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Relativistic correction of $(v/c)^2$ to the collective Thomson scattering for high-temperature high-density plasma*

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Collective Thomson scattering is theoretically investigated with the inclusion of the relativistic correction of $(v/c)^2$. The correction is rather small for the plasma parameters inferred from the spectra of the thermal electron plasma waves in the plasma. Since the full formula of the corrected result is rather complicated, a simplified one is derived for practical use, which is shown to be in good agreement with the un-simplified one.

Keywords: Thomson scattering, relativistic effect

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1. Introduction

Thomson scattering,[1,2] which can provide highly reliable time- and space-resolved measurements of plasma parameters, including electron density, temperature and plasma flow, is now widely utilized to measure the laser-produced plasma relevant to inertial confinement fusion (ICF).[3–11] Thomson scattering is usually operated in the collective regime in the field of ICF. In most of those experiments, only the ion-acoustic features of the Thomson scattering spectra are detected and the following equation is usually adopted to infer plasma parameters from experimental data,[1,2]

$$\frac{\mathrm{d}^2 P}{\mathrm{d}\omega_\mathrm{s}\,\mathrm{d}\Omega} = r_\mathrm{e}^2 I_0 V_\mathrm{s} n_\mathrm{e} (\boldsymbol{e}_0 \times \boldsymbol{n}_\mathrm{s})^2 S(k,\omega), \tag{1}$$

where P is the power of the scattered light, r_e is the classical electron radius, I_0 is the intensity of the probe light, V_s is the scattering volume, n_e is the electron density, $\boldsymbol{e}_0$ is the polarization of the probe light, $\boldsymbol{n}_\mathrm{s}$ is the scattering direction and $S(k,\omega)$ is the so-called dynamic form factor. The ω and $\boldsymbol{k}$ are the differential frequency and the wave vector, respectively, given by

$$\omega = \omega_\mathrm{s} - \omega_0, \tag{2a}$$

$$\boldsymbol{k} = \boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_0, \tag{2b}$$

where $\omega_{0,\mathrm{s}}$ is the frequency of the probe/scattering light and $\boldsymbol{k}_{0,\mathrm{s}}$ is the wave vector of the probe/scattering light. The wave number of the electromagnetic wave depends on the plasma density, $k_{0,\mathrm{s}} = (\omega_{0,\mathrm{s}}/c)(1-\omega_\mathrm{pe}^2/\omega_{0,\mathrm{s}}^2)^{1/2}$where ω_pe is the Langmuir frequency of the plasma. When the plasma density is dilute and the differential frequency is small, the differential wave vector is usually approximated by[1,2]

$$\boldsymbol{k} = \frac{\omega_0}{c}\boldsymbol{n}_\mathrm{s} - \boldsymbol{k}_0. \tag{3}$$

In previous experiments, it is Eq. (3) that is used to calculate the differential wave vector instead of the more accurate Eq. (2b). For collisionless non-relativistic plasma in the quasi-equilibrium state, the dynamic form factor is given by

$$S(k,\omega) = \frac{1}{n_\mathrm{e}}\left(\left|1-\frac{\chi_\mathrm{e0}}{\epsilon_\mathrm{l0}}\right|^2 F_\mathrm{e0} + Z^2\left|\frac{\chi_\mathrm{e0}}{\epsilon_\mathrm{l0}}\right|^2 F_\mathrm{i0}\right), \tag{4}$$

where ϵ_l0 is the non-relativistic longitudinal permittivity of the plasma, χ_e0 is the non-relativistic electron susceptibility, Z is the charge state of the ions and $F_\mathrm{e0,i0}$ are given by

$$\begin{aligned} F_\mathrm{e0,i0} &= \int f_\mathrm{e0,i0}(\boldsymbol{p})\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\,\mathrm{d}^3p. \\ &= \frac{n_\mathrm{e,i}}{\sqrt{2\pi}kv_\mathrm{e,i}}\exp\left(-\frac{\omega^2}{2k^2v_\mathrm{e,i}^2}\right), \end{aligned} \tag{5}$$

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with $f_{\rm e0,i0}$ being the Maxwellian momentum distribution function of the electrons or the ions, $\delta(x)$ the δ function, $v_{\rm e,i} = \sqrt{T_{\rm e,i}/m_{\rm e,i}}$ the electron/ion thermal speed, $T_{\rm e,i}$ the electron/ion temperature and $m_{\rm e,i}$ the electron/ion mass.

The theories of Thomson scattering are thus developed to include various effects that may have important effects on the dynamic form factor $S(k,\omega)$, such as the electron–positron plasma,[12] the super-Gaussian electron velocity distribution due to the strong inverse bremsstrahlung absorption,[13−14] the frequent Coulomb collisions[15,16] and the plasma inhomogeneity.[17,18] The scattering spectra from the thermal electron plasma waves, which allow the direct measurement of the electron density and the temperature, were also successfully detected by Glenzer *et al.*[6] Noticing that the plasma temperature was rather high in that experiment and that the differential frequency ω was no longer negligible in comparison with the probe frequency ω_0, we revisited the first order correction of v/c (where v and c are the electron and the light speed, respectively) to the scattering power spectrum[19] and obtained the same result as that presented by Sheffield,[2]

$$\frac{\mathrm{d}^2P}{\mathrm{d}\omega_{\rm s}\mathrm{d}\Omega} = r_{\rm e}^2 I_0 V_{\rm s} n_{\rm e}(\boldsymbol{e}_0\times\boldsymbol{n}_{\rm s})^2 S(k,\omega)\left(1+\frac{2\omega}{\omega_0}\right). \quad (6)$$

The $S(k,\omega)$ in Eq. (6) is the same as the one in Eq. (1) and the new term $2\omega/\omega_0$ comes from the first order correction of v/c. Another correction to Eq. (1) is implied in the exact differential wave vector,

$$\boldsymbol{k} = \frac{\omega_{\rm s}}{c}\boldsymbol{n}_{\rm s} - \boldsymbol{k}_0. \quad (7)$$

Equations (3) and (7) are different from each other by a term proportional to ω/ω_0. With the inclusion of the first order of v/c, we find that not only the intensity but also the spectral profile of the scattering light of the electron plasma waves changes significantly.[19] With Eq. (6), the inferred electron temperature from the scattering spectra in Ref. [6] is about 30% higher than that obtained with Eq. (1).[19] With the inclusion of the dependence of wave number on the plasma density, the differential wave vector becomes a little smaller, leading to a smaller wavelength of the resonance peak of the scattering spectrum.[19]

As indicated in Eq. (6), the correction term of $2\omega/\omega_0$ can become of order 1 when the light scattering from the electron plasma waves is detected. This fact means that the first order correction may not be enough for the fitting of the Thomson scattering spectra obtained from the electron plasma waves in high-temperature high-density plasma. However, the electron temperature in the coronal region of the laser-produced plasma may reach 5 keV and the plasma becomes mildly relativistic in the following ignition experiment.[20] Therefore, a theory of collective Thomson scattering including the relativistic correction, i.e., $(v/c)^2$, may be necessary for such high-temperature plasma. In fact, relativistic corrections to Thomson scattering were addressed many years ago.[21−26] However, these previous studies concentrated on the relativistic effect for incoherent Thomson scattering. In this paper, we calculate the power spectrum of collective Thomson scattering with the correction up to $(v/c)^2$. When the ratio of the electron temperature to the electron rest energy is over a few percent or when the phase velocity of the detected fluctuation is larger than $0.2c$, the relativistic effect should be included in the theory of Thomson scattering. The electron temperature obtained with the corrected theory is a little higher than that obtained with the theory only including terms of v/c order. Since the full formula for the corrected spectrum is rather lengthy and complicated, a simplified one is derived for routine applications. The results may make collective Thomson scattering a more accurate method for diagnosing plasma parameters in laser-produced high-temperature plasma.

2. Basic equations

In the wave zone, the spectral energy emitted from N accelerated electrons into the solid angle $\mathrm{d}\Omega$ is approximately given by[19]

$$\frac{\mathrm{d}^2\mathcal{E}_{\boldsymbol{n}_{\rm s},\omega_{\rm s}}}{\mathrm{d}\Omega\mathrm{d}\omega_{\rm s}} = \frac{e^2}{8\pi^2c^3}\bigg|\sum_{i=1}^{N}\int_{-\infty}^{\infty}\frac{\mathrm{d}}{\mathrm{d}\tau}\left[\frac{\boldsymbol{v}_i(\tau)\times\boldsymbol{n}_{\rm s}}{1-\boldsymbol{n}_{\rm s}\cdot\boldsymbol{v}_i(\tau)/c}\right] \times \mathrm{e}^{\mathrm{i}\omega_{\rm s}\tau-\mathrm{i}(\omega_{\rm s}/c)\boldsymbol{n}_{\rm s}\cdot\boldsymbol{r}_i(\tau)}\mathrm{d}\tau\bigg|^2, \quad (8)$$

where $-e$ is the electron charge, $\boldsymbol{v}_i$ and $\boldsymbol{r}_i$ are the velocity and the coordinate of the i-th electron, respectively, $\boldsymbol{R}$ is the observation coordinate, τ is the retardation time defined as $t = \tau + R/c - \boldsymbol{n}_{\rm s}\cdot\boldsymbol{r}_i(\tau)/c$ and $\omega_{\rm s}$ denotes the frequency of the emitted electromagnetic wave.

The incident wave is assumed to be plane, monochromatic and linearly polarized,

$$\boldsymbol{E}_0 = \boldsymbol{e}_0 E_0 \cos(\boldsymbol{k}_0\cdot r - \omega_0 t), \quad (9)$$

$$\boldsymbol{B}_0 = (\boldsymbol{n}_0\times\boldsymbol{e}_0)E_0\cos(\boldsymbol{k}_0\cdot r - \omega_0 t), \tag{10}$$

where ω_0 and $\boldsymbol{k}_0$ are the frequency and the wave vector of the incident wave, respectively, and $\boldsymbol{n}_0$ is the propagation direction of the incident wave. Here we do not discuss the effect of the plasma polarization on the electromagnetic fields, which has been justified in our previous article.[19] Under the action of the incident wave, the acceleration of an electron is given by

$$\begin{aligned}\dot{\boldsymbol{v}}(\tau) = & -\frac{eE_0}{m_\mathrm{e}}\Bigg[\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)\boldsymbol{e}_0 + \frac{1}{c}\boldsymbol{v}\times(\boldsymbol{n}_0\times\boldsymbol{e}_0) \\ & - \frac{1}{c^2}(\boldsymbol{v}\cdot\boldsymbol{e}_0)\boldsymbol{v}\Bigg]\cos[\boldsymbol{k}_0\cdot\boldsymbol{r}(\tau)-\omega_0\tau] \\ & + O[(v/c)^3].\end{aligned} \tag{11}$$

Here we neglect those terms of orders higher than $(v/c)^2$. For simplicity, we assume that $\boldsymbol{e}_0$ is perpendicular to scattering direction $\boldsymbol{n}_\mathrm{s}$ in the following calculations, i.e., $\boldsymbol{e}_0\cdot\boldsymbol{n}_\mathrm{s}=0$. This condition can easily be achieved by a suitable experimental setup.

Substituting Eq. (11) into Eq. (8) and performing the ensemble average, we can obtain the spectral power of the Thomson scattering via the procedure presented in our previous article,[19] the only difference being that the Klimontovich distribution function in this paper is defined as

$$F_\mathrm{e}(\boldsymbol{r},\boldsymbol{p},\tau) = \sum_{i=1}^{N}\delta[\boldsymbol{r}-\boldsymbol{r}_i(\tau)]\delta[\boldsymbol{p}-\boldsymbol{p}_i(\tau)] \tag{12}$$

instead of $F_\mathrm{e}(\boldsymbol{r},\boldsymbol{v},\tau)$ in Ref. [19]. Here $\boldsymbol{p}$ denotes the momentum. The spectral power is hence given by

$$\begin{aligned}&\frac{\mathrm{d}^2P_{\boldsymbol{n}_\mathrm{s},\omega_\mathrm{s}}}{\mathrm{d}\Omega\mathrm{d}\omega_\mathrm{s}} \\ &= \frac{1}{2\pi n_\mathrm{e}}I_0r_\mathrm{e}^2V_\mathrm{s}n_\mathrm{e}\int\mathrm{d}^3p\mathrm{d}^3p'\left(1+\frac{1}{c}\Xi_1+\frac{1}{c^2}\Xi_2\right) \\ &\quad\times(\delta f_\mathrm{e}^2)^\mathrm{rel}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}.\end{aligned} \tag{13}$$

Here Ξ_1 and Ξ_2 are the first and the second order corrections, respectively, given by

$$\Xi_1 = (\boldsymbol{n}_\mathrm{s}-\boldsymbol{n}_0)\cdot(\boldsymbol{v}+\boldsymbol{v}'), \tag{14}$$

$$\Xi_2 = \mathbb{A}\colon(\boldsymbol{v}\boldsymbol{v}+\boldsymbol{v}'\boldsymbol{v}') + \mathbb{B}\colon\boldsymbol{v}\boldsymbol{v}', \tag{15}$$

where velocities $\boldsymbol{v}$ and $\boldsymbol{v}'$ are functions of $\boldsymbol{p}$ and $\boldsymbol{p}'$, respectively, and $\mathbb{A}$ and $\mathbb{B}$ are two tensors defined as

$$\mathbb{A} = -\frac{1}{2}\mathbb{I} + \boldsymbol{n}_\mathrm{s}(\boldsymbol{n}_\mathrm{s}-\boldsymbol{n}_0) - [1-(\boldsymbol{n}_0\cdot\boldsymbol{n}_\mathrm{s})]\boldsymbol{e}_0\boldsymbol{e}_0, \tag{16}$$

$$\mathbb{B} = (\boldsymbol{n}_\mathrm{s}-\boldsymbol{n}_0)(\boldsymbol{n}_\mathrm{s}-\boldsymbol{n}_0) + \left[1-(\boldsymbol{n}_0\cdot\boldsymbol{n}_\mathrm{s})^2\right]\boldsymbol{e}_0\boldsymbol{e}_0, \tag{17}$$

with $\mathbb{I}$ being the unit tensor of rank two. The $(\delta f_\mathrm{e}^2)^\mathrm{rel}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}$ in the integrand of Eq. (13) is the spectral density of the correlation function of the Klimontovich electron distribution functions, which has already been obtained.[27] It should be pointed out that with the inclusion of the relativistic effects, $(\delta f_\mathrm{e}^2)^\mathrm{rel}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}$ is different from the non-relativistic one $(\delta f_\mathrm{e}^2)^\mathrm{non}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}$ presented in Ref. [28], because both fluctuating electric and magnetic fields have an effect on the particle fluctuation.

Introducing the notations,

$$\frac{1}{2\pi n_\mathrm{e}}\int(\delta f_\mathrm{e}^2)^\mathrm{rel}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}\mathrm{d}^3p\mathrm{d}^3p' = A(k,\omega), \tag{18a}$$

$$\frac{1}{2\pi n_\mathrm{e}}\int\boldsymbol{v}(\delta f_\mathrm{e}^2)^\mathrm{rel}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}\mathrm{d}^3p\mathrm{d}^3p' = B(k,\omega)\frac{\boldsymbol{k}}{k}, \tag{18b}$$

$$\begin{aligned}&\frac{1}{2\pi n_\mathrm{e}}\int(\boldsymbol{v}\boldsymbol{v}+\boldsymbol{v}'\boldsymbol{v}')\,(\delta f_\mathrm{e}^2)^\mathrm{non}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}\mathrm{d}^3p\mathrm{d}^3p' \\ &= C_1(k,\omega)\frac{\boldsymbol{k}\boldsymbol{k}}{k^2} + D_1(k,\omega)\left(\mathbb{I}-\frac{\boldsymbol{k}\boldsymbol{k}}{k^2}\right),\end{aligned} \tag{18c}$$

$$\begin{aligned}&\frac{1}{2\pi n_\mathrm{e}}\int\boldsymbol{v}\boldsymbol{v}'(\delta f_\mathrm{e}^2)^\mathrm{non}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}\mathrm{d}^3p\mathrm{d}^3p' \\ &= C_2(k,\omega)\frac{\boldsymbol{k}\boldsymbol{k}}{k^2} + D_2(k,\omega)\left(\mathbb{I}-\frac{\boldsymbol{k}\boldsymbol{k}}{k^2}\right),\end{aligned} \tag{18d}$$

Eq. (13) can now be written as

$$\frac{\mathrm{d}^2P_{\boldsymbol{n}_\mathrm{s},\omega_\mathrm{s}}}{\mathrm{d}\Omega\mathrm{d}\omega_\mathrm{s}} = I_0r_\mathrm{e}^2N_\mathrm{s}\,(\sigma_1+\sigma_2+\sigma_3+\sigma_4)\,, \tag{19}$$

where

$$\sigma_1 = A(k,\omega) \tag{20a}$$

$$\sigma_2 = \frac{2}{c}(\boldsymbol{n}_\mathrm{s}-\boldsymbol{n}_0)\cdot\frac{\boldsymbol{k}}{k}B(k,\omega), \tag{20b}$$

$$\begin{aligned}\sigma_3 = \frac{1}{c^2}\Bigg\{&\frac{(\boldsymbol{k}\cdot\boldsymbol{n}_\mathrm{s})\,[\boldsymbol{k}\cdot(\boldsymbol{n}_\mathrm{s}-\boldsymbol{n}_0)]}{k^2}(C_1-D_1) \\ &+ [1-(\boldsymbol{n}_\mathrm{s}\cdot\boldsymbol{n}_0)]D_1 - \frac{C_1+2D_1}{2}\Bigg\},\end{aligned} \tag{20c}$$

$$\begin{aligned}\sigma_4 = \frac{1}{c^2}\Bigg\{&\frac{[\boldsymbol{k}\cdot(\boldsymbol{n}_\mathrm{s}-\boldsymbol{n}_0)]^2}{k^2}(C_2-D_2) \\ &+ (1-\boldsymbol{n}_\mathrm{s}\cdot\boldsymbol{n}_0)\,(3+\boldsymbol{n}_\mathrm{s}\cdot\boldsymbol{n}_0)\,D_2\Bigg\}.\end{aligned} \tag{20d}$$

It should be pointed out that Eqs. (18a)–(18d) are valid only if the fluctuation in the plasma is statistically isotropic.

3. Relativistic corrections

The integral $A(k,\omega)$ in Eq. (18a) is actually the auto-correlation function of the electron density, which essentially has the same form as that for the non-relativistic plasma,

$$A(k,\omega) \equiv S^{\rm rel}(k,\omega) = \frac{1}{n_{\rm e}}\left(\left|1-\frac{\chi_{\rm e}}{\epsilon_{\rm l}}\right|^2 F_{\rm e} + Z^2\left|\frac{\chi_{\rm e}}{\epsilon_{\rm l}}\right|^2 F_{\rm i}\right), \quad (21)$$

where $\epsilon_{\rm l}$ is the relativistic longitudinal permittivity of the plasma, $\chi_{\rm e}$ is the relativistic electron susceptibility and $F_{\rm e,i}$ are given by

$$F_{\rm e,i} = \int f_{\rm e,i}(\boldsymbol{p})\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\,{\rm d}^3 p, \quad (22)$$

with $f_{\rm e,i}$ being the momentum distribution function of the electrons or the ions. The relativistic dynamic forming factor Eq. (21) is different from the non-relativistic one given in Eq. (4), as the relativistic momentum distribution functions are used instead of the non-relativistic ones.

For relativistic plasma, the momentum distribution function of the electrons is given by the Maxwell–Jüttner function,

$$f_{\rm e}(\boldsymbol{p}) = \frac{n_{\rm e}\alpha^{-1}}{4\pi m_{\rm e}^3 c^3 {\rm K}_2(\alpha^{-1})} \times \exp\left[-\alpha^{-1}(1+\boldsymbol{p}^2/m_{\rm e}^2c^2)^{1/2}\right], \quad (23)$$

where $\alpha = T_{\rm e}/m_{\rm e}c^2$ and ${\rm K}_2(x)$ is the modified Bessel function. With the relativistic distribution function given in Eq. (23), the function $F_{\rm e}$ can be written as

$$F_{\rm e} = \frac{n_{\rm e}}{kv_{\rm e}}\frac{\alpha^{1/2}}{2{\rm K}_2(\alpha^{-1})}\left(\frac{1}{1-2\alpha\xi^2} + \frac{2\alpha}{\sqrt{1-2\alpha\xi^2}} + 2\alpha^2\right) \times \exp\left(-\frac{\alpha^{-1}}{\sqrt{1-2\alpha\xi^2}}\right), \quad (24)$$

where $\xi = \omega/\sqrt{2}kv_{\rm e}$. The function $F_{\rm e}$ becomes zero when $2\alpha\xi^2 \geqslant 1$, as no particle can move faster than light. In Fig. 1, we compare function $F_{\rm e}$ with the non-relativistic function $F_{\rm e0}$ given in Eq. (5) in the case of $\alpha = 0.01$. As seen in Fig. 1, the two functions are different from each other only when ω/k becomes closer to the speed of light. The two curves are not separable when $\xi < 2$.

The real part of the full relativistic electron susceptibility $\chi_{\rm e}$ is very complicated and no analytical form is available at present. Detailed discussion on this function can be found in recent articles on the linear wave dispersion relation of the relativistic plasma.[29,30] For laser-produced plasma, parameter α is usually less than 0.01. Hence the relativistic correction to the real part of $\chi_{\rm e}$ is small. The real part of $\chi_{\rm e}$ can be approximately written as (see Appendix A)

$$\operatorname{Re}\chi_{\rm e} = \operatorname{Re}\chi_{\rm e0} + \frac{\alpha}{k^2\lambda_{\rm D}^2}\left[\left(\frac{5}{4}-\frac{3}{2}\xi^2\right)\xi^2 + \left(\frac{1}{8}+2\xi^2-\frac{3}{2}\xi^4\right)\xi\operatorname{Re}Z(\xi)\right], \quad (25)$$

where $\lambda_{\rm D}$ is the Debye length of the plasma, $\operatorname{Re}\chi_{\rm e0} = (k\lambda_{\rm D})^{-2}[1+\xi\operatorname{Re}(\xi)]$ is the real part of the non-relativistic electron susceptibility and $\operatorname{Re}Z(x)$ is the real part of the plasma dispersion function. When $\xi \gg 1$, the asymptotic expansion of $\operatorname{Re}\chi_{\rm e}$ is given by

$$\operatorname{Re}\chi_{\rm e} = -\frac{1}{(k\lambda_{\rm D})^2}\left[\left(1-\frac{5}{2}\alpha\right)\frac{1}{2\xi^2} + (1-8\alpha)\frac{3}{4\xi^4}\right] + \cdots. \quad (26)$$

With Eq. (26), we can obtain the relativistic correction to the dispersion relation of the electron plasma wave as

$$\omega^2 = \omega_{\rm p}^2\left(1-\frac{5}{2}\alpha\right) + 3k^2v_{\rm e}^2, \quad (27)$$

where $\omega_{\rm p} = (4\pi n_{\rm e}e^2/m_{\rm e})^{1/2}$ is the Langmiur frequency. Equation (27) is the same as that obtained by Buti.[31] As indicated by Bergman and Eliasson,[29] Eq. (26) is accurate enough when $\alpha \lesssim 0.01$.

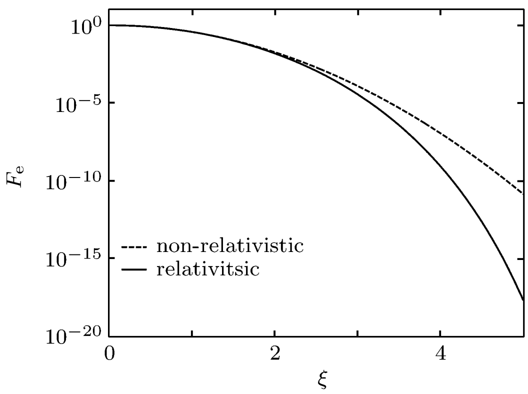

Fig. 1. Comparison between $F_{\rm e}$ and $F_{\rm e0}$ in the case with parameter $\alpha = 0.01$, i.e., $T_{\rm e} = 5.11$ keV.

The imaginary part of $\chi_{\rm e}$ can be analytically calculated and is given by

$$\operatorname{Im}\chi_{\rm e} = \frac{1}{k^2\lambda_{\rm D}^2}\frac{\sqrt{2}\alpha}{2{\rm K}_2(\alpha^{-1})}\xi \times \left(\frac{1}{1-2\alpha\xi^2} + \frac{2\alpha}{\sqrt{1-2\alpha\xi^2}} + 2\alpha^2\right) \times \exp\left(-\frac{\alpha^{-1}}{\sqrt{1-2\alpha\xi^2}}\right). \quad (28)$$

When $2\alpha\xi^2 \geqslant 1$, i.e., $\omega/k \geqslant c$, $\mathrm{Im}\,\chi_{\rm e} = 0$. In this case, the Landau damping of the electron plasma wave disappears, as no particle can catch up with a wave with a phase velocity faster than light.

Since the relativistic corrections are proportional to $(v/c)^2$ in the case of $(v/c)^2 \ll 1$, the integrals of $C_{1,2}(k,\omega)$ and $D_{1,2}(k,\omega)$, which appear in the terms proportional to $1/c^2$, can be calculated with the non-relativistic spectral density $(\delta f_{\rm e}^2)^{\rm non}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}$. The calculations are straight forward and similar to those presented in the Appendix of Ref. [19]. Since the first correction term is of order 1, the integral $B(k,\omega)$ in the first correction term should be calculated with the relativistic spectral density $(\delta f_{\rm e}^2)^{\rm rel}_{\boldsymbol{k},\omega,\boldsymbol{p},\boldsymbol{p}'}$. Its calculation is also similar to that presented in Ref. [19]. These integrals are given by

$$B = \frac{\omega}{k} S^{\rm rel}(k,\omega), \tag{29}$$

$$C_1 = 2\frac{\omega^2}{k^2} S(k,\omega) + \frac{\omega_{\rm p}^2}{k^2}\frac{1}{n_{\rm e}}\left[\frac{\chi_{\rm e0}^* + \chi_{\rm e0}}{|\epsilon_{l0}|^2}\sum_{\alpha={\rm e,i}} (e_\alpha/e) F_{\alpha 0} - \left(\frac{1}{\epsilon_{l0}} + \frac{1}{\epsilon_{l0}^*}\right) F_{\rm e0}\right], \tag{30}$$

$$D_1 = 2\frac{T_{\rm e}}{m_{\rm e}} S(k,\omega), \tag{31}$$

$$C_2 = \frac{\omega^2}{k^2} S(k,\omega), \tag{32}$$

$$D_2 = \frac{T_{\rm e}}{m_{\rm e}} F_{\rm e0}, \tag{33}$$

where e_α is the charge of the α-like particle and $F_{\alpha 0}$ are the non-relativistic limits of functions F_α defined by Eq. (22).

In Eq. (20b), the unit vector $\boldsymbol{k}/k$ should be approximated by

$$\frac{\boldsymbol{k}}{k} = \frac{1}{\sqrt{2(1-\boldsymbol{n}_0\cdot\boldsymbol{n}_{\rm s})}}\left[(\boldsymbol{n}_0 - \boldsymbol{n}_{\rm s}) + \frac{\omega}{2\omega_0}(\boldsymbol{n}_0 + \boldsymbol{n}_{\rm s})\right]. \tag{34}$$

Then the second term in Eq. (19) can be approximately written as

$$\frac{2}{c} B(k,\omega)(\boldsymbol{n}_{\rm s} - \boldsymbol{n}_0)\cdot\frac{\boldsymbol{k}}{k} = \left(\frac{2\omega}{\omega_0} - \frac{\omega^2}{\omega_0^2}\right) S^{\rm rel}(k,\omega).$$

In Eqs. (20c) and (20d), the unit vector $\boldsymbol{k}/k$ can be approximated by

$$\frac{\boldsymbol{k}}{k} = \frac{(\boldsymbol{n}_0 - \boldsymbol{n}_{\rm s})}{\sqrt{2(1-\boldsymbol{n}_0\cdot\boldsymbol{n}_{\rm s})}}. \tag{35}$$

Thus we have

$$\frac{(\boldsymbol{k}\cdot\boldsymbol{n}_{\rm s})\,[\boldsymbol{k}\cdot(\boldsymbol{n}_{\rm s}-\boldsymbol{n}_0)]}{k^2} = 1 - \boldsymbol{n}_{\rm s}\cdot\boldsymbol{n}_0,$$

$$\frac{[\boldsymbol{k}\cdot(\boldsymbol{n}_{\rm s}-\boldsymbol{n}_0)]^2}{k^2} = 2(1 - \boldsymbol{n}_{\rm s}\cdot\boldsymbol{n}_0).$$

Finally, we obtain the spectral power of the Thomson scattering with the corrections of $(v/c)^2$,

$$\frac{{\rm d}^2 P_{\boldsymbol{n}_{\rm s},\omega_{\rm s}}}{{\rm d}\varOmega\,{\rm d}\omega_{\rm s}} = r_{\rm e}^2 I_0 V_{\rm s} n_{\rm e}\left\{S^{\rm rel} + \frac{2\omega}{\omega_0}S^{\rm rel} - \frac{\omega^2}{\omega_0^2}S^{\rm rel} + \frac{1}{c^2}\left[2(C_1 + 2C_2 - D_1)\sin^2\frac{\theta_{\rm s}}{2} - \frac{C_1 + 2D_1}{2} + D_2\sin^2\theta_{\rm s}\right]\right\}, \tag{36}$$

where $\theta_{\rm s}$ is the scattering angle given by $\cos\theta_{\rm s} = \boldsymbol{n}_{\rm s}\cdot\boldsymbol{n}_0$.

In Fig. 2(a), we plot the three spectral powers respectively given by Eqs. (1), (6), and (36). The parameters that we take in the calculations are the same as those used by Glenzer *et al*,[6] i.e., $T_{\rm e} = 2$ keV, $n_{\rm e} = 2.1\times10^{20}$ cm^{-3}, $\lambda_0 = 0.5266$ μm, and $\theta_{\rm s} = 104°$. Equation (7) is used in the computations of Eqs. (6) and (36), while Eq. (2b) is used in the computation of Eq. (1). It is shown in Fig. 2(a) that the maximum intensity given by the corrected formula Eq. (36) is higher than that given by Eq. (6), and that the peak position predicted by Eq. (36) is slightly blue shifted in comparison with that predicted by Eq. (6). The physical reason is that both the damping rate and the frequency of the electron plasma waves become a little smaller when the relativistic effect is included.[29,30] As seen in Eq. (27), when the relativistic effect is taken into account, the frequency of the electron plasma wave is a little downshifted through the factor $(1-5\alpha/2)$, making the peak of the electron plasma wave blueshift as shown in Fig. 2. The damping of the electron plasma wave also becomes lighter because of the relativistic effect. As seen in Fig. 1, the imaginary part of electron susceptibility $\mathrm{Im}\,\chi_{\rm e}$, which is proportional to $F_{\rm e}$, is smaller with the inclusion of the relativistic effect, leading to slightly higher peak of the electron plasma wave.

The profile of Eq. (36) is rather close to that of Eq. (6). This result means that the relativistic corrections are not significant for this example. In order to show the relativistic effect more clearly, we plot in Fig. 2(b) the profiles of the three equations in the case of $T_{\rm e} = 3$ keV, $n_{\rm e} = 4\times10^{20}$ cm^{-3}, $\lambda_0 = 0.5266$ μm and $\theta_{\rm s} = 104°$. In Fig. 2(b), we can clearly see the difference between the first and the second order theories. The relativistic effect could become more notable when the electron temperature becomes even higher, say 5 keV, that may be achievable at the National Ignition Facility or when the phase velocity of the measured electron plasma wave is closer to the speed of light.

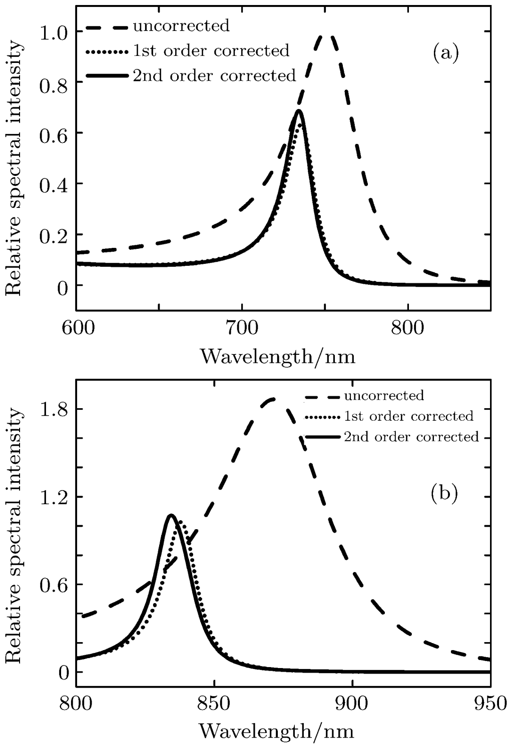

Fig. 2. Profiles of the scattering spectral power. The parameters used in calculations are (a) $T_{\rm e} = 2$ keV, $n_{\rm e} = 2.1 \times 10^{20}$ cm^{-3} , $\lambda_0 = 0.5266$ μm, and $\theta_{\rm s} = 104°$, and (b) $T_{\rm e} = 3$ keV, $n_{\rm e} = 4 \times 10^{20}$ cm^{-3} , $\lambda_0 = 0.5266$ μm, and $\theta_{\rm s} = 104°$.

We pointed out in our previous article[19] that in comparison with the fitting results obtained with the uncorrected theory, the inferred electron temperature could be significantly underestimated and the inferred electron density is nearly unchanged when the corrections of v/c to Thomson scattering are included. We now investigate the influence of the relativistic correction on the inference of the plasma parameters. We still use the uncorrected theory to generate an artificial scattering spectrum and fit it with Eqs. (6) and (36), respectively. One of the comparisons is shown in Fig. 3(a), where the plasma parameters for the dashed line are $T_{\rm e} = 2$ keV and $n_{\rm e} = 2.1 \times 10^{20}$ cm^{-3}, while those for the first order fitting curve (dotted line) are $T_{\rm e} = 2.7$ keV, $n_{\rm e} = 2.15 \times 10^{20}$ cm^{-3}, and for the second order fitting curve (solid line) are $T_{\rm e} = 2.75$ keV and $n_{\rm e} = 2.15 \times 10^{20}$ cm^{-3}. Another comparison is shown in Fig. 3(b), where the plasma parameters for the dashed line are $T_{\rm e} = 3$ keV and $n_{\rm e} = 4 \times 10^{20}$ cm^{-3}, while those for the first order fitting curve (dotted line) are $T_{\rm e} = 4.44$ keV and $n_{\rm e} = 4.08 \times 10^{20}$ cm^{-3}, and for the second order fitting curve (solid line) are $T_{\rm e} = 4.48$ keV and $n_{\rm e} = 4.10 \times 10^{20}$ cm^{-3}. From these results, we conclude that the first order corrected theory can very slightly underestimate the electron temperature.

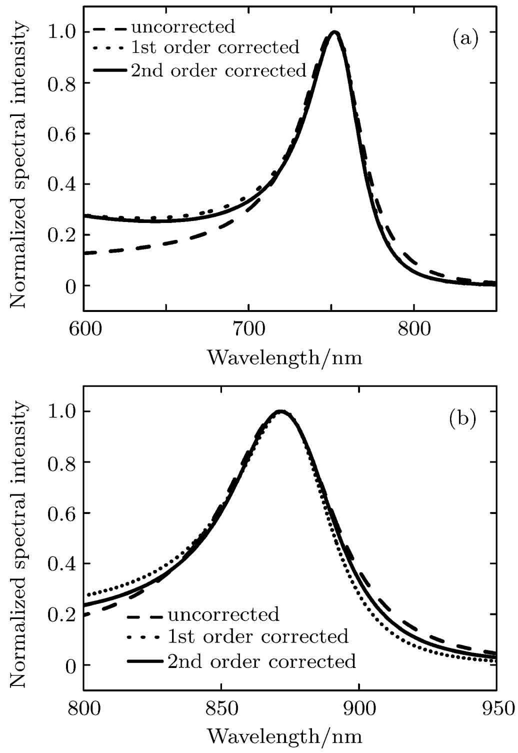

Fig. 3. Fitting of the first and the second order corrected theories to the uncorrected one. The plasma parameters used to calculate the uncorrected spectrum are (a) $T_{\rm e} = 2$ keV, $n_{\rm e} = 2.1 \times 10^{20}$ cm^{-3}, $\lambda_0 = 0.5266$ μm, and $\theta_{\rm s} = 104°$; and (b) $T_{\rm e} = 3$ keV, $n_{\rm e} = 4 \times 10^{20}$ cm^{-3}, $\lambda_0 = 0.5266$ μm, and $\theta_{\rm s} = 104°$.

Equation (36) is rather lengthy and complicated. For routine applications, we can further simplify it. Noticing that the relativistic corrections are important only in the high-frequency part of the Thomson scattering spectrum, we can neglect the low-frequency response of the electrons that screen the ion motion in the plasma, i.e., we approximately have $\chi_{\rm i} \approx 0$ and $F_{\rm i} \approx 0$ when $\omega/kc_{\rm s} \gg 1$, where $c_{\rm s}$ is the ion-acoustic speed of the plasma. Then, the normal dynamic form factor can be written as

$$S(k,\omega) \approx \frac{1}{|\epsilon_{10}|^2}\frac{1}{n_{\rm e}}F_{\rm e0}.$$

Also, C_1 and D_2 can be simplified into

$$C_1(k,\omega) \approx 2\frac{\omega^2}{k^2}\left(1 - \frac{\omega_{\rm p}^2}{\omega^2}\right) S(k,\omega),$$

$$D_2(k,\omega) \approx \frac{T_{\rm e}}{m_{\rm e}}|\epsilon_{10}|^2 S(k,\omega).$$

With the above approximations, Eq. (36) can be written as

$$\frac{\mathrm{d}^2 P_{\boldsymbol{n}_\mathrm{s},\omega_\mathrm{s}}}{\mathrm{d}\Omega\,\mathrm{d}\omega_\mathrm{s}} = r_\mathrm{e}^2 I_0 V_\mathrm{s} n_\mathrm{e} \left\{ \left[S^\mathrm{rel}(k,\omega) + \frac{2\omega}{\omega_0} S^\mathrm{rel}(k,\omega) \right] + \left[\left(1 - \frac{\omega_\mathrm{p}^2}{\omega^2}\right) \left(1 - \frac{1}{4\sin^2(\theta_\mathrm{s}/2)}\right) \frac{\omega^2}{\omega_0^2} + \alpha \left(-4\sin^2\frac{\theta_\mathrm{s}}{2} - 1 + |\epsilon_{10}| \sin^2\theta_\mathrm{s} \right) \right] S(k,\omega) \right\}. \tag{37}$$

Here we have replaced $(\omega/\omega_0)^2 S^\mathrm{rel}(k,\omega)$ with $(\omega/\omega_0)^2 S(k,\omega)$, since $(\omega/\omega_0)^2$ is small in the second order correction. The term proportional to α can be neglected, because $\alpha \lesssim 10^{-2}$ for the laser-produced plasma with temperature lower than 5 keV. The term proportional to $(1 - \omega_\mathrm{p}^2/\omega^2)$ can also be neglected, because $\omega \approx \omega_\mathrm{p}$ around the maximum of the collective scattering spectrum. Therefore, we approximately have

$$\frac{\mathrm{d}^2 P_{\boldsymbol{n}_\mathrm{s},\omega_\mathrm{s}}}{\mathrm{d}\Omega\,\mathrm{d}\omega_\mathrm{s}} = r_\mathrm{e}^2 I_0 V_s n_\mathrm{e} \left(1 + \frac{2\omega}{\omega_0}\right) S^\mathrm{rel}(k,\omega). \tag{38}$$

In Fig. 4, we plot the profiles of the scattering spectral power separately given by Eqs. (36) and (38). The parameters used in the calculation are $T_\mathrm{e} = 2$ keV, $n_\mathrm{e} = 2.1 \times 10^{20}$ cm^{-3}, $\lambda_0 = 0.5266$ μm and $\theta_\mathrm{s} = 104°$. As seen in the figure, the difference between the two curves is really very small, indicating that Eq. (38) is a good approximation to Eq. (36). The accuracy of the approximation becomes worse when the plasma density becomes higher, because the difference between $(\omega/\omega_0)^2 S^\mathrm{rel}(k,\omega)$ and $(\omega/\omega_0)^2 S(k,\omega)$ increases with the plasma density increasing.

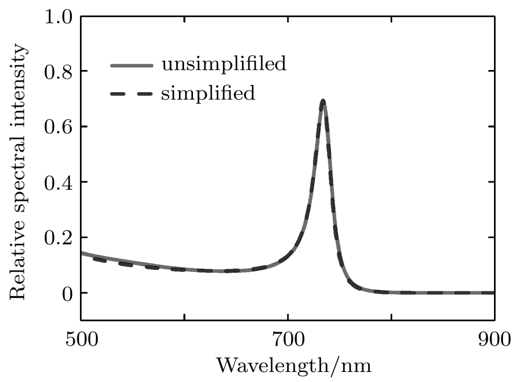

Fig. 4. Comparison between the unsimplified result from Eq. (36) and the simplified result from Eq. (38). The parameters used in the calculation are $T_\mathrm{e} = 2$ keV, $n_\mathrm{e} = 2.1 \times 10^{20}$ cm^{-3} , $\lambda_0 = 0.5266$ μm and $\theta_\mathrm{s} = 104°$.

4. Conclusion

We revisit the theory of collective Thomson scattering for high-temperature high-density plasma. The spectral power of the Thomson scattering with the inclusion of the relativistic corrections of $(v/c)^2$ is derived with the aid of the fluctuation theory. It is found that with the inclusion of the relativistic corrections, the inferred electron temperature is even higher than that obtained with the theory with the inclusion of only the first order corrections of v/c. The full formula (Eq. (36)) for the scattering spectrum with the inclusion of the relativistic corrections is rather lengthy and complicated. For practical use, a simplified formula (Eq. (38)) is given. A comparison between the two equations shows that the latter accords well with the former.

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Appendix A: the derivation of Eq. (25)

The real part of the relativistic electron susceptibility can be written as[31]

$$\mathrm{Re}\,\chi_\mathrm{e} = \frac{1}{k^2\lambda_\mathrm{D}^2} \left[1 + \frac{\omega}{kc} \frac{\alpha^{-1}}{4\mathrm{K}_2(\alpha^{-1})} \int_0^\infty \exp\left(-\alpha^{-1}\cosh x\right) \ln\frac{(\tanh x - \omega/kc)^2}{(\tanh x + \omega/kc)^2} \sinh x \cosh^2 x \,\mathrm{d}x \right]. \tag{A1}$$

After integrating by part, we have

$$\mathrm{Re}\,\chi_\mathrm{e} = \frac{1}{k^2\lambda_\mathrm{D}^2} \left\{ 1 + \frac{\omega}{kc} \frac{1}{4\mathrm{K}_2(\alpha^{-1})} \int_0^\infty \mathrm{e}^{-\alpha^{-1}\cosh x} \frac{\mathrm{d}}{\mathrm{d}x} \left[\ln\frac{(\tanh x - \omega/kc)^2}{(\tanh x + \omega/kc)^2} \cosh^2 x \right] \mathrm{d}x \right\}. \tag{A2}$$

Introducing a new variable $t=\alpha^{-1}\cosh x$, we can rewrite the second term of Eq. (A2) as

$$\frac{\omega}{kc}\frac{\alpha}{\mathrm{K}_2(\alpha^{-1})\exp(\alpha^{-1})}\int_0^\infty \mathrm{e}^{-t}\left\{-\frac{1}{\sqrt{2\alpha t+\alpha^2t^2}}\frac{(\omega/kc)}{(\omega/kc)^2-(2\alpha t+\alpha^2t^2)/(1+\alpha t)^2}\right.$$
$$\left.+\frac{(1+\alpha t)}{2}\ln\frac{[\sqrt{(2\alpha t+\alpha^2t^2)}/(1+\alpha t)-\omega/kc]^2}{[\sqrt{(2\alpha t+\alpha^2t^2)}/(1+\alpha t)+\omega/kc]^2}\right\}\mathrm{d}t.$$

Due to the rapidly decreasing factor $\exp(-t)$ in the integrand, the important contribution to the integral comes from $t<1$. When $\alpha\ll 1$, we can expand the bracketed part in the integrand in series of α and obtain

$$\mathrm{Re}\,\chi_\mathrm{e}=\frac{1}{k^2\lambda_\mathrm{D}^2}\left\{1+\frac{\sqrt{2\alpha}\xi^2}{\mathrm{K}_2(\alpha^{-1})\exp(\alpha^{-1})}\int_0^\infty \mathrm{e}^{-x^2}\left[\frac{1}{x^2-\xi^2}+\alpha\left(\frac{2}{x^2-\xi^2}-\frac{1}{4}\frac{x^2}{x^2-\xi^2}+\frac{3}{2}\frac{x^4}{(x^2-\xi^2)^2}\right)\right]\mathrm{d}x\right\}.$$

Noticing that the real part of the plasma dispersion function can be written as

$$\mathrm{Re}\,Z(\xi)=\frac{2\xi}{\sqrt{\pi}}\int_0^\infty\frac{1}{x^2-\xi^2}\,\mathrm{e}^{-x^2}\,\mathrm{d}x,$$

we have

$$\mathrm{Re}\,\chi_\mathrm{e}=\mathrm{Re}\,\chi_\mathrm{e0}+\frac{\alpha}{k^2\lambda_\mathrm{D}^2}\left[\left(\frac{5}{4}-\frac{3}{2}\xi^2\right)\xi^2+\left(\frac{1}{8}+2\xi^2-\frac{3}{2}\xi^4\right)\xi\,\mathrm{Re}\,Z(\xi)\right]. \quad \text{(A3)}$$

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