Sorting
Overview

We spend a lot of time sorting things

- it makes searching easier
- many problems in computer science are functionally just searching problems

In this module, we will learn about how to sort elements stored inside an array

Example:

- Sort the cats stored inside an array by their name alphabetically
Learning Objectives

- To be able to use **insertion sort** to sort elements inside an array
- To be able to use **selection sort** to sort elements inside an array
- To be able to use Java methods to sort an array or a list
**MergeSort**

We'll study two sorting algorithms today: Insertion Sort and Selection Sort.

In recitation, we'll reference **MergeSort**, which is a recursive sorting algorithm that usually runs faster than Insertion or Selection Sort. It is **not** considered testable material for this course, and is only included for your reference.
**Insertion Sort: High Level View**

- Maintain a sorted sub-section of the array starting at the beginning
  - This section doesn’t account for elements outside of the section
  - This section starts as just the first element
- Continually add the next element of overall array to the sorted sub-section, shifting the elements in the subsection to maintain order
- After transferring the last unsorted element to the sorted subsection and adjusting it, sorting is done!
**Insertion Sort**

- Insertion sort compares the first two elements and put them in order.
- Insertion sort then takes the third element and put it into the right position with respect to the first two.
- Insertion sort then takes the fourth element and put it into the right position with respect to the first three.
- And so on, until the entire array is sorted.
# Insertion Sort

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10</td>
<td>15</td>
<td>54</td>
<td>55</td>
<td>11</td>
<td>78</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>
**Insertion Sort**

We start to process index 1

<table>
<thead>
<tr>
<th>20</th>
<th>10</th>
<th>15</th>
<th>54</th>
<th>55</th>
<th>11</th>
<th>78</th>
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</tr>
</thead>
<tbody>
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<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
Insertion Sort

We add 10 to the sorted sub-section. To do so, we swap 10 and 20 since 10 < 20.

Since there are no elements smaller than 10 in the sub-section, we are done processing 10 and can move on to the next element.
Insertion Sort

We start to process index 2
**Insertion Sort**

We compare 15 to 20 and swap them since 15 < 20

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>54</th>
<th>55</th>
<th>11</th>
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</tr>
</thead>
<tbody>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
Insertion Sort

We compare 15 to the value at index 0

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>54</th>
<th>55</th>
<th>11</th>
<th>78</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 15 to the value at index 0

Since 10 < 15, we don’t swap.

We can now consider 15 as "integrated" into the sorted subsection
Insertion Sort

We start to process index 3
**Insertion Sort**

We compare 54 to the value at index 2

<table>
<thead>
<tr>
<th>10</th>
<th>15</th>
<th>20</th>
<th>54</th>
<th>55</th>
<th>11</th>
<th>78</th>
<th>14</th>
</tr>
</thead>
<tbody>
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<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
Insertion Sort

We compare 54 to the value at index 2

Since 20 < 54, we don't swap.

We can now consider 54 as "integrated" into the sorted subsection

<table>
<thead>
<tr>
<th>10</th>
<th>15</th>
<th>20</th>
<th>54</th>
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<th>11</th>
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<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
Insertion Sort

We start to process index 4

<table>
<thead>
<tr>
<th>10</th>
<th>15</th>
<th>20</th>
<th>54</th>
<th>55</th>
<th>11</th>
<th>78</th>
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<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 55 to the value at index 3

<table>
<thead>
<tr>
<th>10</th>
<th>15</th>
<th>20</th>
<th>54</th>
<th>55</th>
<th>11</th>
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<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
Insertion Sort

We compare 55 to the value at index 3

Since 54 < 55, we don’t swap.

We can now consider 55 as "integrated" into the sorted subsection

<table>
<thead>
<tr>
<th>10</th>
<th>15</th>
<th>20</th>
<th>54</th>
<th>55</th>
<th>11</th>
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<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
Insertion Sort

We start to process index 5

<table>
<thead>
<tr>
<th>10</th>
<th>15</th>
<th>20</th>
<th>54</th>
<th>55</th>
<th>11</th>
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<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
Insertion Sort

We compare 11 to the value at index 4
Insertion Sort

We compare 11 to the value at index 4

Since 11 < 55, we swap
**Insertion Sort**

We compare 11 to the value at index 3

Since 11 < 54, we swap

<table>
<thead>
<tr>
<th></th>
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<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 11 to the value at index 2

Since 11 < 20, we swap

<table>
<thead>
<tr>
<th>10</th>
<th>15</th>
<th>11</th>
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<th>55</th>
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</tr>
</tbody>
</table>
**Insertion Sort**

We compare 11 to the value at index 1

Since 11 < 15, we swap
**Insertion Sort**

We compare 11 to the value at index 0.

Since 10 < 11, we stop and consider the sub-section sorted.
**Insertion Sort**

We start to process index 6

<table>
<thead>
<tr>
<th>10</th>
<th>11</th>
<th>15</th>
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<td>7</td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 78 to the value at index 5

<table>
<thead>
<tr>
<th>10</th>
<th>11</th>
<th>15</th>
<th>20</th>
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<th>55</th>
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<td>7</td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 78 to the value at index 5

Since 55 < 78, we don’t swap.

We can now consider 78 as "integrated" into the sorted subsection

| 10 | 11 | 15 | 20 | 54 | 55 | 78 | 14 |

| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
Insertion Sort

We start to process index 7
**Insertion Sort**

We compare 14 to the value at index 6

<table>
<thead>
<tr>
<th>10</th>
<th>11</th>
<th>15</th>
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<td>7</td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 14 to the value at index 6

Since $14 < 78$, we swap the values

<table>
<thead>
<tr>
<th>10</th>
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<td>7</td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 14 to the value at index 5

Since 14 < 55, we swap the values

<table>
<thead>
<tr>
<th>10</th>
<th>11</th>
<th>15</th>
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<th>14</th>
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<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 14 to the value at index 4

Since 14 < 54, we swap the values
**Insertion Sort**

We compare 14 to the value at index 3

Since 14 < 20, we swap the values

<table>
<thead>
<tr>
<th>10</th>
<th>11</th>
<th>15</th>
<th>14</th>
<th>20</th>
<th>54</th>
<th>55</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
**Insertion Sort**

We compare 14 to the value at index 2.

Since 14 < 15, we swap the values.
Insertion Sort

We compare 14 to the value at index 1

Since 11 < 14, we stop

Since 14 was the last value, the array is now sorted

\[
\begin{array}{cccccccc}
10 & 11 & 14 & 15 & 20 & 54 & 55 & 78 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array}
\]
**Insertion Sort: Summary**

For each unsorted element, swap current element with its predecessor, if out of order. Repeat until the array is sorted.
Insertion Sort

public static void insertionSort(Comparable[] array) {
    for (int i = 1; i < array.length; i++) {
        for (int j = i; (j > 0) && (array[j].compareTo(array[j-1]) < 0); j--) {
            Comparable temp = array[j];
            array[j] = array[j-1];
            array[j-1] = temp;
        }
    }
}

- Why does the outer loop start at 1?
- Why does the inner loop start at \(i\)?
- Why does the inner loop have the continuation condition \((j > 0) && (array[j].compareTo(array[j-1]) < 0)\)?
Insertion Sort

```java
public static void insertionSort(Comparable[] array) {
    for (int i = 1; i < array.length; i++) {
        for (int j = i; (j > 0) && (array[j].compareTo(array[j-1]) < 0); j--) {
            Comparable temp = array[j];
            array[j] = array[j-1];
            array[j-1] = temp;
        }
    }
}
```

- The first element is always sorted with respect to itself.
- We want to compare the first unsorted element to sorted elements only.
- We can stop as soon as we find an element that is smaller than the unsorted element. All other elements to the left will also be smaller.
Poll:

If we are doing Insertion Sort an array of size 5, what is the least number of comparisons that could be done during the sort?

```java
public static void insertionSort(Comparable[] array) {
    for (int i = 1; i < array.length; i++) {
        for (int j = i; (j > 0) && (array[j].compareTo(array[j-1]) < 0); j--) {
            Comparable temp = array[j];
            array[j] = array[j-1];
            array[j-1] = temp;
        }
    }
}
```

What would the input array look like to cause this case?
Poll:

If we are doing Insertion Sort an array of size 5, what is the least number of comparisons that could be done during the sort? **Four.**

```java
public static void insertionSort(Comparable[] array) {
    for (int i = 1; i < array.length; i++) {
        for (int j = i; (j > 0) && (array[j].compareTo(array[j-1]) < 0); j--) {
            Comparable temp = array[j];
            array[j] = array[j-1];
            array[j-1] = temp;
        }
    }
}
```

What would the input array look like to cause this case? **A sorted array!**
Poll:

If we are doing Insertion Sort an array of size 5, what is the **MOST** number of comparisons that could be done during the sort?

```
public static void insertionSort(Comparable[] array) {
    for (int i = 1; i < array.length; i++) {
        for (int j = i; (j > 0) && (array[j].compareTo(array[j-1]) < 0); j--) {
            Comparable temp = array[j];
            array[j] = array[j-1];
            array[j-1] = temp;
        }
    }
}
```

What would the input array look like to cause this case?
Poll:

If we are doing Insertion Sort an array of size 5, what is the MOST number of comparisons that could be done during the sort? **Ten.**

```java
public static void insertionSort(Comparable[] array) {
    for (int i = 1; i < array.length; i++) {
        for (int j = i; (j > 0) && (array[j].compareTo(array[j-1]) < 0); j--) {
            Comparable temp = array[j];
            array[j] = array[j-1];
            array[j-1] = temp;
        }
    }
}
```

What would the input array look like to cause this case? **An array that is in descending order.**
**Selection Sort**

Find the $i$th smallest value, and put it at index $i$.

- Selection sort finds the smallest element in the array and place it at position 0
- then finds the smallest element in the array starting at index 1, and places it at position 1
- then finds the smallest element in the array starting at index 2, and places it at position 2

And so on, until the entire array is sorted
Selection Sort

Start by trying to find the index of the smallest value

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>15</th>
<th>54</th>
<th>55</th>
<th>11</th>
<th>78</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Index of smallest value: 1  
Destination Index: 0
**Selection Sort**

Once the index of smallest is found, swap it with index 0.

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>20</th>
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<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
Selection Sort

Find the next smallest value of the array

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>20</th>
<th>15</th>
<th>54</th>
<th>55</th>
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<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Index of smallest value: 5
Destination Index: 1
Selection Sort

Once the next smallest is found, swap it with index 1

- Repeat until the (length-1)th smallest value is found and swapped
- (We've only sorted two elements—six more to go!)

<table>
<thead>
<tr>
<th>10</th>
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<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
**Selection Sort**

Selection Sort

- We initialize the position of the smallest element
- We update indexOfSmallest if we found a smaller element
- We place the smallest element at the right position
Selection Sort Code

```java
public static void selectionSort(String[] array) {
    for (int i = 0; i < array.length - 1; i++) {
        int indexOfSmallest = i;
        for (int j = i + 1; j < array.length; j++) {
            if (array[j].compareTo(array[indexOfSmallest]) < 0) {
                indexOfSmallest = j;
            }
        }
        String temp = array[indexOfSmallest];
        array[indexOfSmallest] = array[i];
        array[i] = temp;
    }
}
```

Why do we stop the outer loop at `array.length - 1`?
Why do we start the inner loop at `i + 1`?
Selection Sort Code

```java
public static void selectionSort(String[] array) {
    for (int i = 0; i < array.length - 1; i++) {
        int indexOfSmallest = i;
        for (int j = i + 1; j < array.length; j++) {
            if (array[j].compareTo(array[indexOfSmallest]) < 0) {
                indexOfSmallest = j;
            }
        }
        String temp = array[indexOfSmallest];
        array[indexOfSmallest] = array[i];
        array[i] = temp;
    }
}
```

There's no need to "sort" the last element—it'll just be the biggest.
The sorted portion of the array can always be found up to position \(i\), and we start `indexOfSmallest` at position \(i\).
Poll:

If we are doing Selection Sort an array of size 5, what is the least number of comparisons that could be done during the sort? What would the input array look like?

```java
public static void selectionSort(String[] array) {
    for (int i = 0; i < array.length - 1; i++) {
        int indexOfSmallest = i;
        for (int j = i + 1; j < array.length; j++) {
            if (array[j].compareTo(array[indexOfSmallest]) < 0) {
                indexOfSmallest = j;
            }
        }
        String temp = array[indexOfSmallest];
        array[indexOfSmallest] = array[i];
        array[i] = temp;
    }
}
```
These sorting algorithms are actually rather slow in practice. Java's built-in `Arrays.sort()` and `Collections.sort()` (for Lists) are much faster.

- refer to previous slide deck for examples of how to use
- these methods use Timsort, which is like a hybrid of Insertion and Merge sort.