1$ and with payoffs U_k in (6) when $\lambda = 2$. It can be shown that $\mathbf{Q}_k$ is positive definite for all $k = 1, \dots, K$ by looking at the eigenvalues of the matrix $\mathbf{Q}_k$. This implies that Bayesian potential function is strictly concave with a unique maximizer. Hence, both games have unique equilibrium. ■

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