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Hidden flaw jeopardizes millions of online transactions

Mathematicians discover weakness in commonly used encryption schemes

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By Paul Wagenseil

 updated 2/15/2012 6:01:37 PM ET

Mathematicians based in Switzerland and the United States have discovered a small but the most commonly used digital encryption schemes, a flaw that could undermine the secure communication.

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A group of six academic researchers have concluded that real-world RSA encryption keys are riskier than Diffie-Hellman-based ones. It seems that some of the random numbers used to generate them weren't, erm, random. In [IT Blogwatch](#), bloggers wonder if the sky is falling.

The Register

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'Predictably random' public keys can be cracked - crypto boffins

Battling researchers argue over whether you should panic

By **John Leyden** - [Get more from this author](#)

Posted in Enterprise Security, 16th February 2012 13:38 GMT

Analysis Cryptography researchers have discovered flaws in the key generation that underpins the security of important cryptography protocols, including SSL.

Secure Chat

Alice wants to send a secret message to Bob?

- Sometime in the past, they exchange a **one-time pad**
- Alice uses the pad to encrypt the message
- Bob uses the same pad to decrypt the message

"use yT25a5y/8 if I ever send you an encrypted message"

Chat Client 1.0 [Alice]

Alice: Hey Bob
 Bob: Hi Alice
 Alice: gX76W3v7K

Encrypt SENDMONEY with yT25a5y/8

Chat Client 1.0 [Bob]

Alice: Hey Bob
 Bob: Hi Alice
 Alice: gX76W3v7K

Decrypt gX76W3v7K with yT25a5y/8

Key point Without the pad, Eve cannot understand the message

Encryption Machine

Goal Design a machine to encrypt and decrypt data

S	E	N	D	M	O	N	E	Y
---	---	---	---	---	---	---	---	---

↓ encrypt

g	X	7	6	W	3	v	7	K
---	---	---	---	---	---	---	---	---

↓ decrypt

S	E	N	D	M	O	N	E	Y
---	---	---	---	---	---	---	---	---

Encryption Machine

Goal Design a machine to encrypt and decrypt data

S	E	N	D	M	O	N	E	Y
---	---	---	---	---	---	---	---	---

↓ encrypt


g	X	7	6	W	3	v	7	K
---	---	---	---	---	---	---	---	---

↓ decrypt

S	E	N	D	M	O	N	E	Y
---	---	---	---	---	---	---	---	---

Enigma encryption machine

- "Unbreakable" German code during WWII
- Broken by Turing bombe
- One of first uses of computers
- Helped win Battle of Atlantic by locating U-boats



A Digital World

Data is a sequence of bits [bit = 0 or 1]

- **Text**
- Programs, executables
- Documents, pictures, sounds, movies, ...

File formats txt, pdf, java, exe, docx, pptx, jpeg, mp3, divx, ...



computer with a lens



computer with earbuds



computer with a radio

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A Digital World

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File formats txt, pdf, java, exe, docx, pptx, jpeg, mp3, divx, ...



computer with a cash dispenser

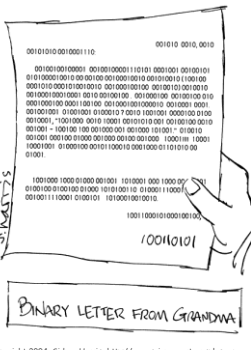


computer with a ballot box



computer with a heating element

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A Digital World

Data is a sequence of bits [bit = 0 or 1]

- **Text**
- Programs, executables
- Documents, pictures, sounds, movies, ...

Base64 encoding Use 6 bits to represent each alphanumeric symbol.

very weak type of encryption

Binary Char	Binary Char	Binary Char	Binary Char	Binary Char	Binary Char
000000	A	001011	L	010110	W
000001	B	001100	M	010111	X
000010	C	001101	N	011000	Y
000011	D	001110	O	011001	Z
000100	E	001111	P	011010	a
000101	F	010000	Q	011011	b
000110	G	010001	R	011010	c
000111	H	010010	S	011101	d
001000	I	010011	T	011110	e
001001	J	010100	U	011111	f
001010	K	010101	V	100000	g
				101011	h
				101100	i
				101101	j
				101110	k
				101111	l
				110000	m
				110001	n
				110010	o
				110011	p
				110100	q
				110101	r
				110110	s
				110111	t
				111000	u
				111001	v
				111010	w
				111011	x
				111100	y
				111101	z
				111110	+
				111111	/

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One-Time Pad Encryption

Encryption

- Convert text message to N bits

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...
M	12	001100
...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64

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One-Time Pad Encryption

Encryption

- Convert text message to N bits
- Generate N random bits (one-time pad)

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits

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One-Time Pad Encryption

Encryption

- Convert text message to N bits
- Generate N random bits (one-time pad)
- Take bitwise XOR of two bitstrings

XOR Truth Table

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

sum corresponding pair of bits: 1 if sum is odd, 0 if even

S	E	N	D	M	O	N	E	Y	
010010	000100	001101	000011	001100	001110	001101	000100	011000	message
110010	010011	110110	111001	011010	111001	100010	111111	010010	base64
100000	010111	111011	111010	010110	110111	101111	111011	001010	random bits
									XOR

$0 \oplus 1 = 1$

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One-Time Pad Encryption

Encryption

- Convert text message to N bits
- Generate N random bits (one-time pad)
- Take bitwise XOR of two bitstrings
- Convert binary back into text

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...
W	22	010110
...

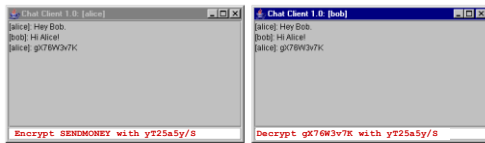
S	E	N	D	M	O	N	E	Y	
010010	000100	001101	000011	001100	001110	001101	000100	011000	message
110010	010011	110110	111001	011010	111001	100010	111111	010010	base64
100000	010111	111011	111010	010110	110111	101111	111011	001010	random bits
									XOR
g	x	7	6	w	3	v	7	k	encrypted

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Secure Chat (review)

Alice wants to send a secret message to Bob?

- Sometime in the past, they exchange a **one-time pad**
- Alice uses the pad to **encrypt** the message
- Bob uses the same pad to **decrypt** the message



Key point Without the pad, Eve cannot understand the message



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One-Time Pad Decryption

Decryption

- Convert encrypted message to binary

g	x	7	6	w	3	v	7	k	encrypted
---	---	---	---	---	---	---	---	---	-----------

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One-Time Pad Decryption

Decryption

- Convert encrypted message to binary

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...
W	22	010110
...

g	x	7	6	w	3	v	7	k	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64

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One-Time Pad Decryption

Decryption

- Convert encrypted message to binary
- Use **same** N random bits (one-time pad)

g	x	7	6	w	3	v	7	k	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits

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One-Time Pad Decryption

Decryption

- Convert encrypted message to binary
- Use same N random bits (one-time pad)
- Take bitwise XOR of two bitstrings

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

g	x	7	6	w	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR

1 ^ 1 = 0

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One-Time Pad Decryption

Decryption

- Convert encrypted message to binary
- Use same N random bits (one-time pad)
- Take bitwise XOR of two bitstrings
- Convert back into text

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...
M	12	001100
...

g	x	7	6	w	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message

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Why Does It Work?

Crucial property Decrypted message = original message

Notation	Meaning
a	original message bit
b	one-time pad bit
^	XOR operator
a ^ b	encrypted message bit
(a ^ b) ^ b	decrypted message bit

Why is crucial property true?

- Use properties of XOR.
 - $(a \wedge b) \wedge b = a \wedge (b \wedge b) = a \wedge 0 = a$
- ↙ associativity of ^
↙ always 0
↙ identity

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

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One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary

g	x	7	6	w	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted

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One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary

g	x	7	6	w	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64

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One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary
- Use **wrong** N bits (bogus one-time pad)

g	x	7	6	w	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits

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One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary
- Use **wrong** N bits (bogus one-time pad)
- Take bitwise XOR of two bitstrings

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR

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One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary
- Use **wrong** N bits (bogus one-time pad)
- Take bitwise XOR of two bitstrings
- Convert back into text: **Oops**

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR
I	L	O	V	E	O	K	R	A	wrong message

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Goods and Bads of One-Time Pads

Good

- Easily computed by hand
- Very simple encryption/decryption processes
- Provably unbreakable if bits are truly random [Shannon, 1940s]

← eavesdropper Eve sees only random bits

Bad

- Easily breakable if pad is re-used
- Pad must be as long as the message
- Truly random bits are very hard to come by
- Pad must be distributed securely

← impractical for Web commerce



a Russian one-time pad

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Pseudo-Random Bit Generator

Practical middle-ground

- Let's make a "random"-bit generator gadget
- Alice and Bob each get identical small gadgets

← instead of identical large one-time pads

How to make small gadget that produces "random" bits

- Enigma machine
- Linear feedback shift register
- Linear congruential generator
- Blum-Blum-Shub generator
- ...

"Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin."
 — Jon von Neumann (left)
 — ENIAC (right)

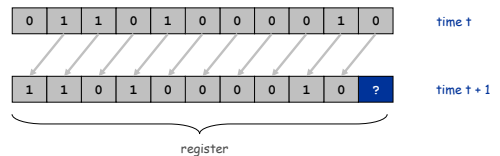


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Shift Register

Shift register terminology.

- Bit: 0 or 1
- Cell: storage element that holds one bit
- Register: sequence of cells
- Seed: initial sequence of bits
- Shift register: when clock ticks, bits propagate one position to left

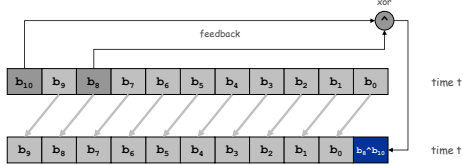


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Linear Feedback Shift Register (LFSR)

{8, 10} linear feedback shift register

- Shift register with 11 cells
- Bit b_0 is XOR of previous bits b_8 and b_9
- Pseudo-random bit = b_0



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Random Numbers

Q. Are these 2000 numbers random? If not, what is the pattern?

```
110010010011101101100101101011001100001011110100000100110100101110011000100101111
101110000010101100000000111010010100000111000010011001101111010100000000000001010
01010000100000011100001001001010111001101001100110011011001100110011001100110011001
011010000010100100010001010101011000000010100001011000010110101010101010010000
111111001100000011110001000010111001101001101001100100110101101010101010101010000
0000001000000010000001000001010101001000001101000001100100001010101010101010000
010100001000100010001010110100001100001110001011000110001101011001001001011011
0000101100100001011010010010011011001101101001010111000000010010000010111
100100100001101010101000110001101101101101010101010101010000101100011011011100001010
0110000011111010100001000110010101011000011001100011100111010110000100110101010
01101010011100001100110111111010000000100100001011010001001100101110001011100001
00001001001100110000110001101010110110101010101010101010000010110001101010101000011
0110001111111001010011100000011100001101011100010101010101000000010000000111110
10011000010111010110000001010101010000001110000100011001101110111001100100000
1100010011010101101100000010110101001000011000010011001100111001100001110011000011110
1100110010111110010000011101000010100001100101101110101010000100000010100000
100001001000010100001100110011100111001100110011001111110000000010000000
11100000100100001111110100000010100000010100100110011100111001110001110001110011
0100111101111000010000101000011100100001100100010101100010100001100101010001010
001100011011011101010010000100110011100111001000000001010000110000010101000010
1110111010010010010001010110110101000010001000011010101010010000001
110100010010010111101000001110101001001000011010011010011010110101000010
01010101010100000001100000111000011011001101010111000010001100010101101010
```

A. No. This is output of {8, 10} LFSR with seed 01101000010!

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LFSR Encryption

LFSR encryption

- Convert text message to N bits
- Initialize LFSR with small seed
- Generate N bits with LFSR
- Take bitwise XOR of two bitstrings
- Convert binary back into text

Base64 Encoding		
char	dec	binary
A	0	000000
B	1	000001
...
W	22	010110
...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
100000	010111	111011	111010	010110	110111	111011	111011	001010	XOR
g	X	7	6	W	3	v	7	K	encrypted

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LFSR Decryption

LFSR Decryption

- Convert encrypted message to N bits
- Initialize identical LFSR with same seed
- Generate N bits with LFSR
- Take bitwise XOR of two bitstrings
- Convert binary back into text

Base64 Encoding		
char	dec	binary
A	0	000000
B	1	000001
...
M	12	001100
...

g	X	7	6	W	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	010110	111001	100010	111111	010010	LFSR bits
010010	000100	001101	000011	001100	001110	001011	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message

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Goods and Bads of LFSR Encryption

Goods

- Easily computed with simple machine
- Very simple encryption/decryption process
- Scalable: 20 cells for 1 million bits; 30 cells for 1 billion bits [but need theory of finite groups to know where to put taps]



a commercially available LFSR

Bads

- Still need secure, independent way to distribute LFSR seed
- The bits are not truly random [bits in our 11-bit LFSR cycle after $2^{11} - 1 = 2047$ steps]
- Experts have cracked LFSR [more complicated machines needed]

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Other LFSR Applications

What else can we do with a LFSR?

- DVD encryption with CSS
- DVD decryption with DeCSS!
- Subroutine in military cryptosystems



DVD Jon (Norwegian hacker)

```
/* edft.c Author: Charles M. Hannum croot@ihack.net */
/* Usage is: cat title=key scrambled.vob | edft.c unlar.vob */

#define M (1+(1)*8)

unsigned char x[M], .y,*[2048],min(
n){for( read(0,w,3 );read(0,w ,w=2048
); w+=1; .w=n) i=i^x
{y=x [13]*8+201 /164 w=1 }i=i^x
w=( 117 *256 w+(0) S,x w=(2)
0,3= m(4) 17* m(3) 9*8* 2-318
"8,w w=0; c =25; for( a[0] i=-1;
--i; )w+=m w^2=14 i,i=1 /2^31
<24;for(j= 127; ++j;n;w+=
)
c
w+=m*/8^1>>>i>12;
i=308*yc17, a=w>>4,y=w^8*cc6,w=
>>8*cc6,w=i^1,i, a =78w;g, U15^1
47)*2"ca3a4w; *k>>7,n. {k>4)*2^3*257/
8,w(3)*w*(k^k^134) *e^e^y
}}

http://www.ca.umd.edu/~dft/deCSS/gallery
```

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A Closing Profound Question

Q. What is a random number?

LFSR does not produce random numbers

- It is a very simple deterministic machine
- But not obvious how to distinguish the bits it produces from random

Q. Are truly random processes found in nature?

- Motion of cosmic rays or subatomic particles?
- Mutations in DNA?

Q. Or, is the natural world a (not-so-simple) deterministic machine?

"God does not play dice."
- Albert Einstein



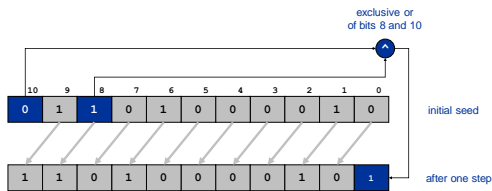
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Extra Slides

Introduction to Programming in Java: An Interdisciplinary Approach · Robert Sedgwick and Kevin Wayne · Copyright © 2008 · *

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Linear Feedback Shift Register



One step of an 11-bit LFSR with initial seed 0110100010 and tap at position 8

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