Algorithmic Causal Inference

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Equivalence of Algorithmic Markov Conditions

Given strings $x_1, \ldots, x_n$ and directed acyclic graph G, the following conditions are equivalent:

1. **Recursive Form (Markov Factorization):**

$$K(x_1, \ldots, x_n) = \sum_{j=1}^{n} K(x_j | pa_j^*)$$

2. **Local Markov Conditions:**

$$I(x_j : nd_j | pa_j^*) = 0$$

3. **Global Markov Conditions:**

$$I(S : T | R) = 0$$

if R d-separates S and T.

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Note!

the Turing machine simulating the process would not necessarily halt on all inputs pa_j, n_j.

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Let $x_1, \ldots, x_n$ be generated by the algorithmic model. Then they satisfy the algorithmic Markov condition with respect to G.

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Note!

In the statistical version of the link between causality and dependence, the relevance of the background information is less obvious because it is evident that statistical methods are always applied to a given statistical ensemble.

From Algorithmic to Statistical

- Consider random variables X and Y.
- The machine S, generates samples of X according to $P(X)$.
- M that generates y-values according to $P(Y|X)$.
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Postulate

A causal hypothesis G (i.e., a DAG) is only acceptable if the shortest description of the joint density P is given by a concatenation of the shortest description of the Markov kernels, i.e.

$$K(P(X_1, \ldots, X_n)) = \sum_j K(P(X_j|PA_j))$$

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  - In case (I), statistical independence is rejected with high confidence.
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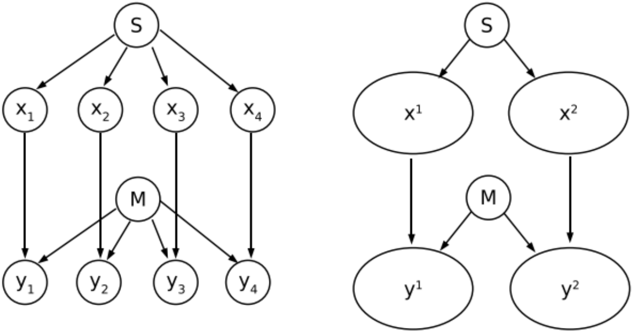
- Algorithmic causal inference, on the other hand, infers a causal link in both cases because the equality $\mathbf{x} = \mathbf{y}$ requires an explanation.

Relation between two scenarios

- Switching between (I) and (II) then consists merely in shifting the causal connection to another level:
  - In the i.i.d setting, every x_j must be causally linked to y_j.
  - In case (II), there must be a connection between the two mechanisms that for instance, be due to the fact that two machines emitting the same string were designed by the same engineer.



$$I(\mathbf{x}^1 : \mathbf{y}^2 | \mathbf{x}^2) = 0 \qquad I(\mathbf{x}^2 : \mathbf{y}^1 | \mathbf{x}^1) = 0$$

The most remarkable property of this, is that they are asymmetric with respect to exchanging the roles of X and Y