Fall, 2013 CIT 592

Mathematical Foundations of Computer Science

Midterm1

Please write your name and email on this first sheet.

Please make at least one of them legible.

Your answer needs to fit into the space provided. You are also allowed to use the back of the page to continue your solution.

Questions.

1. The standard olympic 100 meters race has 8 finalists. In how many possible ways can the gold, silver and bronze medals be handed out(3 pts)

2. Suppose that a buffet restaurant allows you to have exactly 8 items for a family meal. There are 5 different appetizers, 6 different entrees, 3 different desserts. In how many ways can you order a family meal that has at least 4 entrees. (4 pts)

3. For the following 2 relations, tell us whether they are reflexive, symmetric, antisymmetric and transitive. Also say whether or not they are equivalence relations and partial orders (4 pts)

   (a) R_1 defined on the set of integers, where a and b are related if a - b is negative

   (b) 2 students are related if they take any class in common

4. Prove or disprove the following $\forall x \in \mathbb{N}(x^2 \leq 2^x)$ (2 pts)

5. How many pairs of 3 digit numbers exist such that when one is added to the other, we never have to use a 'carry-over'. To clarify the concept of carry over, consider that when you add 75 + 68 you have to do a carry over because 5 + 8 is 13. Note that you need to do another one because 7 + 6 + 1 is 14(6 pts)

6. Define $\downarrow$ to be the NOR operator, meaning $p \downarrow q = \neg(p \vee q)$. Now using just the $\downarrow$ operator, show how to write $p \wedge q$. Please provide some steps to your answer. (2 pts)

7. For all integers a, b, c if we have $a^2 + b^2 = c^2$, then either a or b is even. Prove this statement(6 pts)

8. Determine if function f from natural numbers to natural numbers where $f(x) = x^2$ is one to one. If you think it is false provide a counterexample. If you think it is true prove it.(4 pts)

9. What is the constant term in $(2x + \frac{3}{x})^8$ (2 pts)

10. Use quantifiers, logical connectives, set notation to represent the following statement:

    R is a total order relation on a set X if and only if a) R is a partial order and b) for every pair a and b in X, either a is related to b or b is related to a.

    You do not need to use quantifiers for the partial order part - write that part as $R \in P_X$, where we just understand P_X to be the set of all partial orders on X. You also can use $R \in T_X$, to mean 'R is a total order relation.'

    In other words your answer begins with '$R \in T_X \ldots$' (2pts)