

5. The TOY Machine



Introduction to Programming in Java - An Interdisciplinary Approach - Robert Sedgwick and Kevin Wayne - Copyright © 2002-2011 - 15/10/2012 11:35:57

What is TOY?

- An imaginary machine similar to:
- Ancient computers.
 - Today's microprocessors.



Why Study TOY?

Machine language programming.

- How do Java programs relate to computer?
- Key to understanding Java references.
- Still some situations today where it is really necessary.

multimedia, computer games, scientific computing, SSE, AVX

Computer architecture.

- How does it work?
- How is a computer put together?

TOY machine. Optimized for **simplicity**, not cost or performance.

Data and Programs Are Encoded in Binary

Each bit consists of two states:

- 1 or 0; true or false.
- Switch is on or off; wire has high voltage or low voltage.

Everything stored in a computer is a sequence of bits.

- **Data and programs.**
- Text, documents, pictures, sounds, movies, executables, ...



Binary Encoding

How to represent integers?

- Use **binary** encoding.
- Ex: $6375_{10} = 0001100011100111_2$

Dec	Bin	Dec	Bin
0	0000	8	1000
1	0001	9	1001
2	0010	10	1010
3	0011	11	1011
4	0100	12	1100
5	0101	13	1101
6	0110	14	1110
7	0111	15	1111

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1

$$6375_{10} = +2^{12} + 2^{11} + 2^7 + 2^6 + 2^5 + 2^2 + 2^1 + 2^0$$

$$= 4096 + 2048 + 128 + 64 + 32 + 4 + 2 + 1$$

Hexadecimal Encoding

How to represent integers?

- Use **hexadecimal** encoding.
- Ex: $6375_{10} = 0001100011100111_2 = 18E7_{16}$

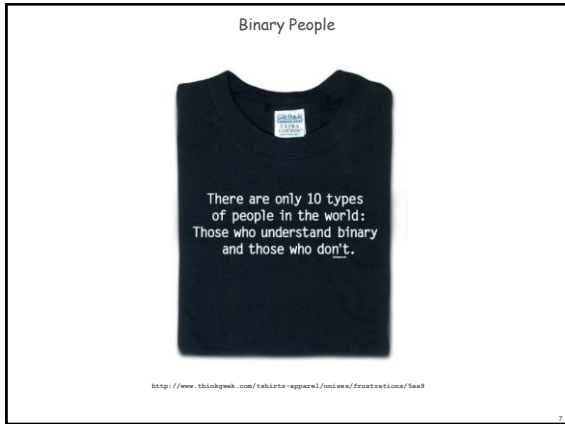
binary code, 4 bits at a time

Dec	Bin	Hex	Dec	Bin	Hex
0	0000	0	8	1000	8
1	0001	1	9	1001	9
2	0010	2	10	1010	A
3	0011	3	11	1011	B
4	0100	4	12	1100	C
5	0101	5	13	1101	D
6	0110	6	14	1110	E
7	0111	7	15	1111	F

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1
1				8				E				7			

$$6375_{10} = 1 \times 16^3 + 8 \times 16^2 + 14 \times 16^1 + 7 \times 16^0$$

$$= 4096 + 2048 + 224 + 7$$



Inside the TOY Box

Switches. Input data and programs.

Lights. View data.

Memory.

- Stores data and programs.
- 256 16-bit "words."
- Special word for stdin / stdout.

Program counter (PC).

- An extra 8-bit register.
- Next instruction to be executed.

Registers.

- Fastest form of storage.
- Scratch space during computation.
- 16 16-bit registers.
- Register 0 is always 0.

Arithmetic-logic unit (ALU). Manipulate data stored in registers.

Standard input, standard output. Interact with outside world.

TOY Machine "Core" Dump

A **core dump** is the contents of machine at a particular place and time.

- Record of what program has done.
- Completely determines what machine will do.

Registers			
R0	R1	R2	R3
0000	0000	0000	0000
R4	R5	R6	R7
0000	0000	0000	0000
R8	R9	RA	RB
0000	0000	0000	0000
RC	RD	RE	RF
0000	0000	0000	0000

pc	
10	

Memory															
00:	0008	0005	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
08:	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
10:	8A00	8B01	1CAB	9C02	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
18:	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
20:	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
28:	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
E8:	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
F0:	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
F8:	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000

Annotations: index of next instruction, data, program, variables

A Sample Program

A sample program. Adds 0008 + 0005 = 000D.

RA	RB	RC
0000	0000	0000

pc
10

TOY memory (program and data)		
00:	0008	8
01:	0005	5
02:	0000	0
10:	8A00	RA ← mem[00]
11:	8B01	RB ← mem[01]
12:	1CAB	RC ← RA + RB
13:	9C02	mem[02] ← RC
14:	0000	halt

add.toy

A Sample Program

Program counter. The pc is initially 10, so the machine interprets 8A00 as an instruction.

RA	RB	RC
0000	0000	0000

pc
10

10:	8A00	RA ← mem[00]
11:	8B01	RB ← mem[01]
12:	1CAB	RC ← RA + RB
13:	9C02	mem[02] ← RC
14:	0000	halt

add.toy

Load

Load. [opcode 8]

- Loads the contents of some memory location into a register.
- 8A00 means load the contents of memory cell 00 into register A.

RA	RB	RC
0000	0000	0000

pc
10

10:	8A00	RA ← mem[00]
11:	8B01	RB ← mem[01]
12:	1CAB	RC ← RA + RB
13:	9C02	mem[02] ← RC
14:	0000	halt

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
8 ₁₆				A ₁₆				00 ₁₆							
opcode				dest d				addr							

Load

Load. [opcode 8]

- Loads the contents of some memory location into a register.
- 8B01 means load the contents of memory cell 01 into register B.

RA	RB	RC
0008	0000	0000

pc
11

00: 0008	8
01: 0005	5
02: 0000	0
10: 8A00	RA ← mem[00]
11: 8B01	RB ← mem[01]
12: 1CAB	RC ← RA + RB
13: 9C02	mem[02] ← RC
14: 0000	halt

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1
B ₁₆				B ₁₆				01 ₁₆							
opcode				dest d				addr							

Add

Add. [opcode 1]

- Add contents of two registers and store sum in a third.
- 1CAB means add the contents of registers A and B and put the result into register C.

RA	RB	RC
0008	0005	0000

pc
12

00: 0008	8
01: 0005	5
02: 0000	0
10: 8A00	RA ← mem[00]
11: 8B01	RB ← mem[01]
12: 1CAB	RC ← RA + RB
13: 9C02	mem[02] ← RC
14: 0000	halt

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	1	0	0	1	0	1	0	1	0	1	1
1 ₁₆				C ₁₆				A ₁₆				B ₁₆			
opcode				dest d				source s				source t			

Store

Store. [opcode 9]

- Stores the contents of some register into a memory cell.
- 9C02 means store the contents of register C into memory cell 02.

RA	RB	RC
0008	0005	000D

pc
13

00: 0008	8
01: 0005	5
02: 0000	0
10: 8A00	RA ← mem[00]
11: 8B01	RB ← mem[01]
12: 1CAB	RC ← RA + RB
13: 9C02	mem[02] ← RC
14: 0000	halt

add.toy

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0
9 ₁₆				C ₁₆				02 ₁₆							
opcode				dest d				addr							

Halt

Halt. [opcode 0]

- Stop the machine.

RA	RB	RC
0008	0005	000D

pc
14

00: 0008	8
01: 0005	5
02: 000D	D
10: 8A00	RA ← mem[00]
11: 8B01	RB ← mem[01]
12: 1CAB	RC ← RA + RB
13: 9C02	mem[02] ← RC
14: 0000	halt

add.toy

Program and Data

Program. Sequence of 16-bit integers, interpreted one way.

Data. Sequence of 16-bit integers, interpreted other way.

Program counter (pc). Holds memory address of the next "instruction" and determines which integers get interpreted as instructions.

16 instruction types. Changes contents of registers, memory, and pc in specified, well-defined ways.

Instructions

0:	halt
1:	add
2:	subtract
3:	and
4:	xor
5:	shift left
6:	shift right
7:	load address
8:	load
9:	store
A:	load indirect
B:	store indirect
C:	branch zero
D:	branch positive
E:	jump register
F:	jump and link

TOY Instruction Set Architecture

TOY instruction set architecture (ISA).

- Interface that specifies behavior of machine.
- 16 register, 256 words of main memory, 16-bit words.
- 16 instructions.

Each instruction consists of 16 bits.

- Bits 12-15 encode one of 16 instruction types or opcodes.
- Bits 8-11 encode destination register d.
- Bits 0-7 encode:
 - [Format 1] source registers s and t
 - [Format 2] 8-bit memory address or constant

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	1	1	1	0	1	0	0	0	0	0	0	0	1	0
opcode				dest d				source s				source t			
opcode				dest d				addr							

Interfacing with the TOY Machine

To enter a program or data:

- Set 8 memory address switches.
- Set 16 data switches.
- Press **Load**: data written into addressed word of memory.

To view the results of a program:

- Set 8 memory address switches.
- Press **Look**: contents of addressed word appears in lights.



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Interfacing with the TOY Machine

To execute the program:

- Set 8 memory address switches to address of first instruction.
- Press **Look** to set **pc** to first instruction.
- Press **Run** to repeat fetch-execute cycle until halt opcode.

Fetch-execute cycle.

- **Fetch**: get instruction from memory.
- **Execute**: update **pc**, move data to or from memory and registers, perform calculations.



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Flow Control

Flow control.

- To harness the power of TOY, need loops and conditionals.
- Manipulate **pc** to control program flow.

```

if (boolean expression) {
    statement 1;
    statement 2;
}

while (boolean expression) {
    statement 1;
    statement 2;
}
    
```

Branch if zero. [opcode C]

- Changes **pc** depending on whether value of some register is **zero**.
- Used to implement: **for, while, if-else**.

Branch if positive. [opcode D]

- Changes **pc** depending on whether value of some register is **positive**.
- Used to implement: **for, while, if-else**.

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An Example: Multiplication

Multiply. Given integers **a** and **b**, compute $c = a \times b$.

TOY multiplication. No direct support in TOY hardware.

Brute-force multiplication algorithm:

- Initialize **c** to 0.
- Add **b** to **c**, **a** times.

```

int a = 3;
int b = 9;
int c = 0;

while (a != 0) {
    c = c + b;
    a = a - 1;
}
    
```

brute force multiply in Java

Issues ignored. Slow, overflow, negative numbers.

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Multiply

```

0A: 0003 3 ← inputs
0B: 0009 9
0C: 0000 0 ← output

0D: 0000 0 ← constants
0E: 0001 1

10: 8A0A RA ← mem[0A]      a = 3;
11: 8B0B RB ← mem[0B]      b = 9;
12: 8C0D RC ← mem[0D]      c = 0;
13: 810E R1 ← mem[0E]      always 1
14: CA18 if (RA == 0) pc ← 18  while (a != 0) {
15: 1CCB RC ← RC + RB        c = c + b;
16: 2AA1 RA ← RA - R1        a = a - 1;
17: C014 pc ← 14             }
18: 9C0C mem[0C] ← RC
19: 0000 halt
    
```



multiply.toy

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Step-By-Step Trace

	R1	RA	RB	RC
10: 8A0A RA ← mem[0A]		0003		
11: 8B0B RB ← mem[0B]			0009	
12: 8C0D RC ← mem[0D]				0000
13: 810E R1 ← mem[0E]	0001			
14: CA18 if (RA == 0) pc ← 18				
15: 1CCB RC ← RC + RB				0009
16: 2AA1 RA ← RA - R1		0002		
17: C014 pc ← 14				
14: CA18 if (RA == 0) pc ← 18				
15: 1CCB RC ← RC + RB				0012
16: 2AA1 RA ← RA - R1		0001		
17: C014 pc ← 14				
14: CA18 if (RA == 0) pc ← 18				
15: 1CCB RC ← RC + RB				001B
16: 2AA1 RA ← RA - R1		0000		
17: C014 pc ← 14				
14: CA18 if (RA == 0) pc ← 18				
18: 9C0C mem[0C] ← RC				
19: 0000 halt				

multiply.toy

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A Little History

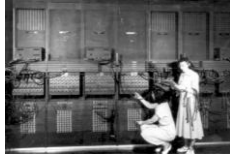
Electronic Numerical Integrator and Calculator (ENIAC).

- First widely known general purpose electronic computer.
- Conditional jumps, programmable.
- Programming: change switches and cable connections.
- Data: enter numbers using punch cards.

30 tons
30 x 50 x 8.5 ft
17,468 vacuum tubes
300 multiply/sec
15,000 watts



John Mauchly (left) and J. Presper Eckert (right)
http://cs.snu.edu/~darkin/articles/history_computing.html



ENIAC Ester Gerston (left), Gloria Gordon (right)
US Army photo: <http://ftp.art.mil/ftp/historic-computers>

Basic Characteristics of TOY Machine

TOY is a general-purpose computer.

- Sufficient power to perform **any** computation.
- Limited only by amount of memory and time.

Stored-program computer. [von Neumann memo, 1944]

- Data and program encoded in binary.
- Data and program stored in **same** memory.
- Can change program without rewiring.

Outgrowth of Alan Turing's work.

All modern computers are general-purpose computers and have same (von Neumann) architecture.



John von Neumann



Maurice Wilkes (left)
EDSAC (right)

TOY Reference Card

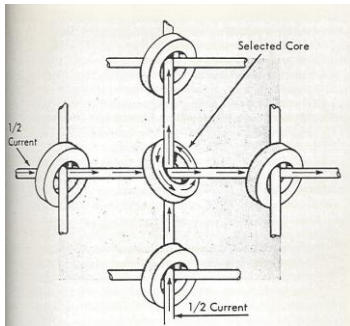
	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
Format 1	opcode				dest d				source s				source t			
Format 2	opcode				dest d				addr							

#	Operation	Fmt	Pseudocode
0:	halt	1	exit(0)
1:	add	1	R[d] ← R[s] + R[t]
2:	subtract	1	R[d] ← R[s] - R[t]
3:	and	1	R[d] ← R[s] & R[t]
4:	xor	1	R[d] ← R[s] ^ R[t]
5:	shift left	1	R[d] ← R[s] << R[t]
6:	shift right	1	R[d] ← R[s] >> R[t]
7:	load addr	2	R[d] ← addr
8:	load	2	R[d] ← mem[addr]
9:	store	2	mem[addr] ← R[d]
A:	load indirect	1	R[d] ← mem[R[t]]
B:	store indirect	1	mem[R[t]] ← R[d]
C:	branch zero	2	if (R[d] == 0) pc ← addr
D:	branch positive	2	if (R[d] > 0) pc ← addr
E:	jump register	2	pc ← R[d]
F:	jump and link	2	R[d] ← pc; pc ← addr

Register 0 always reads 0.
Loads from mem[FF] from stdin.
Stores to mem[FF] to stdout.
16-bit registers.
16-bit memory.
8-bit program counter.

5: Supplemental Notes

Why do They Call it "Core"?



<http://www.columbia.edu/acis/history/core.html>





An Efficient Multiplication Algorithm

Inefficient multiply.

- Brute force multiplication algorithm loops a times.
- In worst case, 65,535 additions!

"Grade-school" multiplication.

- Always 16 additions to multiply 16-bit integers.

Decimal

		1	2	3	4							
*	1	5	1	2								
	2	4	6	8								
		1	2	3	4							
			6	1	7	0						
				1	2	3	4					
					0	1	8	6	5	8	0	8

Binary

				1	0	1	1							
*	1	1	0	1										
				1	0	1	1							
					0	0	0	0						
					1	0	1	1						
						1	0	1	1					
							1	0	0	0	1	1	1	1

Binary Multiplication

Grade school binary multiplication algorithm to compute $c = a \times b$.

- Initialize $c = 0$.
- Loop over i bits of b . $\leftarrow b_i = i^{\text{th}}$ bit of b
 - if $b_i = 0$, do nothing
 - if $b_i = 1$, shift a left i bits and add to c

				1	0	1	1							
*				1	1	0	1							
					1	0	1	1						
						0	0	0	0					
							1	0	1	1				
								1	0	0	1	1	1	1

a

b

$a \ll 0$

$a \ll 2$

$a \ll 3$

c

Implement with built-in TOY shift instructions.

```

int c = 0;
for (int i = 15; i >= 0; i--)
    if (((b >> i) & 1) == 1)
        c = c + (a << i);
    
```

$\leftarrow b_i = i^{\text{th}}$ bit of b

Shift Left

Shift left. (opcode 5)

- Move bits to the left, padding with zeros as needed.
- $1234_{16} \ll 7_{16} = 1A00_{16}$

discard

0	X	X	X	X	X	0	0	0	0	1	1	0	1	0	0	0
1 ₁₆	2 ₁₆	3 ₁₆	4 ₁₆													

$\ll 7$

pad with 0's

0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0
1 ₁₆	2 ₁₆	3 ₁₆	4 ₁₆	0 ₁₆	1 ₁₆	2 ₁₆	3 ₁₆	4 ₁₆								

Shift Right

Shift right. (opcode 6)

- Move bits to the right, padding with sign bit as needed.
- $1234_{16} \gg 7_{16} = 0024_{16}$

discard

0	0	0	1	0	0	1	0	0	0	X	X	X	X	X	X	X
1 ₁₆	2 ₁₆	3 ₁₆	4 ₁₆													

sign bit

pad with 0's

$\gg 7$

0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0 ₁₆	1 ₁₆	2 ₁₆	3 ₁₆	4 ₁₆												

Bitwise AND

Logical AND. (opcode B)

- Logic operations are BITWISE.
- $0024_{16} \& 0001_{16} = 0000_{16}$

x	y	&
0	0	0
0	1	0
1	0	0
1	1	1

0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0 ₂₆	0 ₁₆	2 ₁₆	4 ₁₆													

&

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 ₁₆	0 ₁₆	0 ₁₆	1 ₁₆													

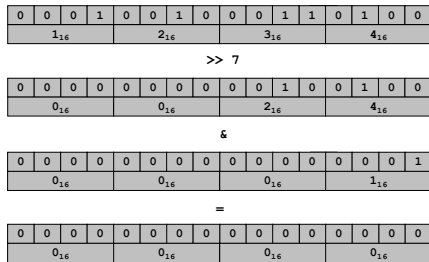
=

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 ₁₆	0 ₁₆	0 ₁₆	0 ₁₆													

Shifting and Masking

Shift and mask: get the 7th bit of 1234.

- Compute $1234_{16} \gg 7_{16} = 0024_{16}$.
- Compute $0024_{16} \& 1_{16} = 0_{16}$.



Binary Multiplication

```

0A: 0003 3
0B: 0009 9 ← inputs
0C: 0000 0 ← output
0D: 0000 0
0E: 0001 1 ← constants
0F: 0010 16

10: 8A0A RA ← mem[0A]
11: 8B0B RB ← mem[0B]
12: 8C0D RC ← mem[0D]
13: 810E R1 ← mem[0E]
14: 820F R2 ← mem[0F]

15: 2221 R2 ← R2 - R1
16: 53A2 R3 ← RA << R2
17: 64B2 R4 ← RB >> R2
18: 3441 R4 ← R4 & R1
19: C41B if (R4 == 0) goto 1B
1A: 1CC3 RC ← RC + R3
1B: D215 if (R2 > 0) goto 15
1C: 9C0C mem[0C] ← RC
    
```

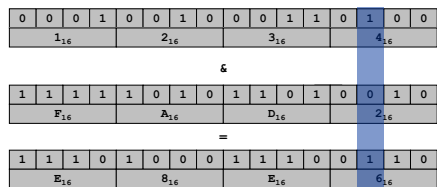
Annotations: "loop" points to lines 15-18, "branch" points to line 19, "discard" points to the rightmost bits of the register A in the diagram below.

Bitwise XOR

Bitwise XOR. (opcode 4)

- Logic operations are BITWISE.
- $1234_{16} \wedge FAD2_{16} = E8E6_{16}$

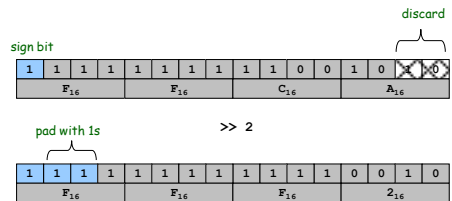
x	y	^
0	0	0
0	1	1
1	0	1
1	1	0



Shift Right (Sign Extension)

Shift right. (opcode 6)

- Move bits to the right, padding with sign bit as needed.
- $FFCA_{16} \gg 2_{16} = FFF2_{16}$
- $-53_{10} \gg 2_{10} = -13_{10}$



5: Extra Slides

Useful TOY "Idioms"

Jump absolute.

- Jump to a fixed memory address.
 - branch if zero with destination
 - register 0 is always 0

```
17: C014 pc ← 14
```

Register assignment.

- No instruction that transfers contents of one register into another.
- Pseudo-instruction that simulates assignment:
 - add with register 0 as one of two source registers

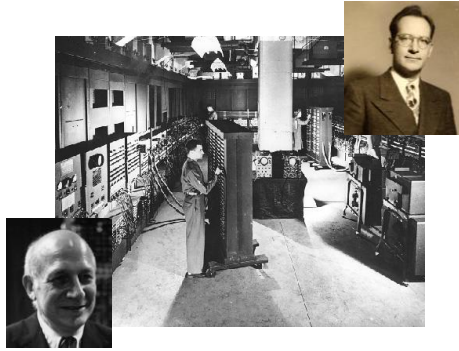
```
17: 1230 R[2] ← R[3]
```

No-op.

- Instruction that does nothing.
- Plays the role of whitespace in C programs.
 - numerous other possibilities!

```
17: 1000 no-op
```

ENIAC



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What is TOY?

An imaginary machine similar to:

- Ancient computers.



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