Programming Languages and Techniques (CIS120)

Lecture 15

Feb 17, 2012

Iteration

Announcements

- Homework 5 is available on the web
  - due February 23rd at 11:59:59pm
  - make sure your version doesn't include "counter"
- Midterm 1 grades can be checked via the "check your grades link" from the HW web pages:
  - Class Average: ~82/100, Median: ~86/100
  - *Rough* grade distribution:

    >= 90 A

    >= 75 B

    >= 60 C
  - Solutions online now
  - Check your exams with Laura Fox starting Monday (Levine 308)
  - Regrades in writing (deadline: March 16th)

Queue Interface

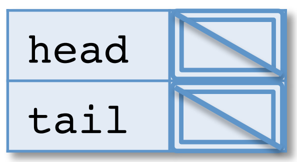
```
module type QUEUE =
sig
  (* type of the data structure *)
  type 'a queue

  (* Make a new, empty queue *)
  val create : unit -> 'a queue

  (* Determine if the queue is empty *)
  val is_empty : 'a queue -> bool

  (* Add a value to the tail of the queue *)
  val enq : 'a -> 'a queue -> unit

  (* Remove the head value and return it (if any) *)
  val deq : 'a queue -> 'a

end
```

Datatypes for Mutable Queues

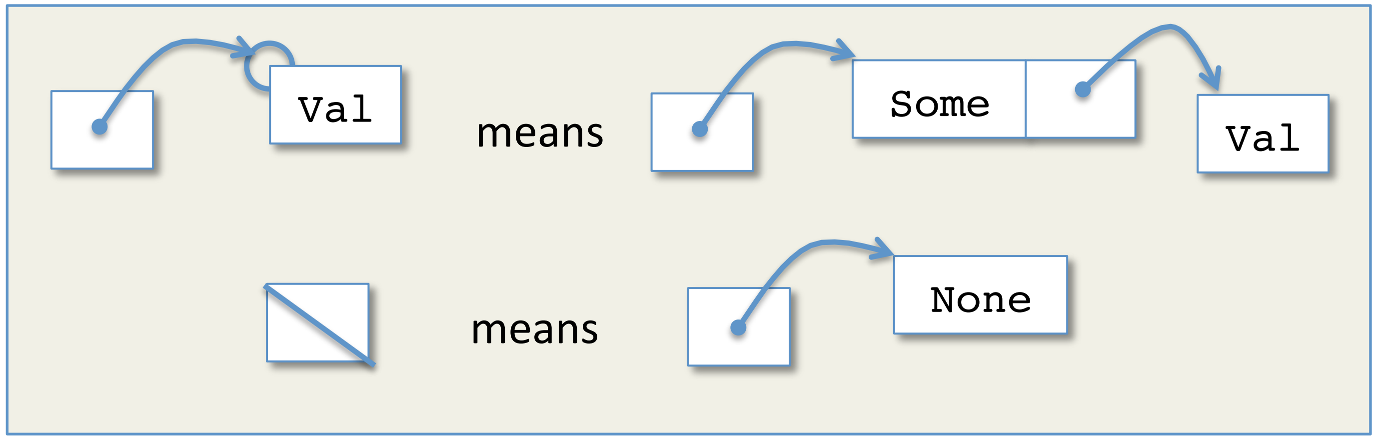
```
type 'a qnode = {
     v: 'a;
     mutable next : 'a qnode option
}

type 'a queue = { mutable head : 'a qnode option;
                  mutable tail : 'a qnode option }
```

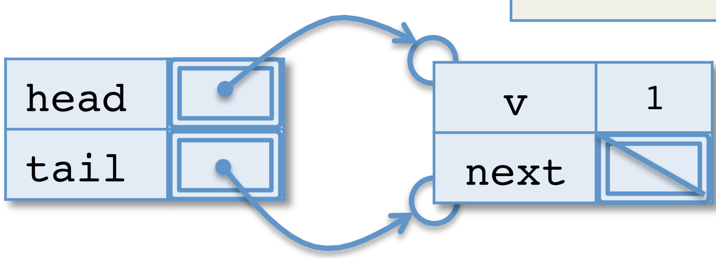
There are two parts to a mutable queue:
- the “internal nodes” of the queue with links from one to the next
- the head and tail references themselves

Example Queues in the Heap

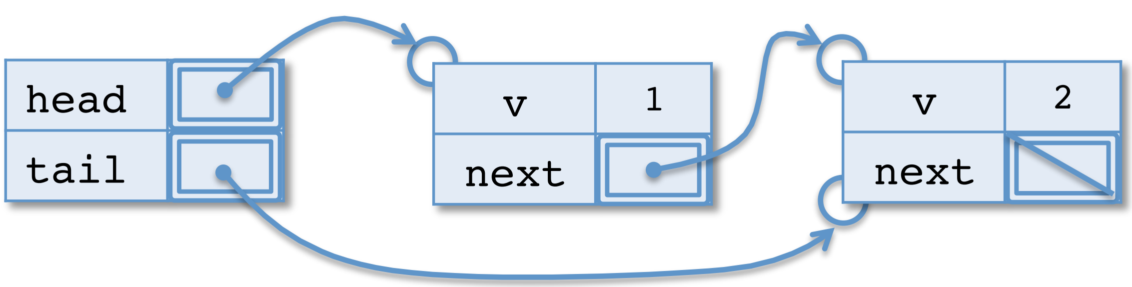


An empty queue





A queue with one element



A queue with two elements

Queue Invariants

- Just as we imposed some restrictions on which trees are legitimate Binary Search Trees, Queues must also satisfy some *invariants*:

Either:
(1) `head` and `tail` are both `None` (i.e. the queue is empty)
or
(2) `head` is `Some n1`, `tail` is `Some n2` and
- `n2` is reachable from `n1` by following 'next' pointers
- `n2.next` is `None`

- We can check that these properties rule out "bogus" examples.
- A queue operation may assume that these queue invariants hold of its inputs, so long as it ensures that the invariants hold when it's done.

enq

```
(* add an element to the tail of a queue *)
let enq (x: 'a) (q: 'a queue) : unit =
   let newnode = {v=x; next=None} in
   begin match q.tail with
     | None ->
         (* Note that the invariant tells us
           that q.head is also None *)
         q.head <- Some newnode;
         q.tail <- Some newnode
     | Some n ->
         n.next <- Some newnode;
         q.tail <- Some newnode
   end
```

- The code for enq is informed by the queue invariant:
  - either the queue is empty, and we just update head and tail, or
  - the queue is non-empty, in which case we have to “patch up” the “next” link of the old tail node to maintain the queue invariant.

Queue Length

- Suppose we want to extend the interface with a length function:

```
module type QUEUE =
sig
  (* type of the data structure *)
  type 'a queue

  …

  (* Get the length of the queue *)
  val length : 'a queue -> int
end
```

- How can we implement it?

`length` (recursively)

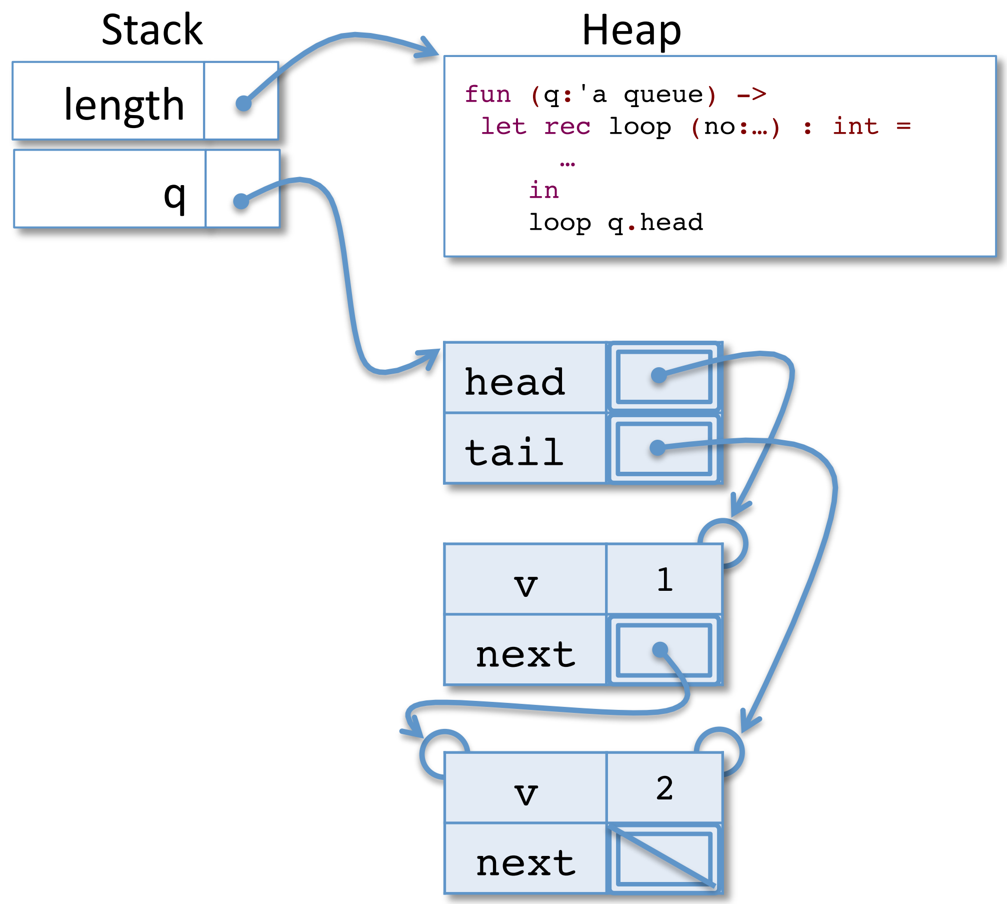
```
(* Calculate the length of the list using recursion *)
let length (q:'a queue) : int =
  let rec loop (no: 'a qnode option) : int =
    begin match no with
      | None -> 0
      | Some n -> 1 + (loop n.next)
    end
  in
  loop q.head
```

- This code for `length` uses a helper function, `loop`:
  - the correctness depends crucially on the queue invariant
  - what happens if we pass in a bogus q that is cyclic?
- The height of the ASM stack is proportional to the length of the queue
  - That seems inefficient… why should it take so much space?

Evaluating length

Workspace

```
length q
```



Evaluating length

Workspace

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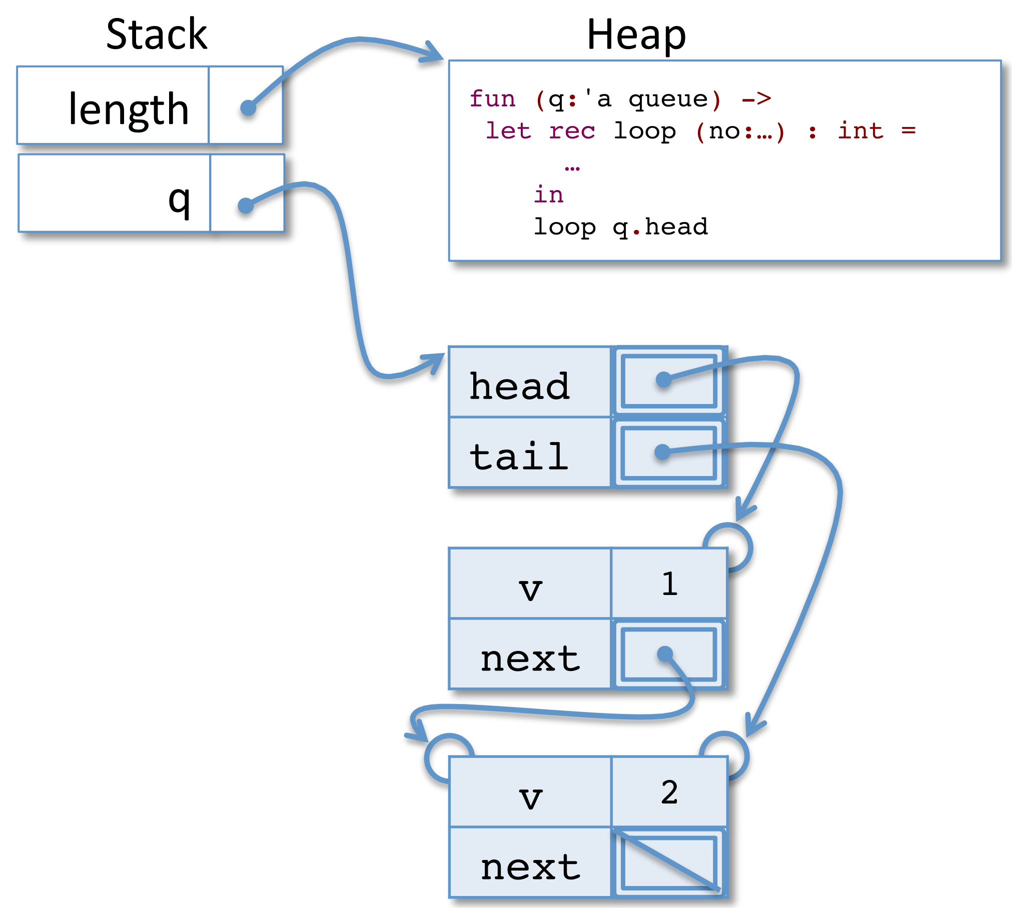
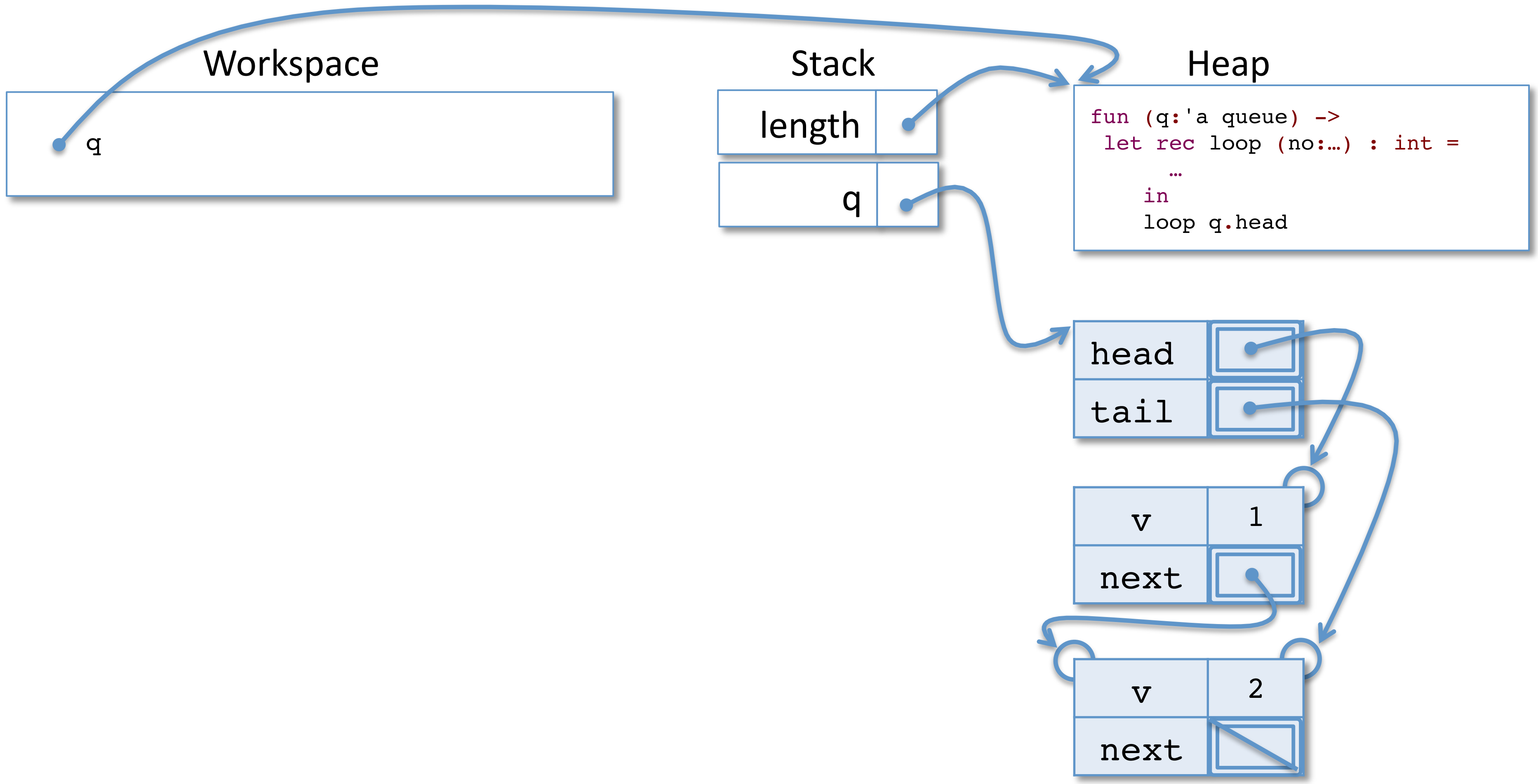
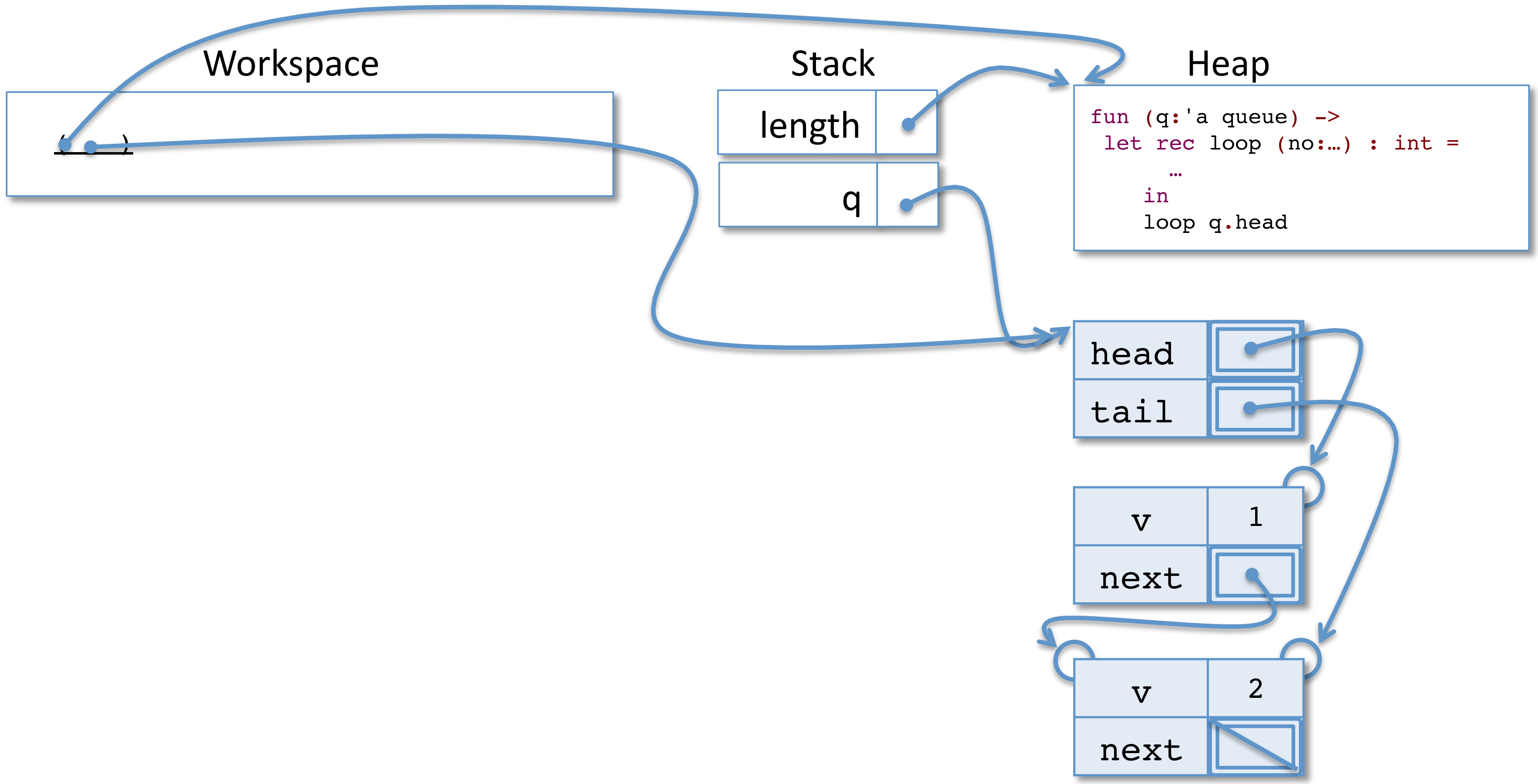


Evaluating length



Evaluating length



Evaluating length

Workspace

```
let rec loop (no: …) : int =
      begin match no with
         | None -> 0
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      end
    in
    loop q.head
```

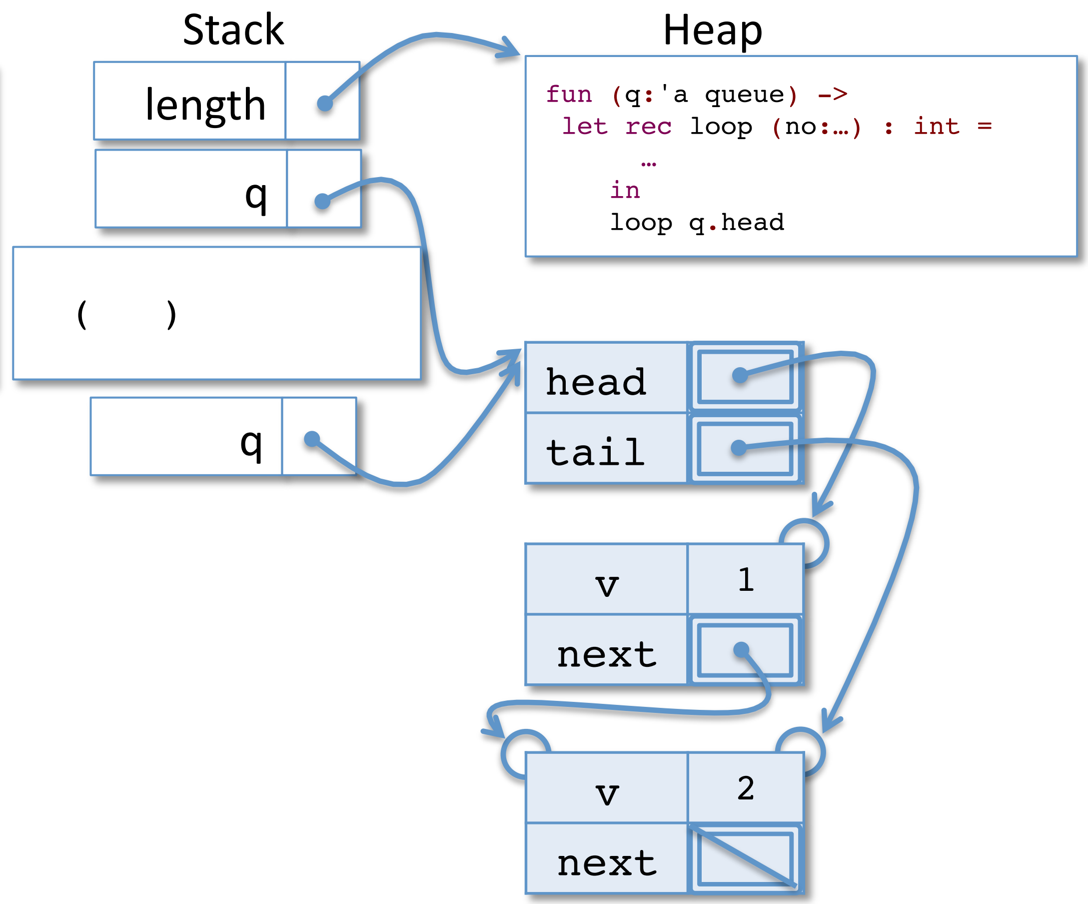

Evaluating length

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let loop = fun (no: …) ->
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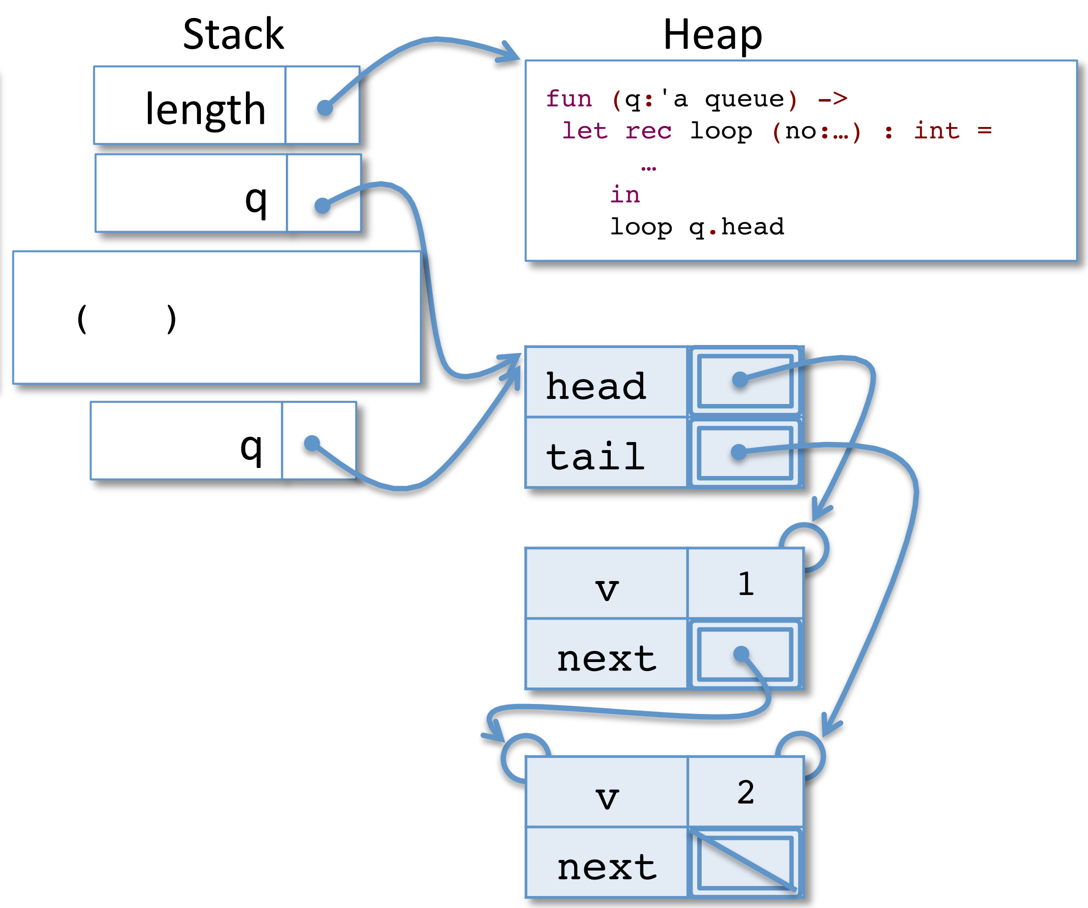

Evaluating length

Workspace

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      end
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    loop q.head
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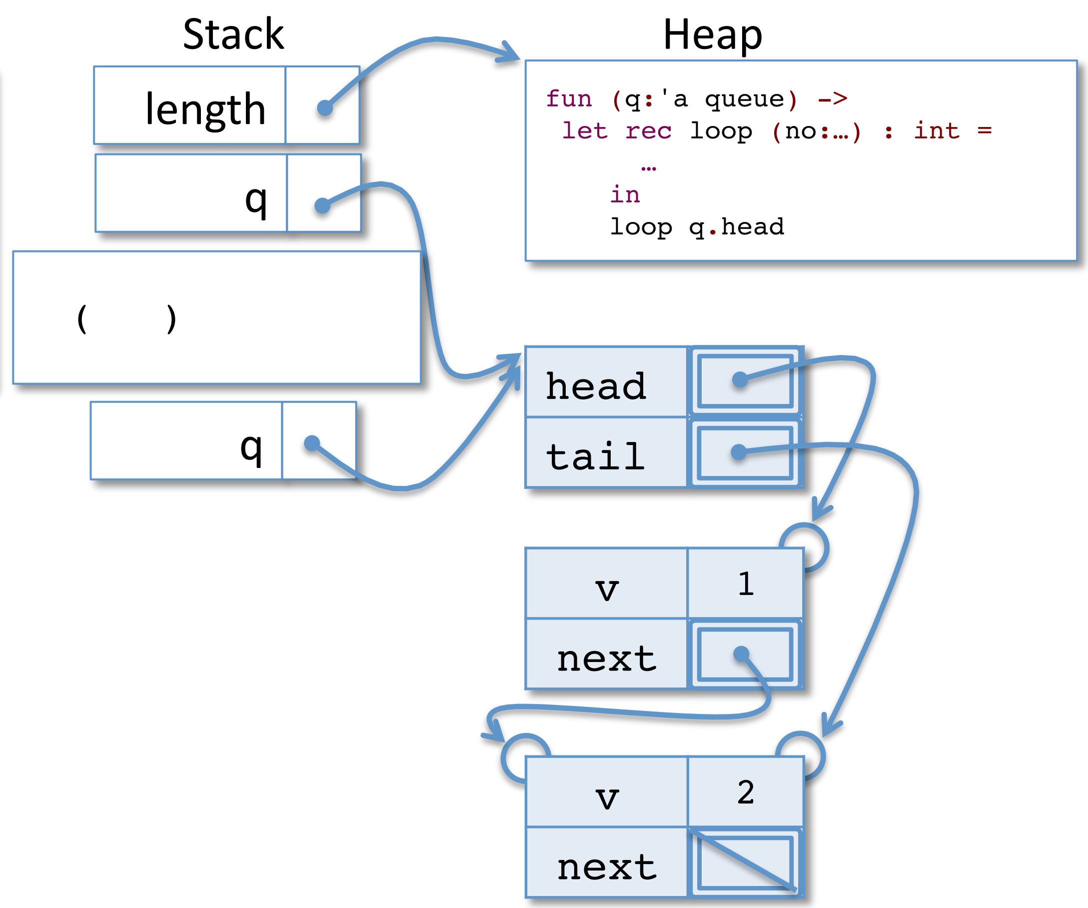
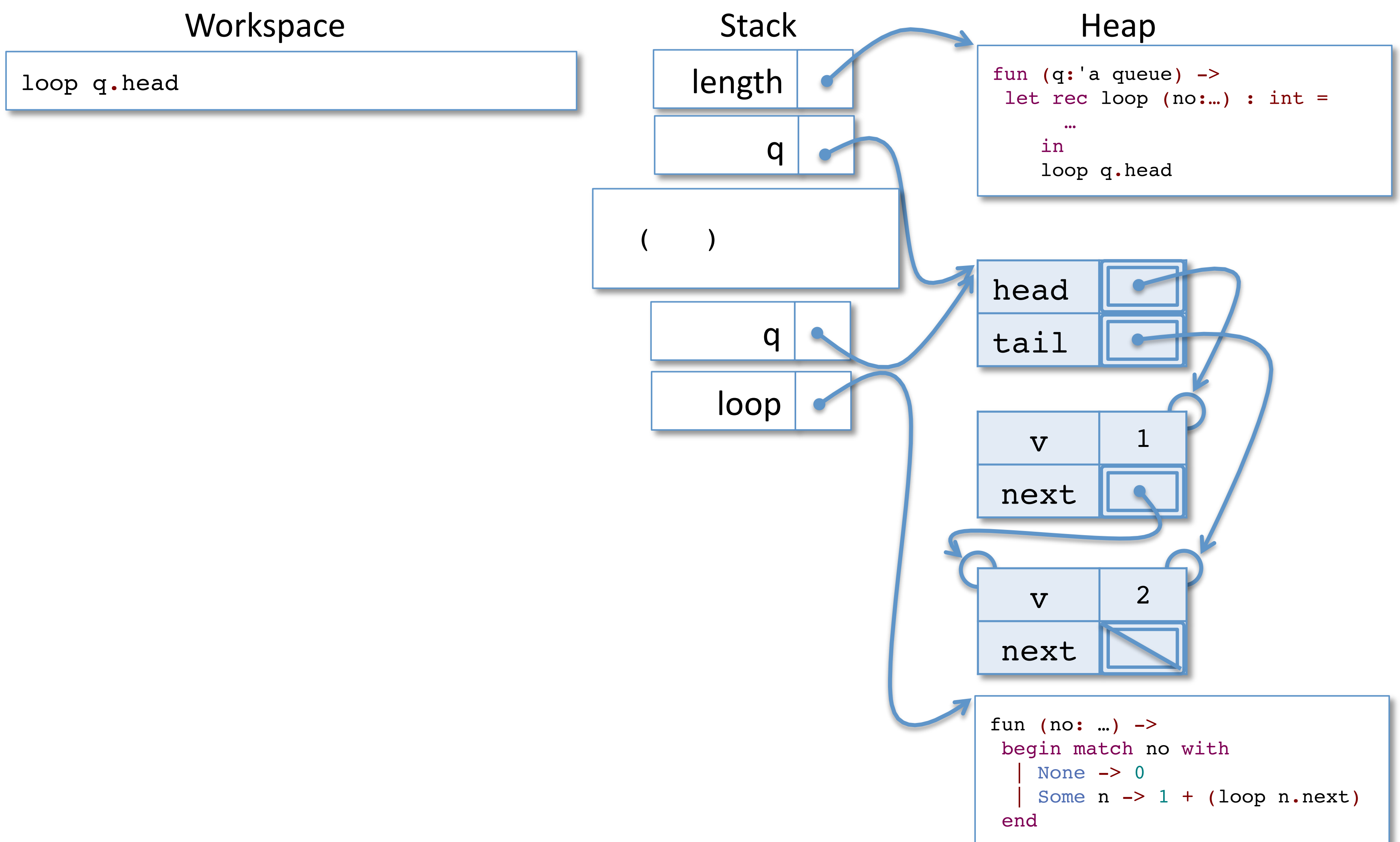
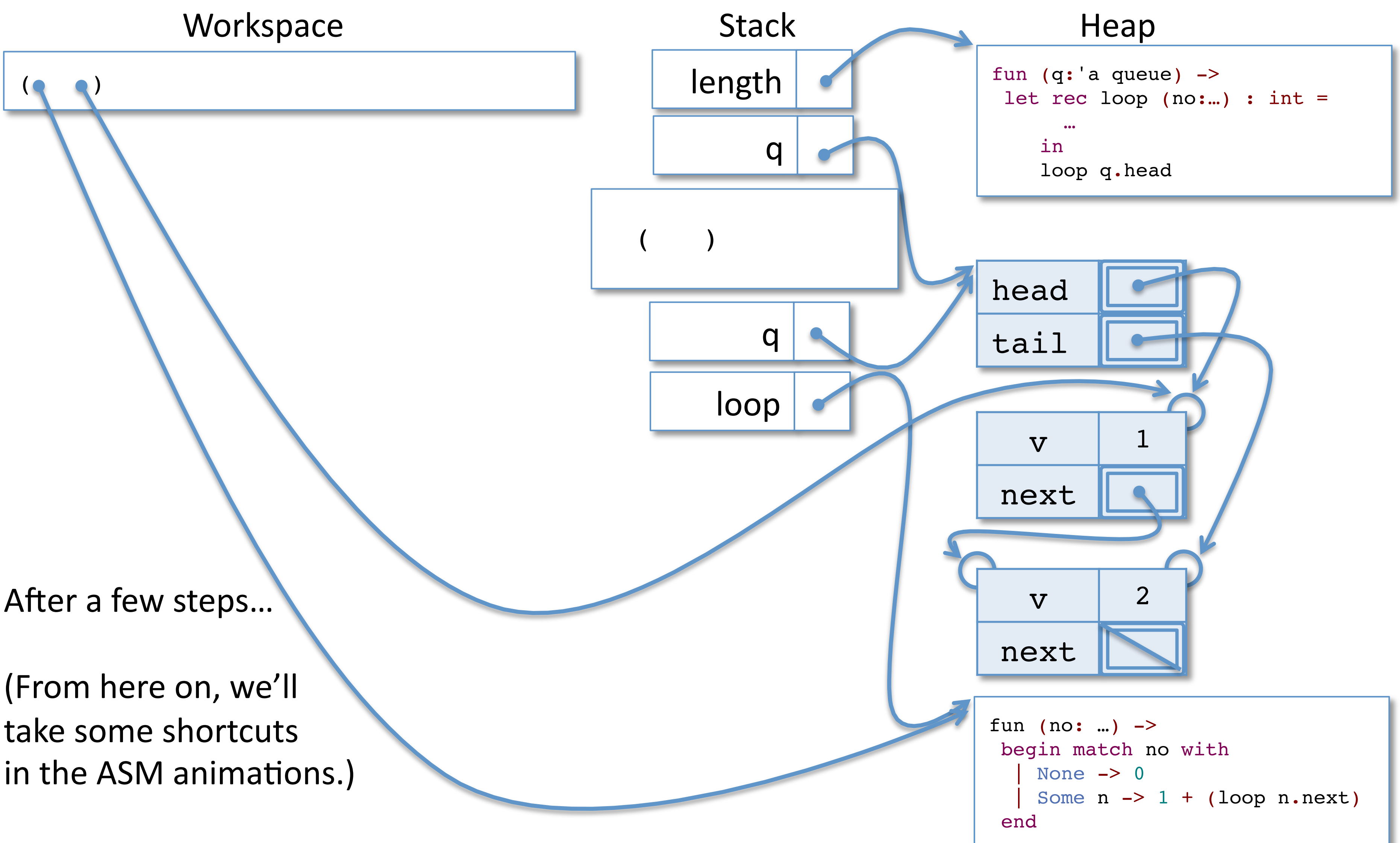


Evaluating length



Evaluating length



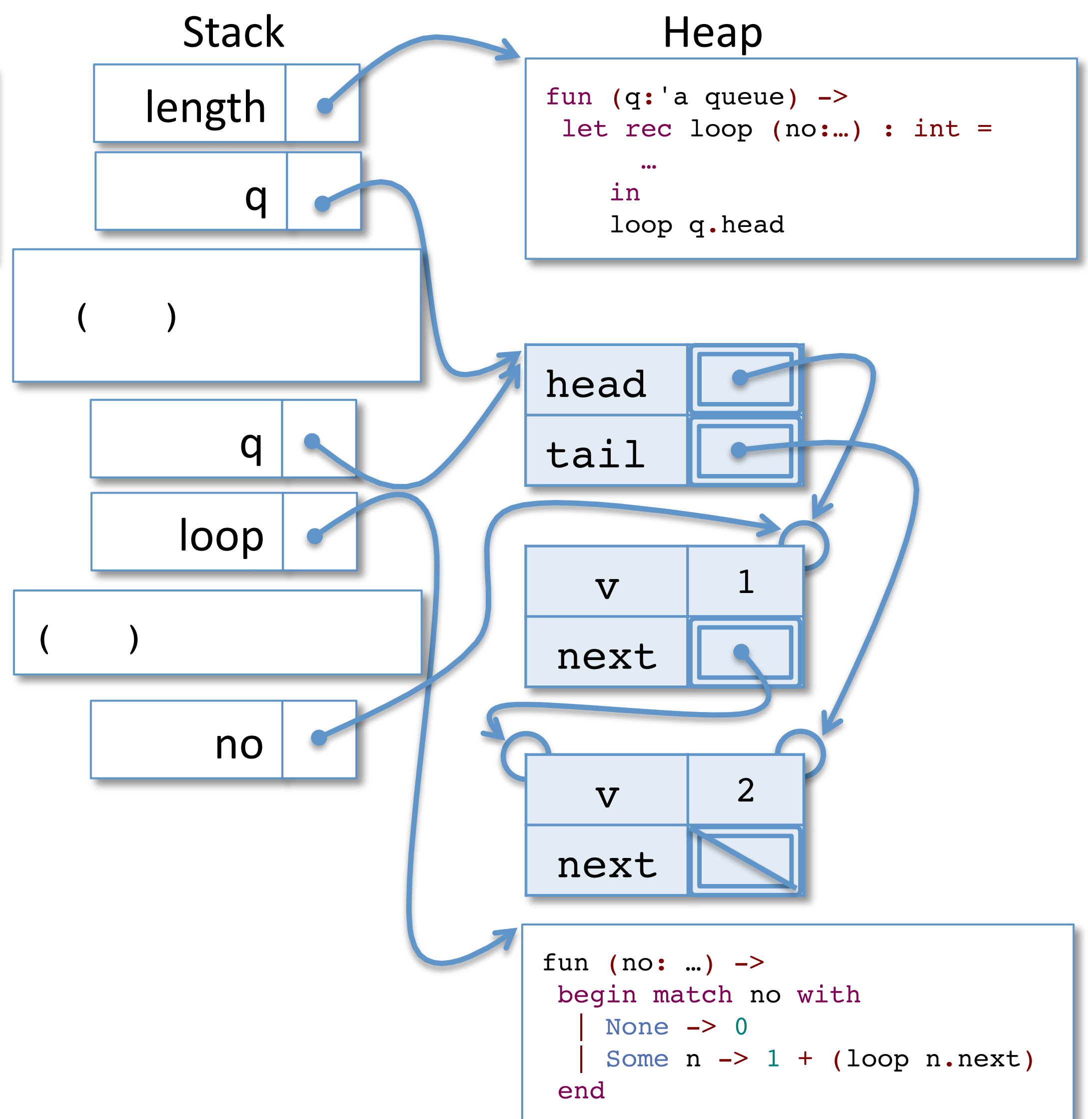
After a few steps…

(From here on, we'll take some shortcuts in the ASM animations.)

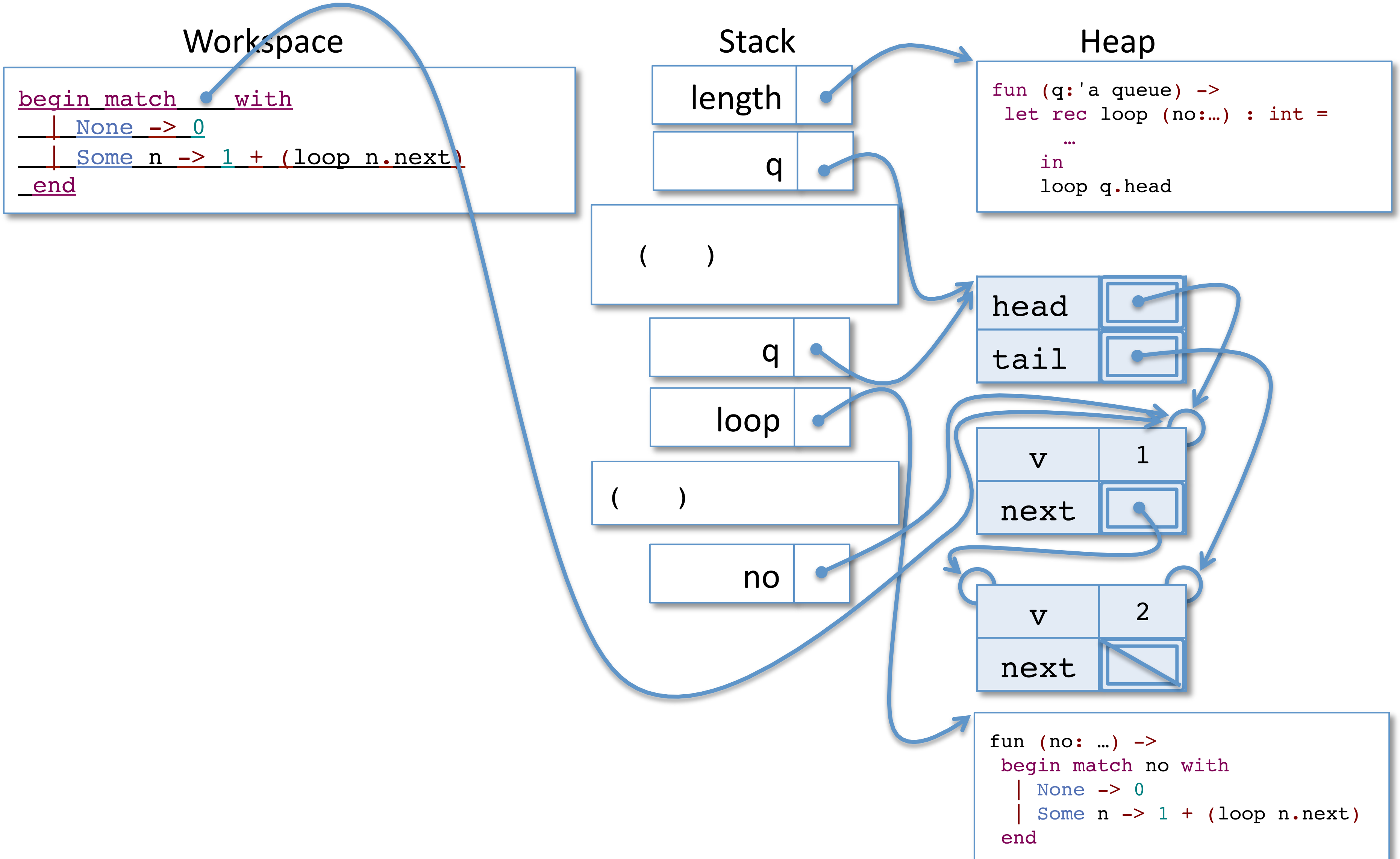
Evaluating length

Workspace

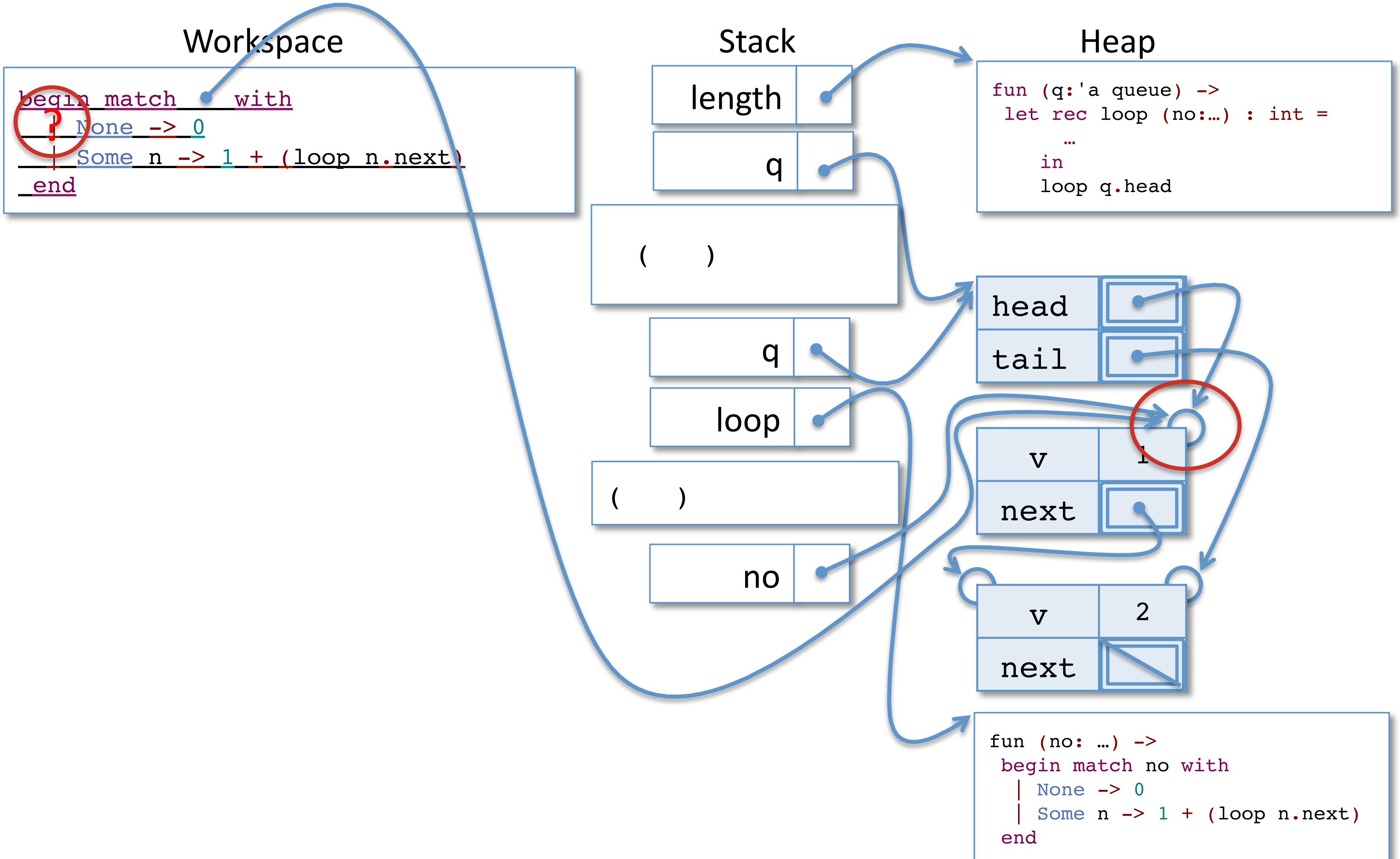
```
begin match no with
  | None -> 0
  | Some n -> 1 + (loop n.next)
 end
```



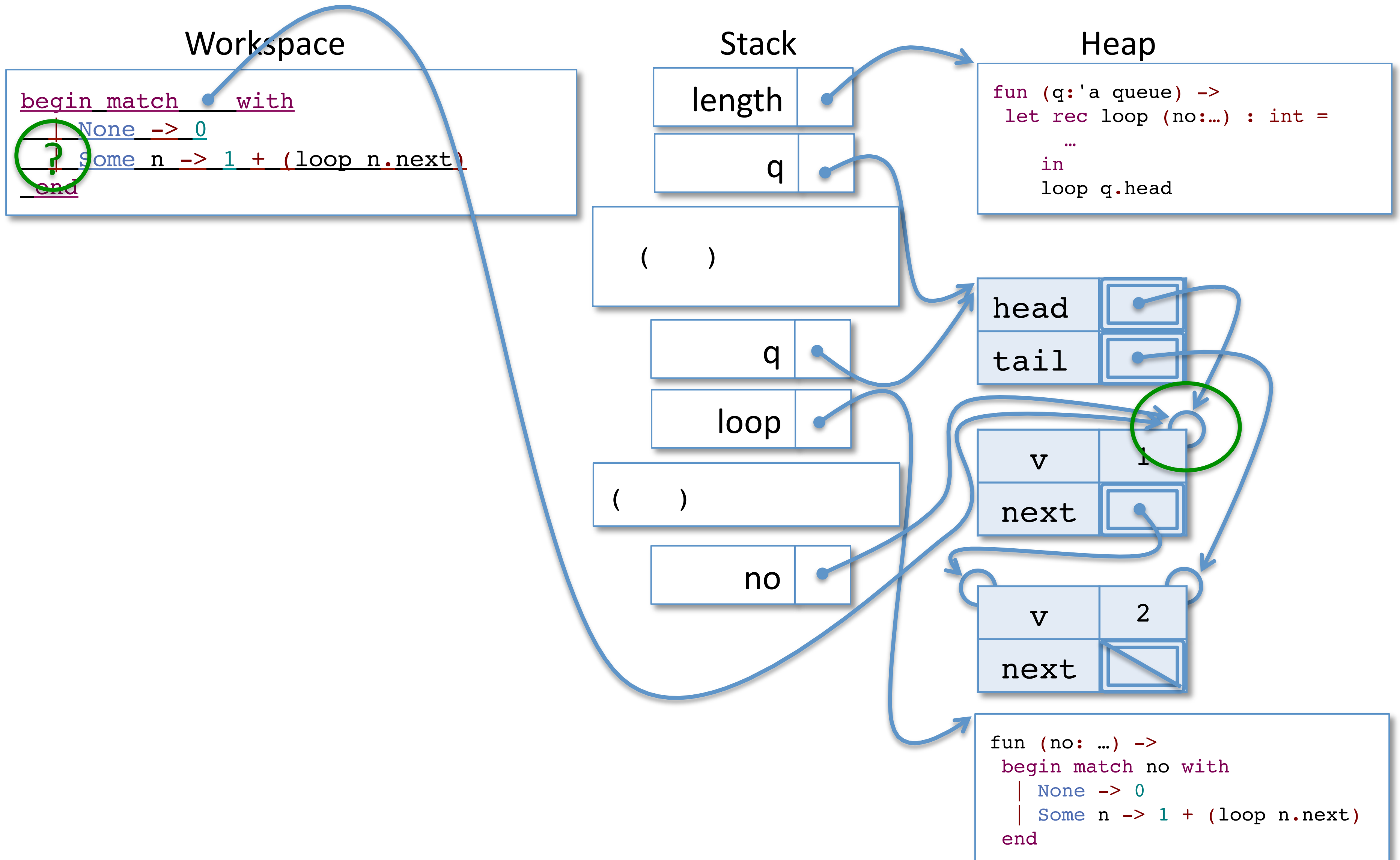
Evaluating length



Evaluating length



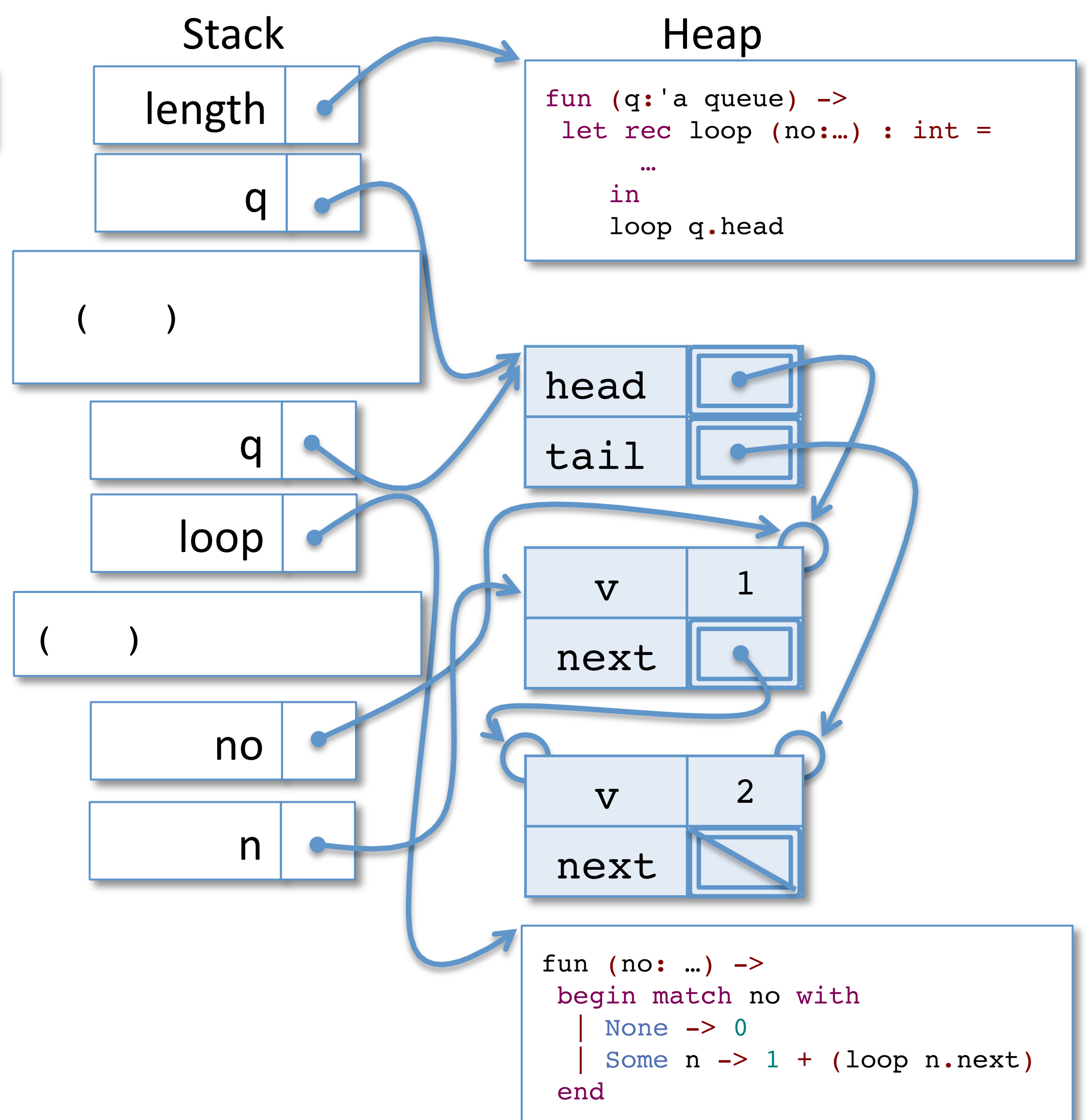
Evaluating length



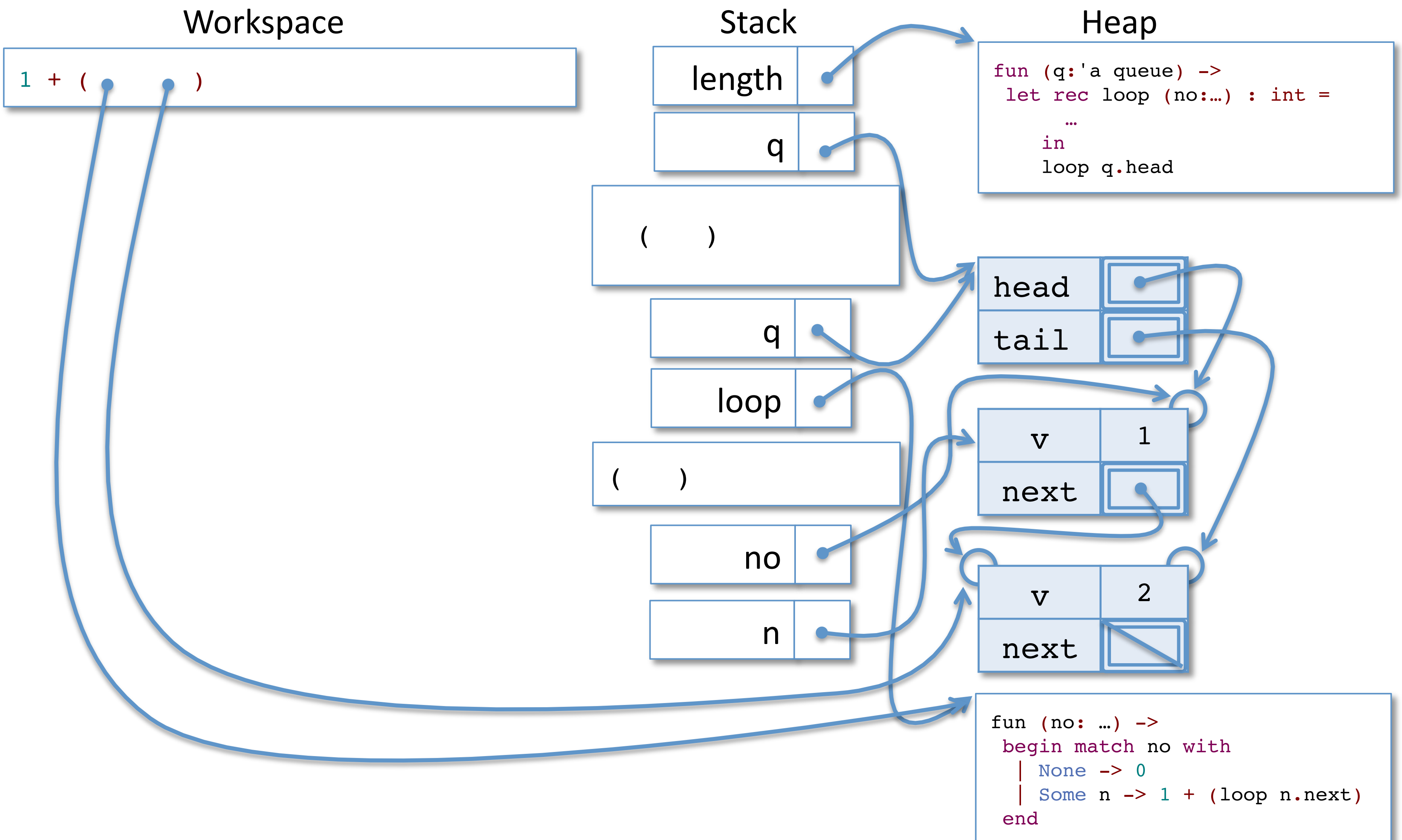
Evaluating length

Workspace

```
1 + (loop n.next)
```



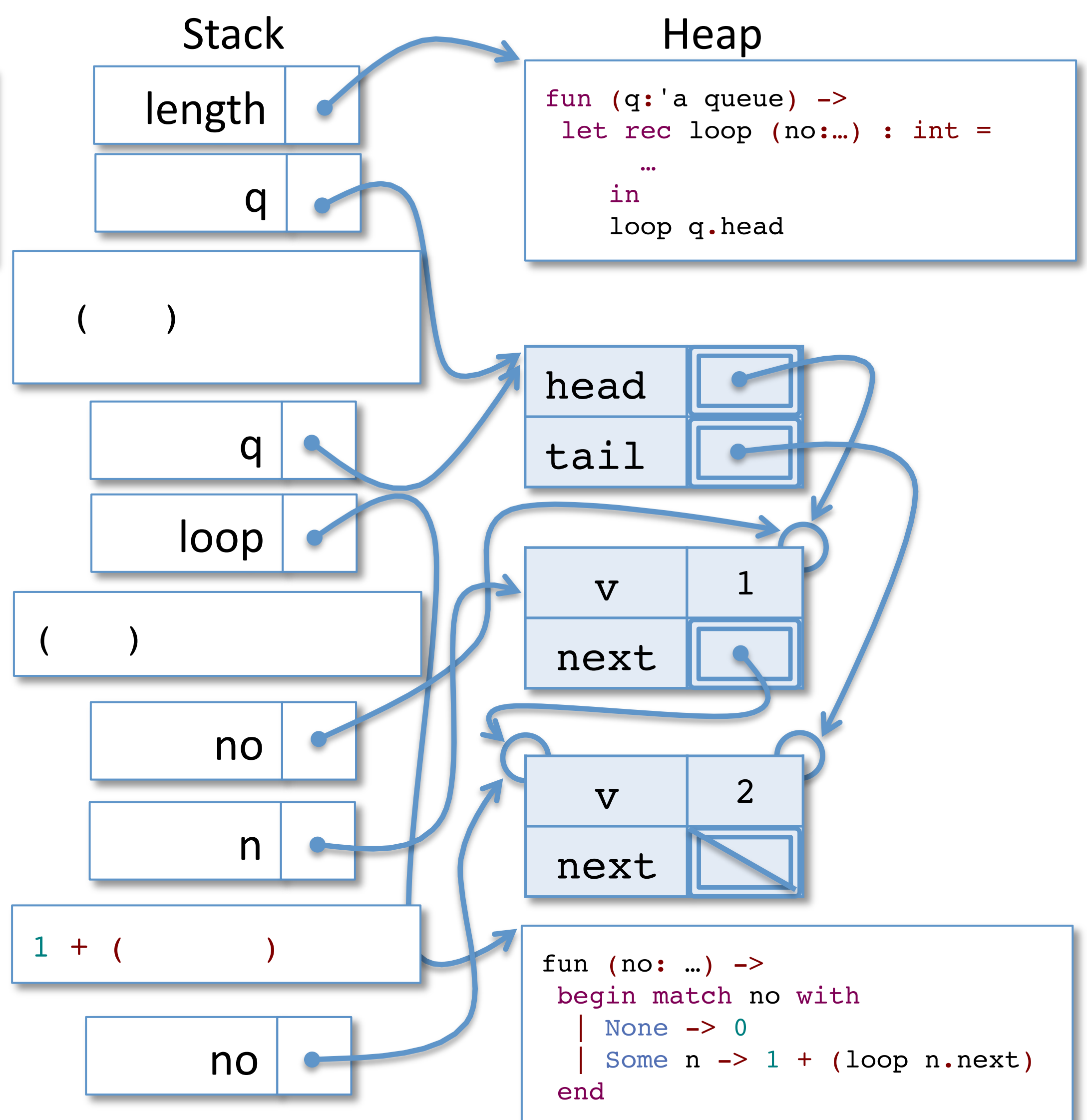
Evaluating length



Evaluating length

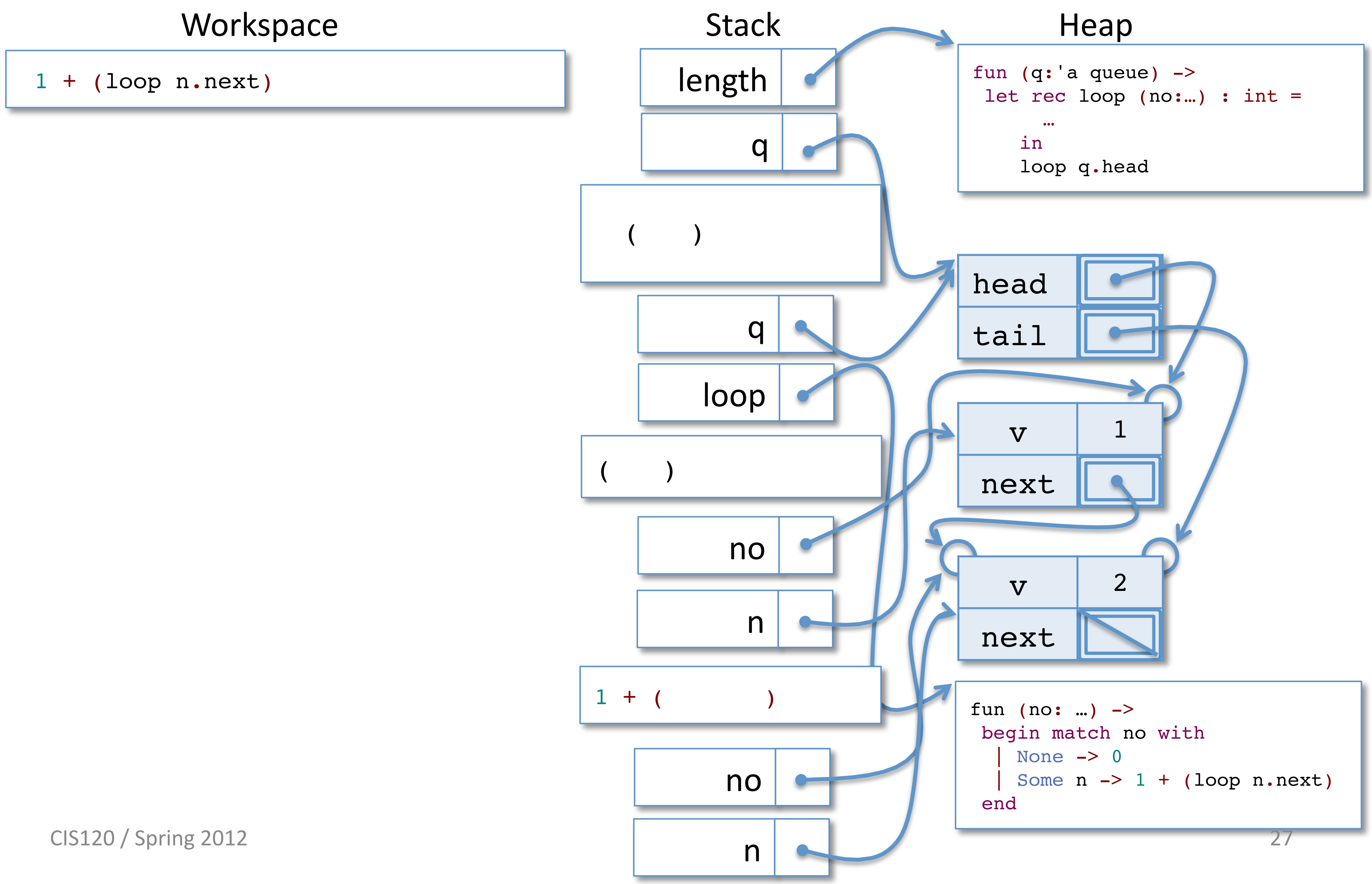
Workspace

```
begin match no with
  | None -> 0
  | Some n -> 1 + (loop n.next)
 end
```



…after a few steps…

Evaluating length

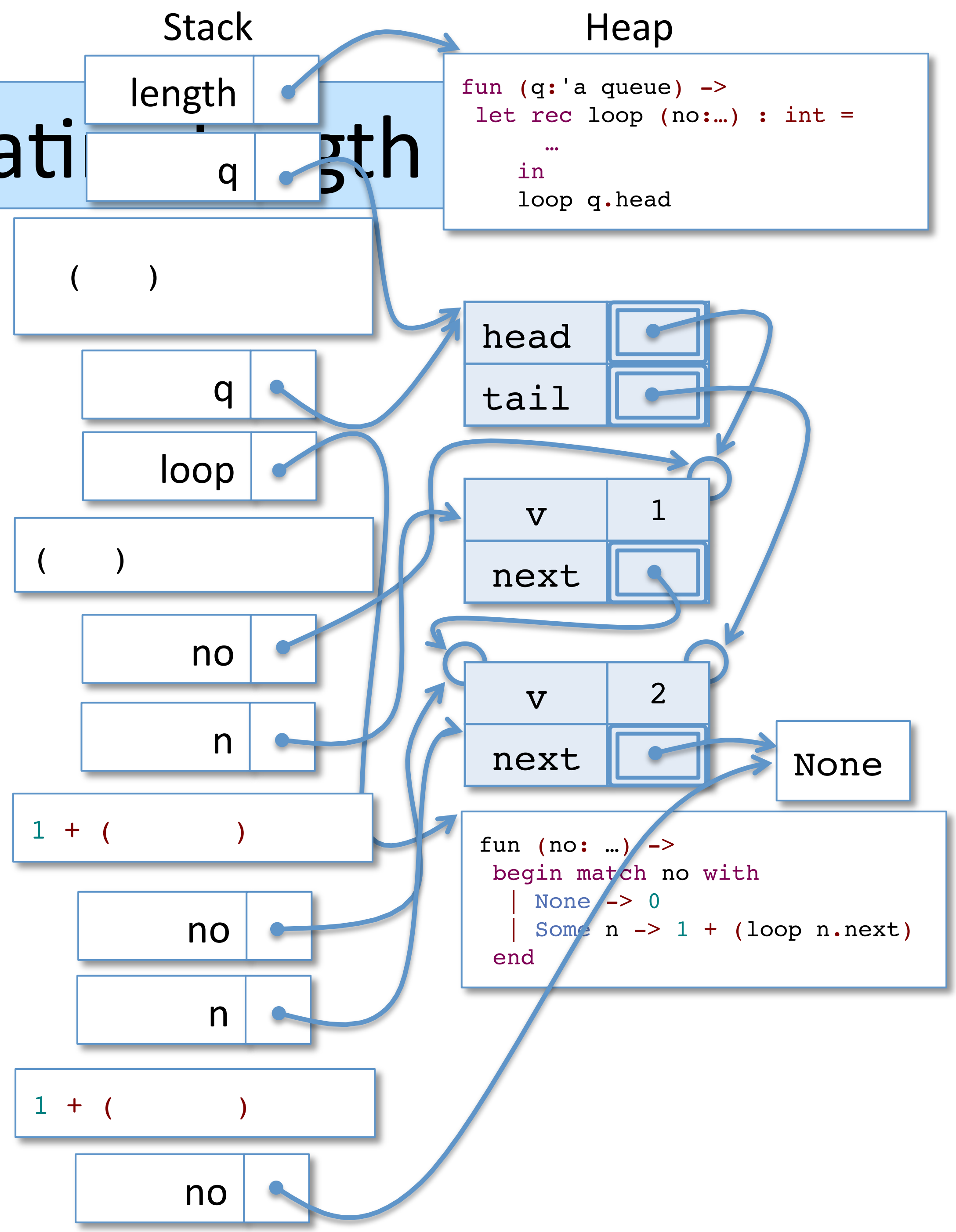


…after a few more steps…

Evaluati gth

Workspace

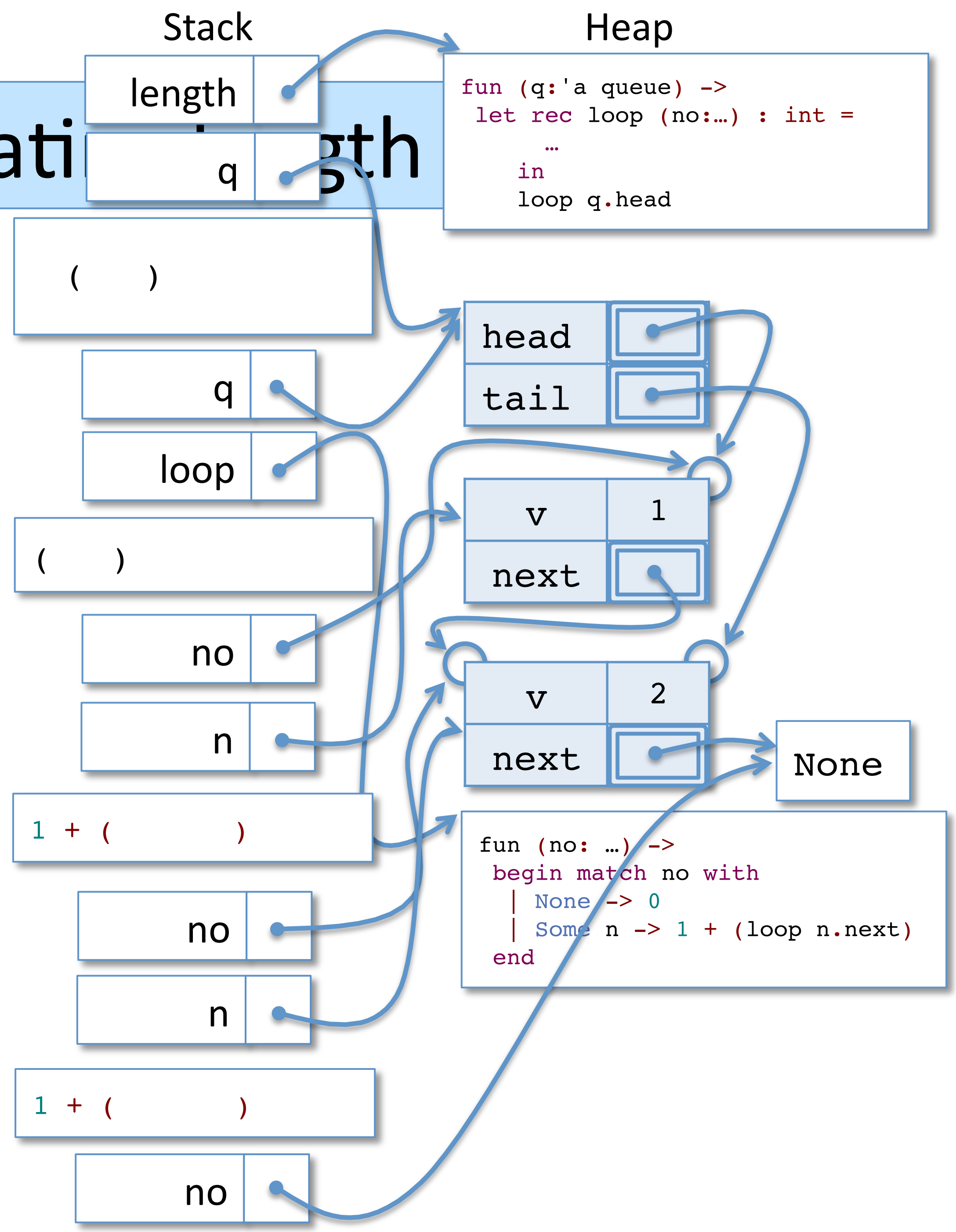
```
begin match no with
  | None -> 0
  | Some n -> 1 + (loop n.next)
end
```



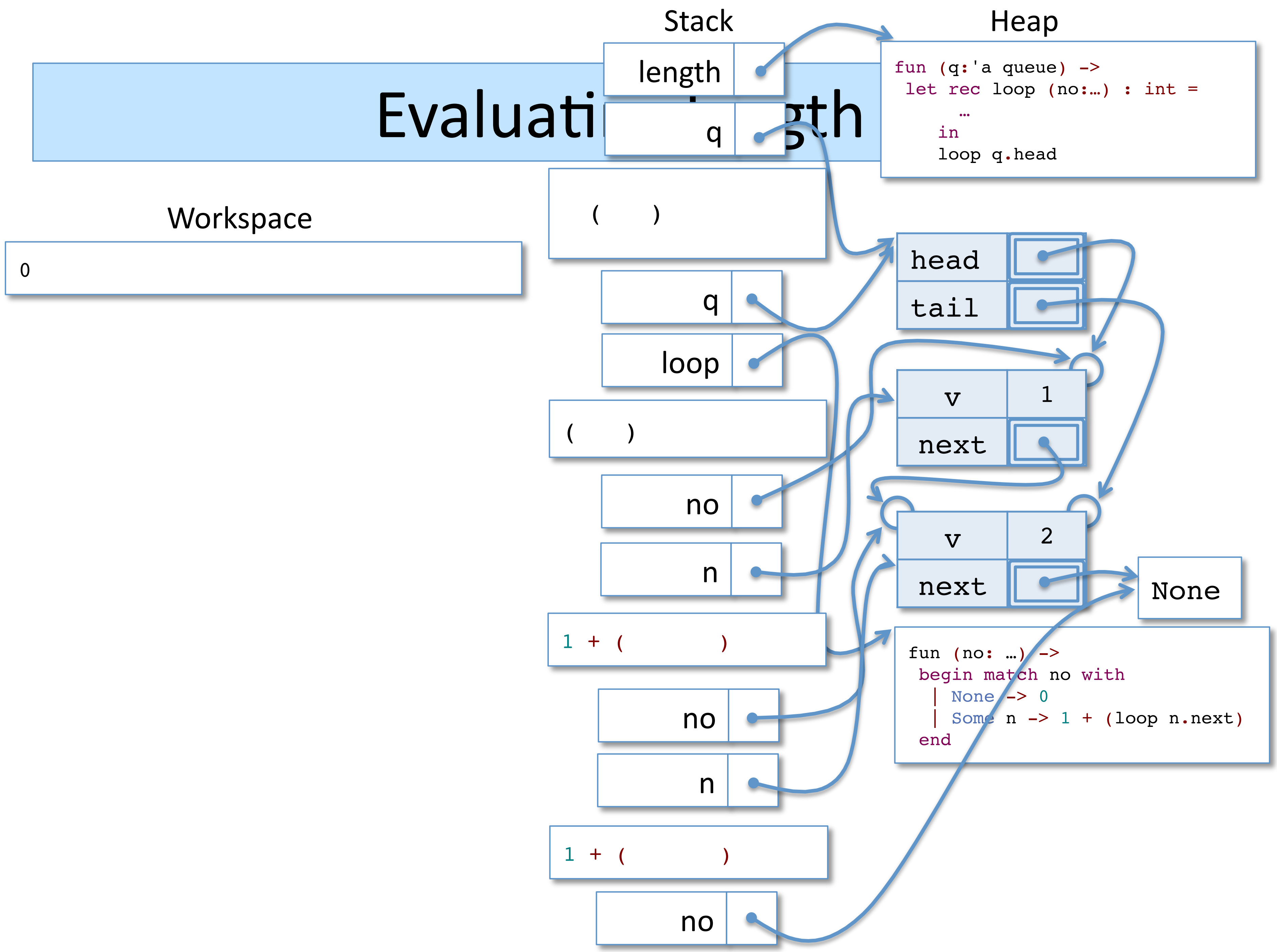
Evaluati gth

Workspace

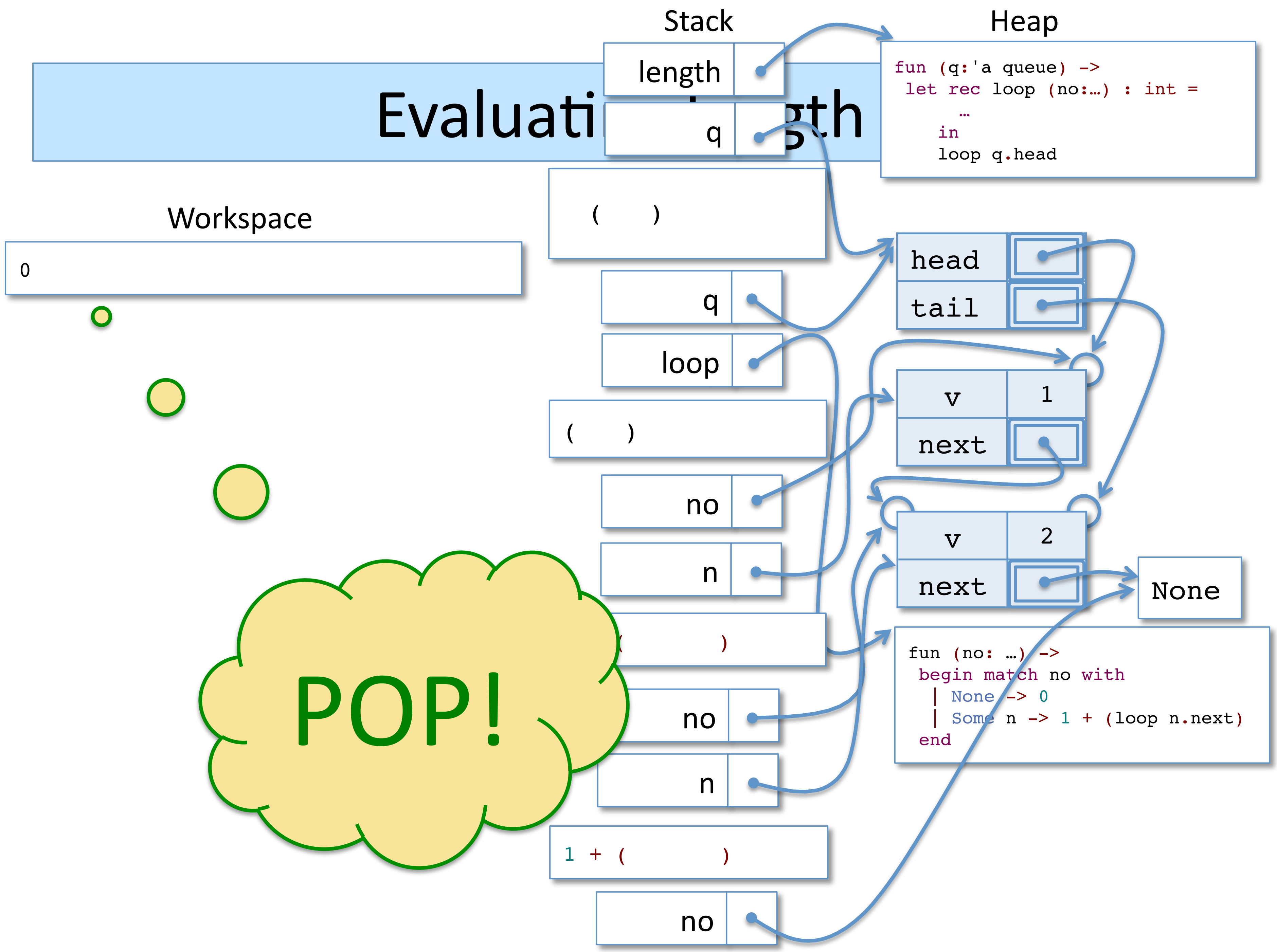
```
begin match no with
  | None -> 0
  | Some n -> 1 + (loop n.next)
 end
```



Evaluati gth



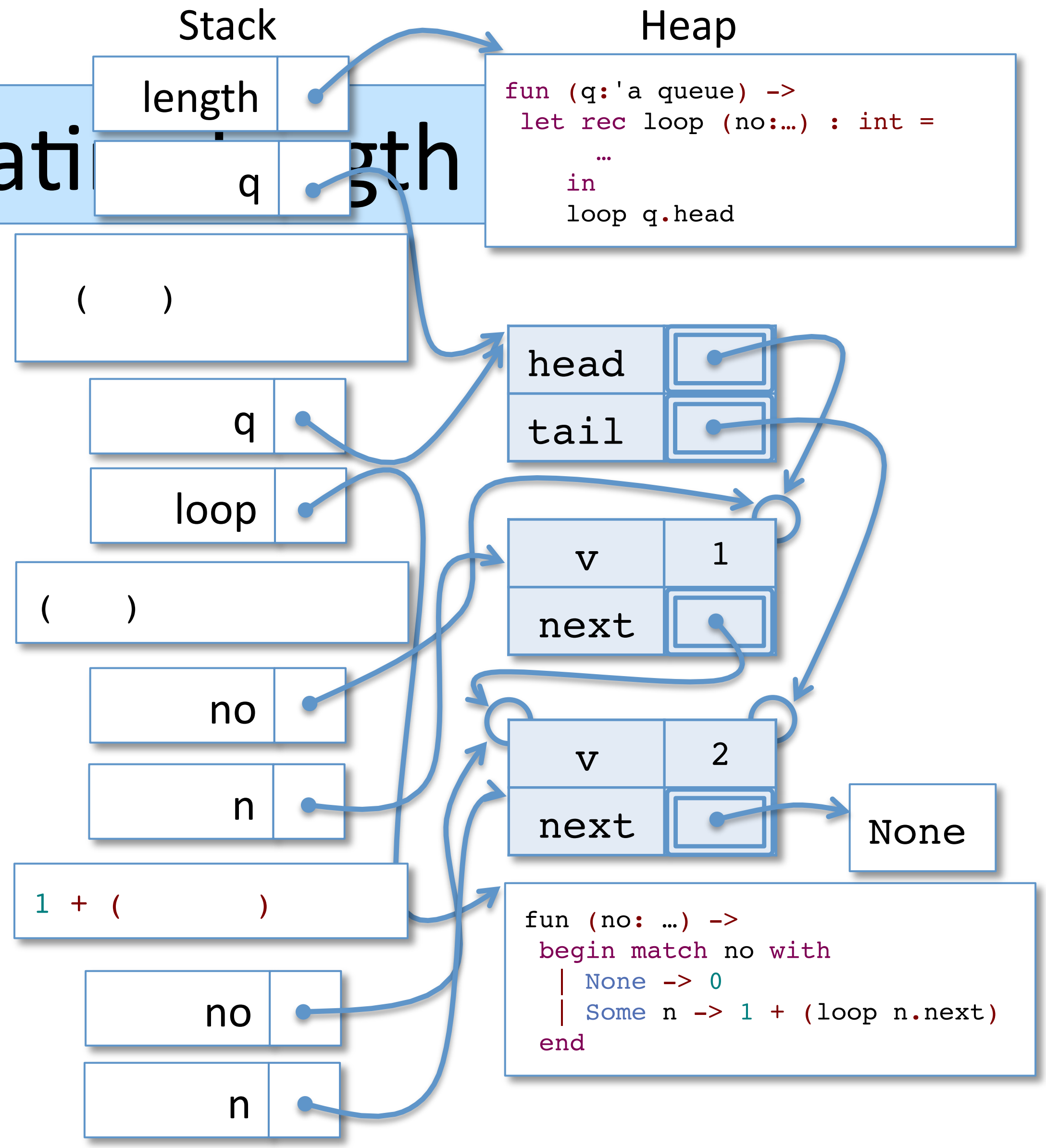
Evaluati… gth



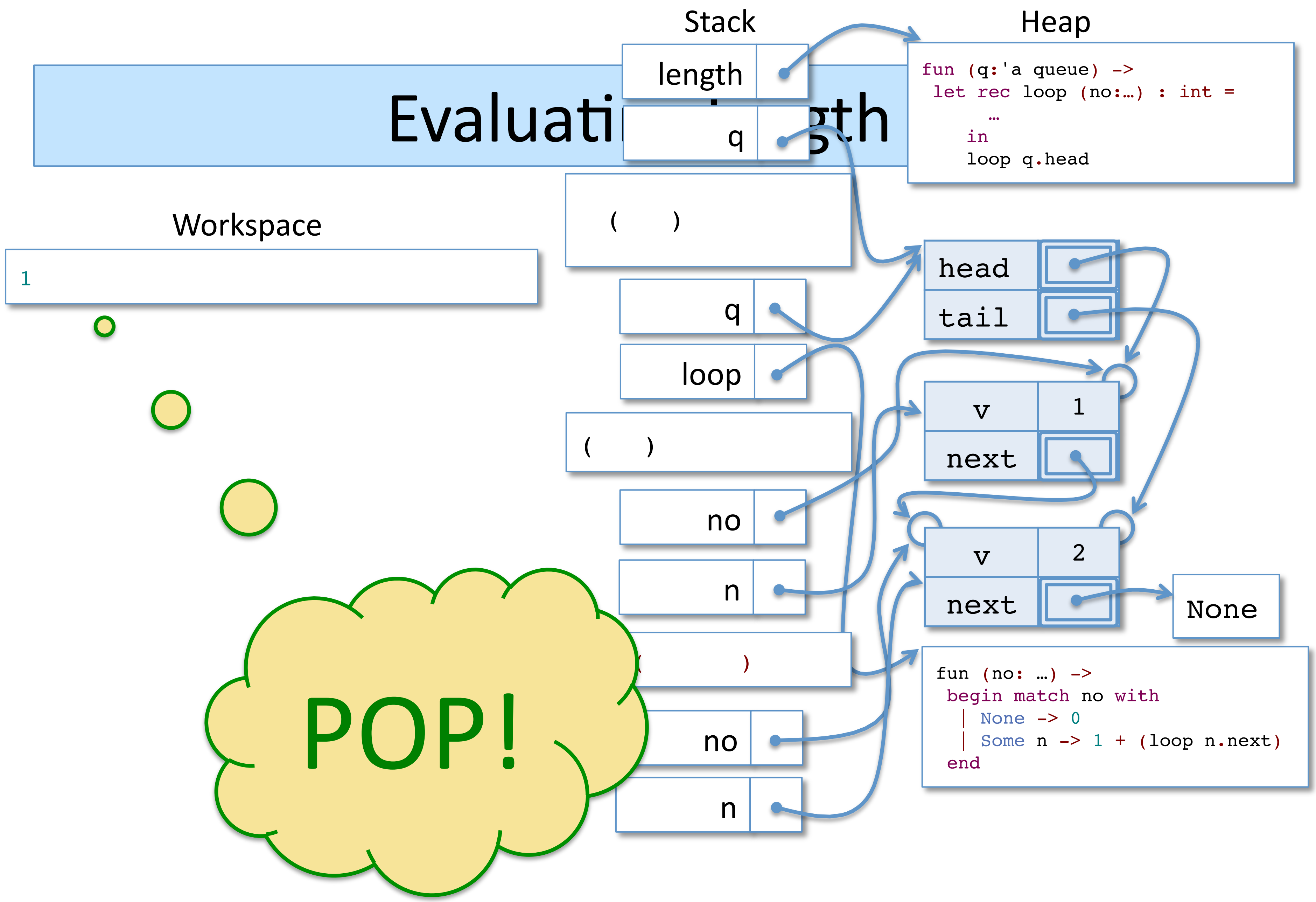
Evaluati gth

Workspace

`1 + 0`



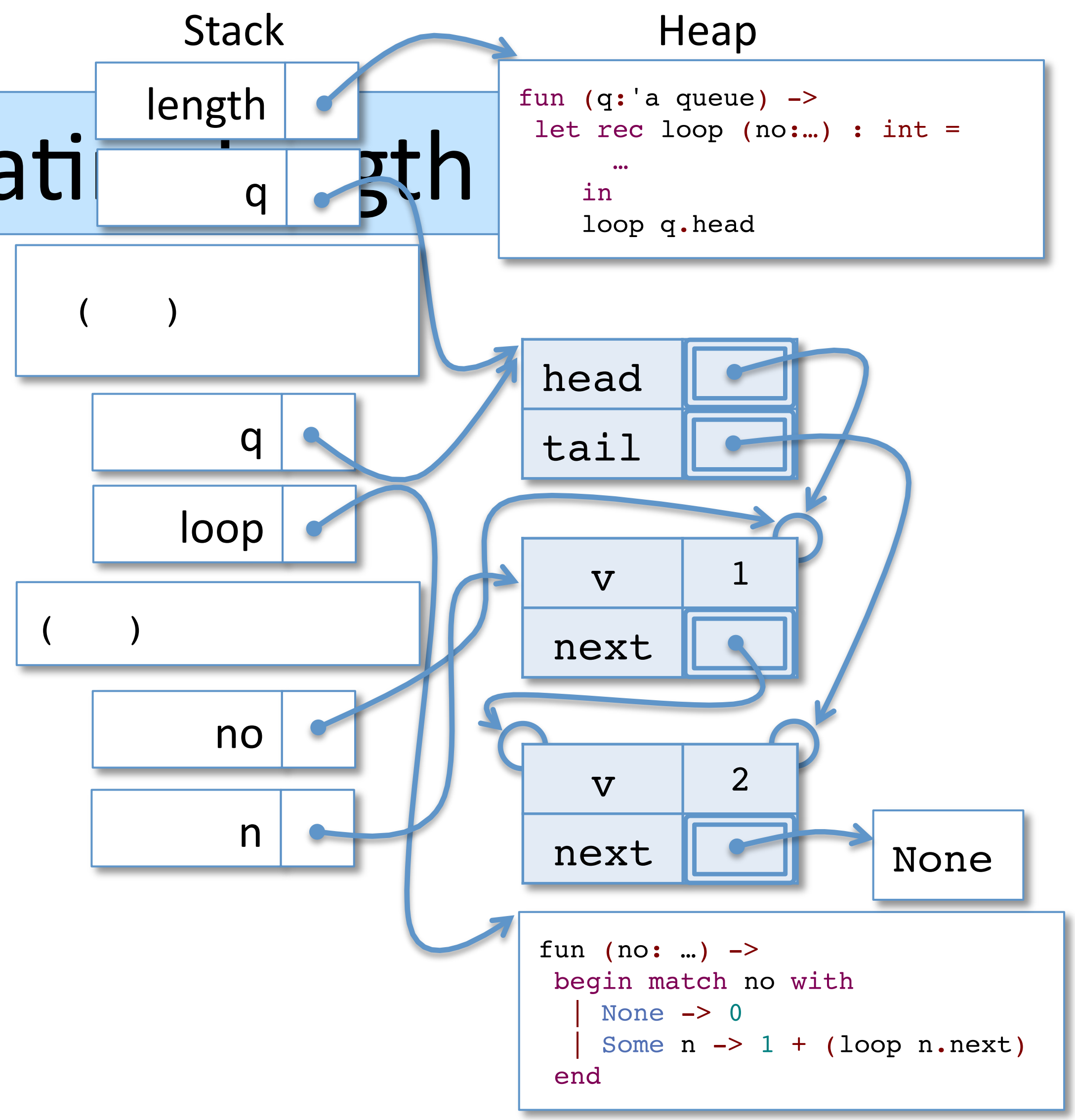
Evaluati gth

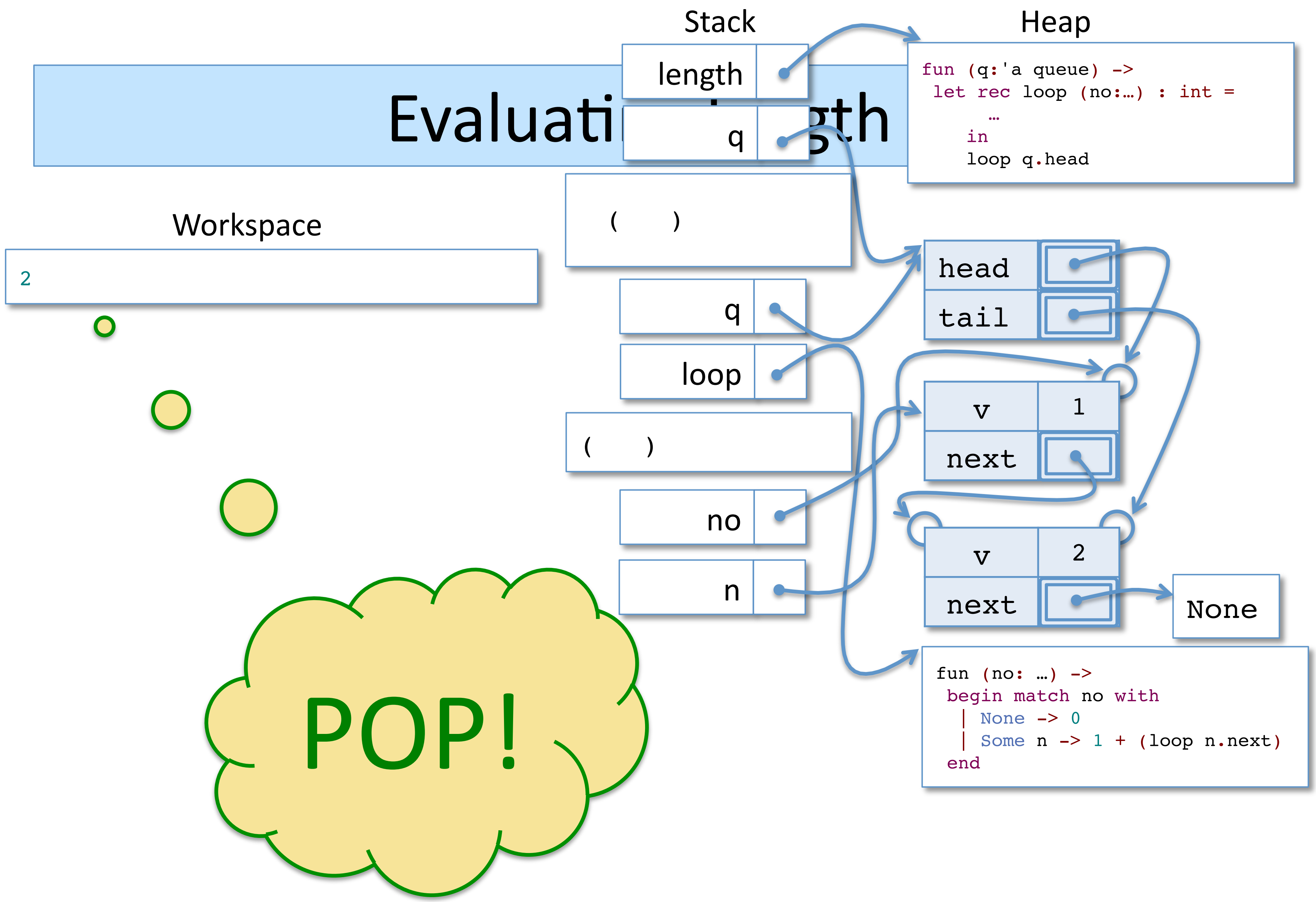


Evaluati gth

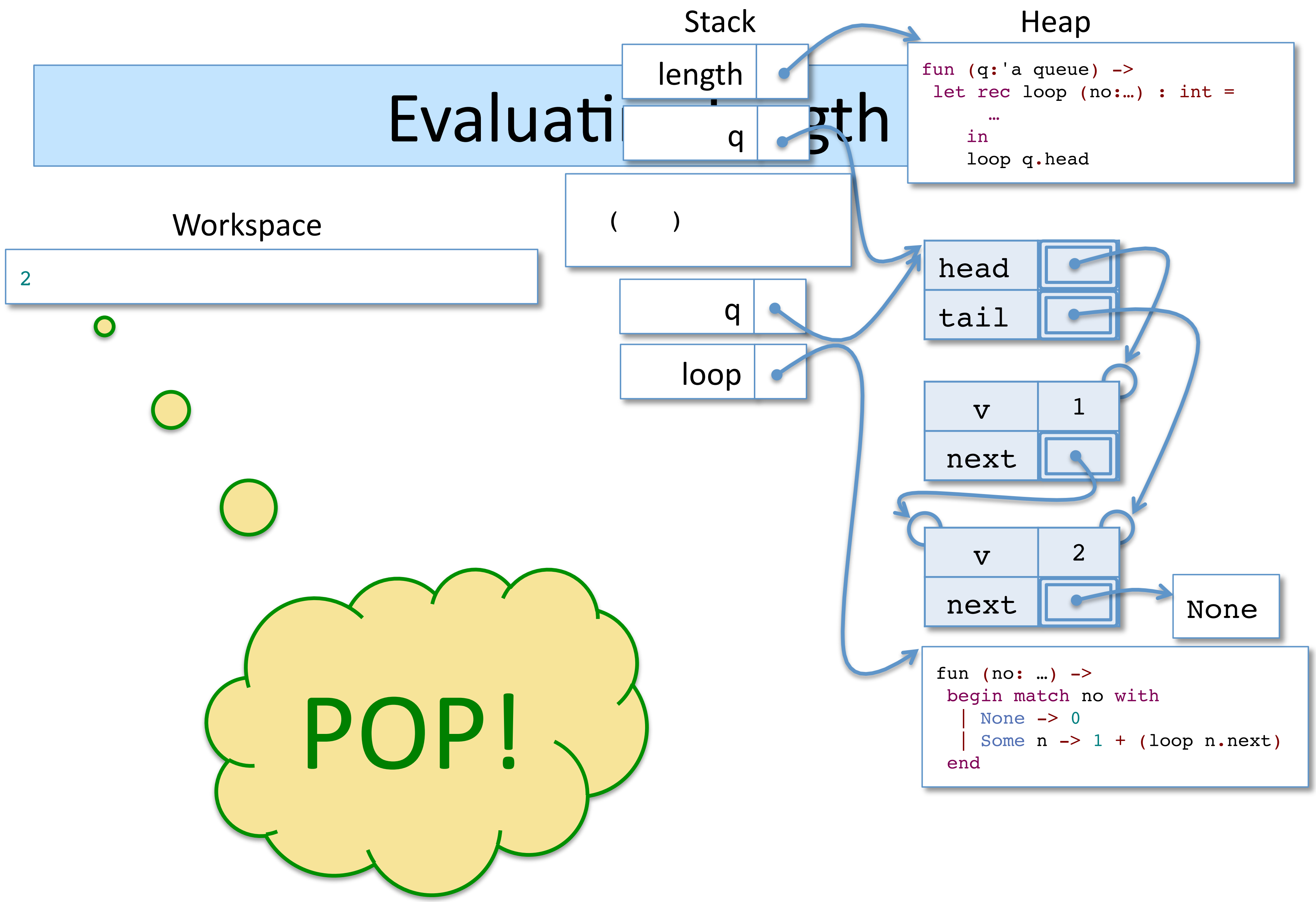
Workspace

1 + 1

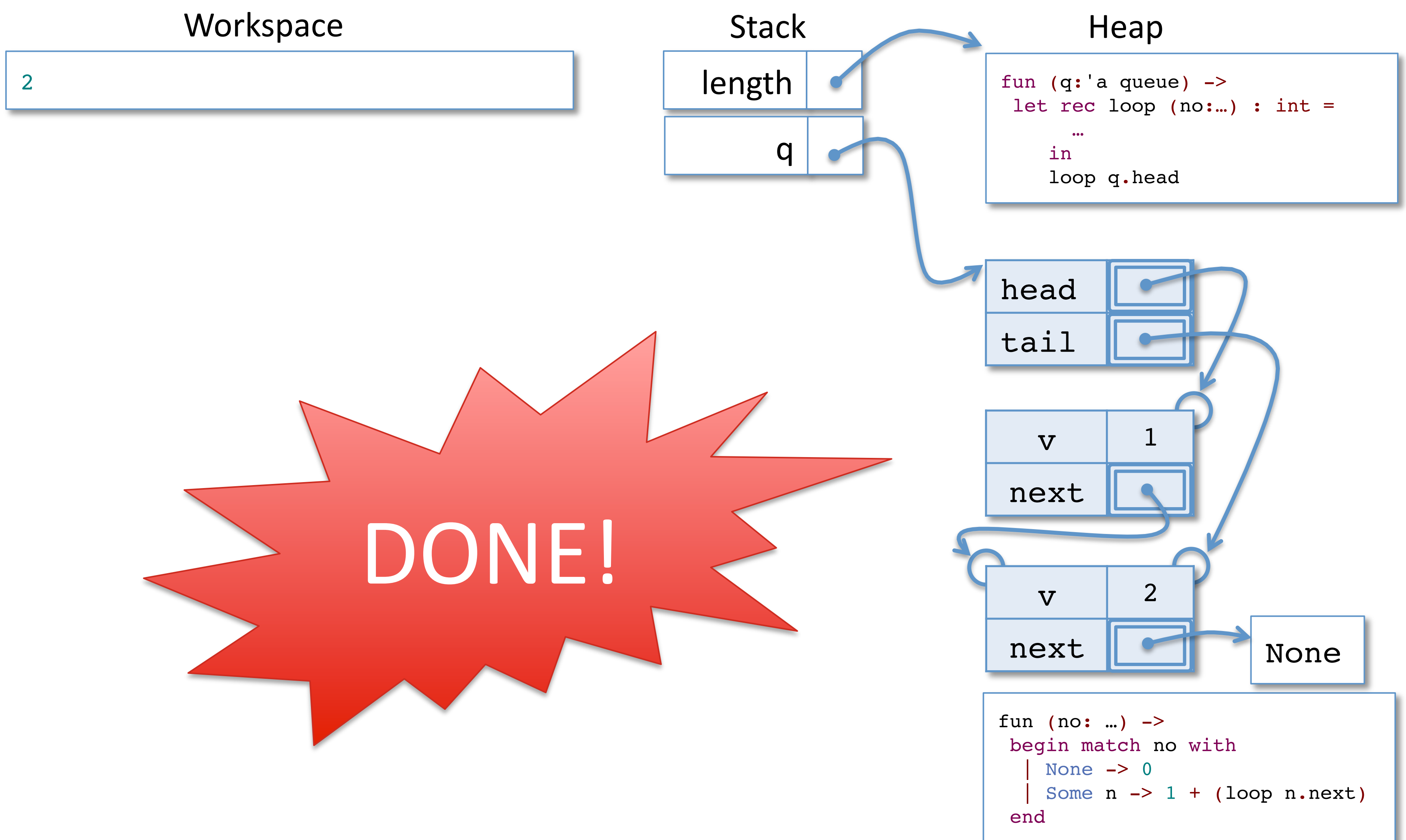



Evaluati
gth
Stack
Heap
length
q
( )
q
loop
( )
no
n
Workspace
2
head
tail
v
1
next
v
2
next
None
fun (q:'a queue) ->
let rec loop (no:…) : int =
…
in
loop q.head
fun (no: …) ->
begin match no with
| None -> 0
| Some n -> 1 + (loop n.next)
end
POP!

Evaluati gth



Evaluating length



Iteration

loops

`length` (using iteration)

```
(* Calculate the length of the list using iteration *)
let length (q:'a queue) : int =
  let rec loop (no:'a qnode option) (len:int) : int =
      begin match no with
        | None -> len
        | Some n -> loop n.next (1+len)
      end
  in
    loop q.head 0
```

- This code for `length` also uses a helper function, `loop`:
  - This loop takes an extra argument, len, called the *accumulator*
  - Unlike the previous solution, the computation happens "on the way down" as opposed to "on the way back up"
  - Note that `loop` will always be called in an empty workspace—the results of the call to `loop` never need to be used to compute another expression. In contrast, we had (1 + (`loop` …)) in the recursive version.

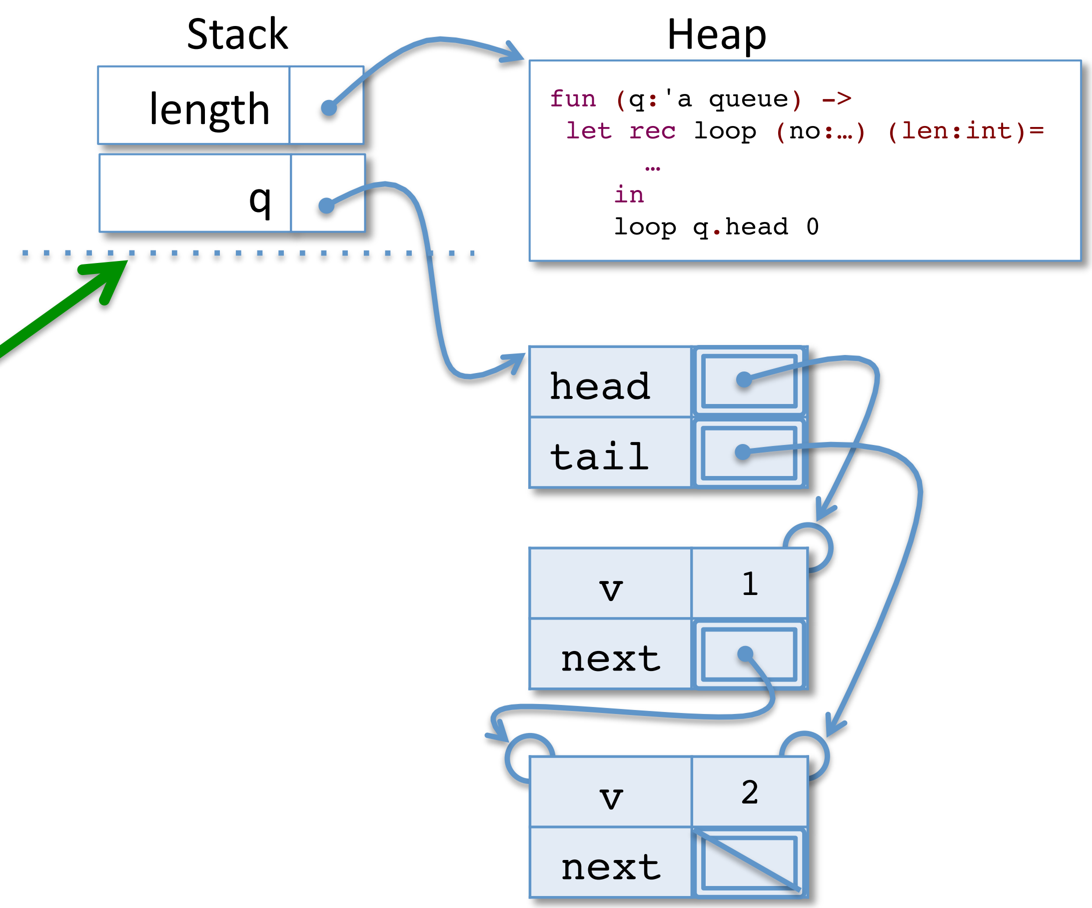
Tail Call Optimization

- Why does it matter that 'loop' is only called in an empty workspace?
- We can *optimize* the abstract stack machine:
  - The workspace pushed onto the stack tells us "what to do" when the function call returns.
  - If the pushed workspace is empty, we will always 'pop' immediately after the function call returns.
  - Therefore, we do not need to save the 'empty' workspace on the stack.
  - Moreover, any local variables that were pushed so that the current workspace could evaluate will no longer be needed, so we can eagerly pop them too.
- The end result is that we have turned *recursion* into a true *loop*. (Just like a 'while' or 'for' loop in Java or C.)

Tail Calls and Iterative length

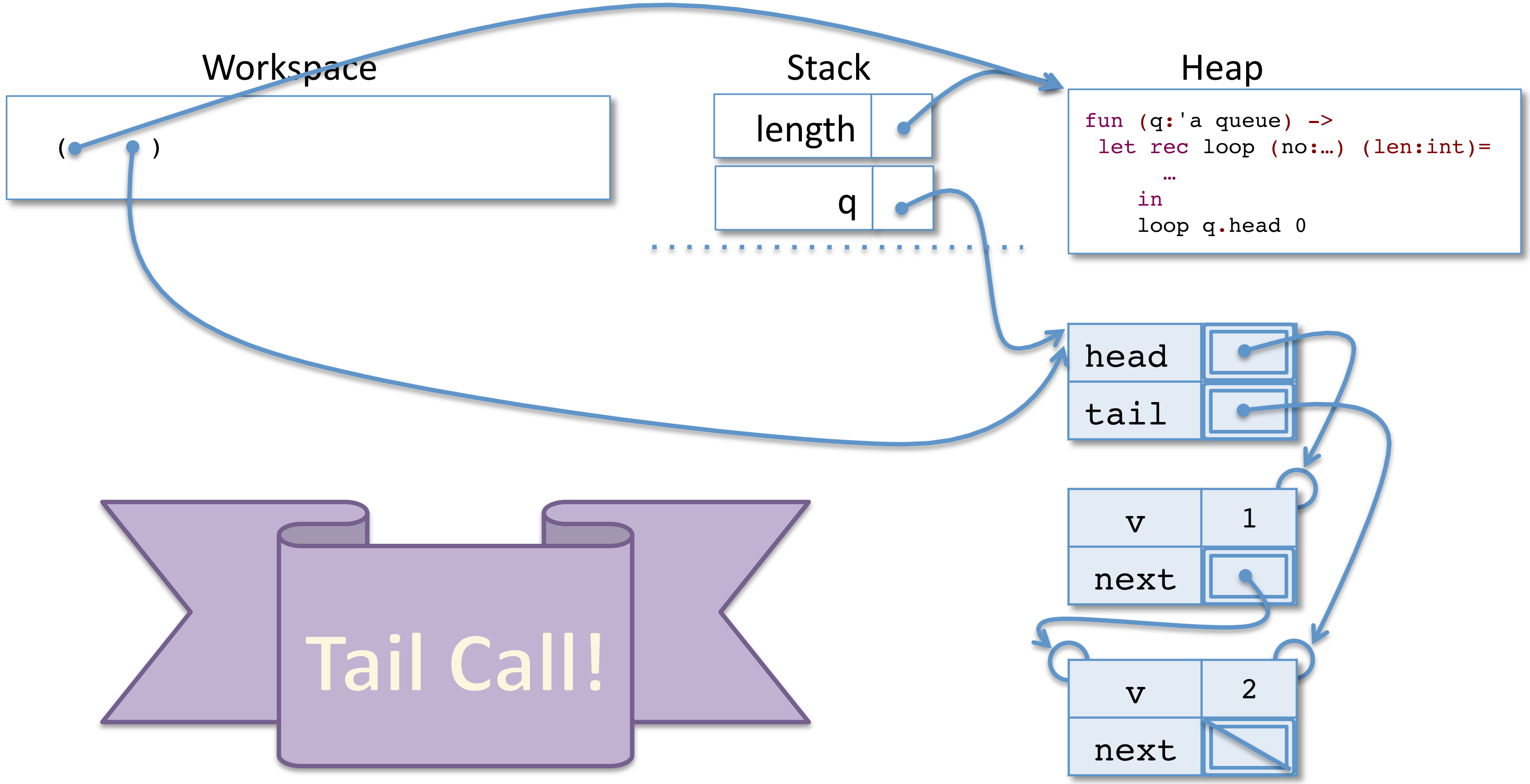
Workspace

```
length q
```



Bindings above this line are top-level declarations.

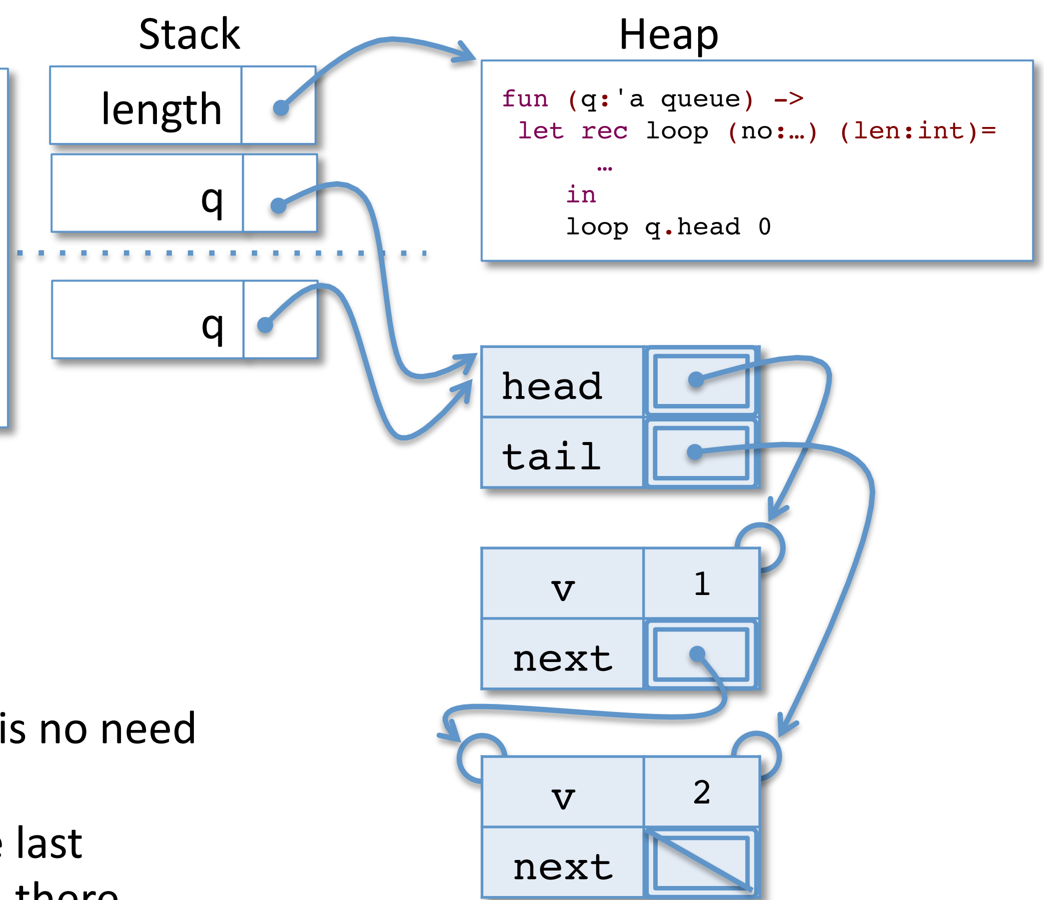
Tail Calls and Iterative length



Tail Calls and Iterative length

Workspace

```
let rec loop (no:'a qnode option)
(len:int) : int =
      begin match no with
        | None -> len
        | Some n -> loop n.next (1+len)
      end
    in
    loop q.head 0
```



Note:

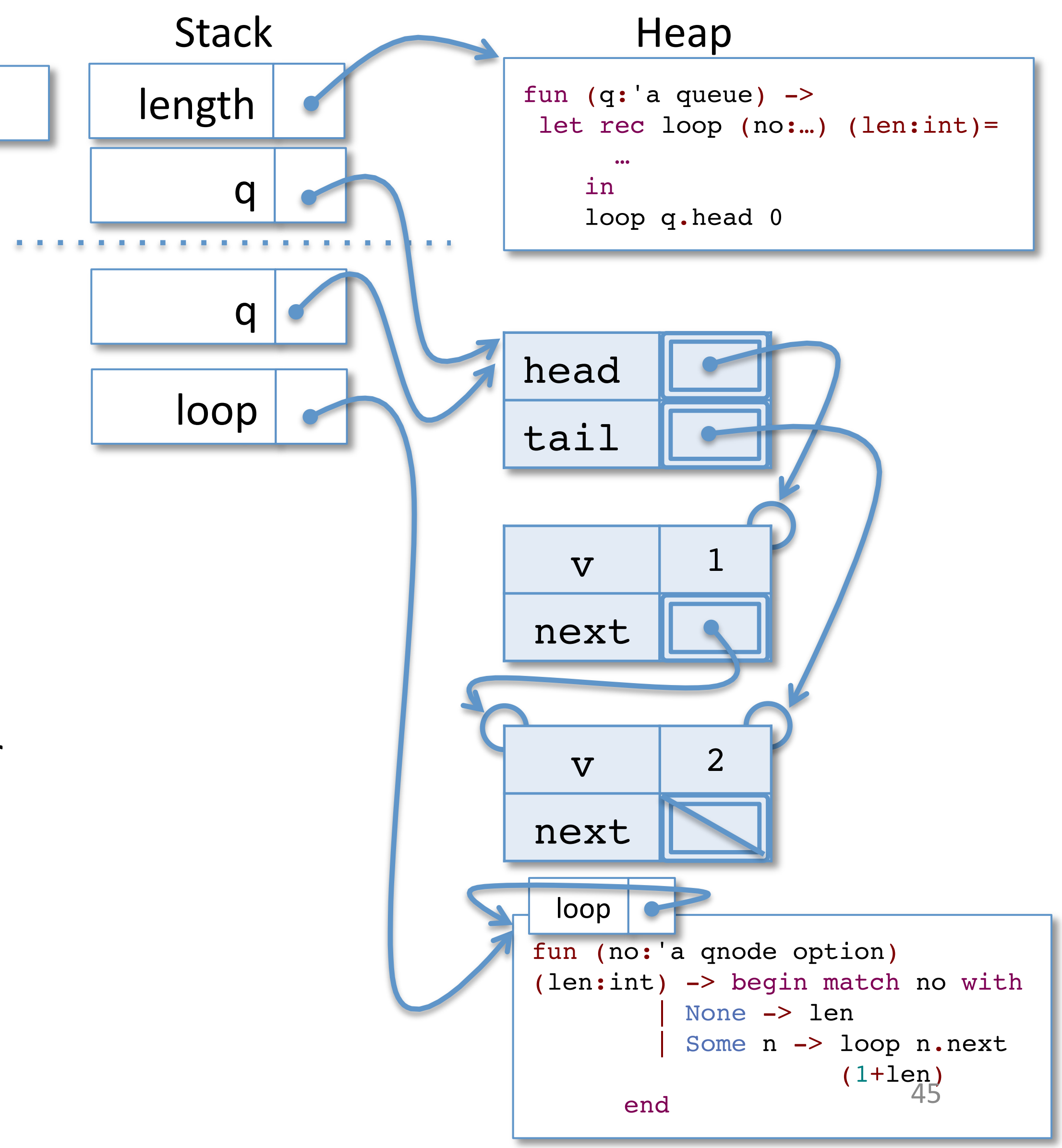
(1) No workspace is saved – there is no need do to that for tail calls

(2) We pop all the locals (up to the last saved workspace). In this case, there are none.

Tail Calls and Iterative length

Workspace

```
(loop q.head 0)
```

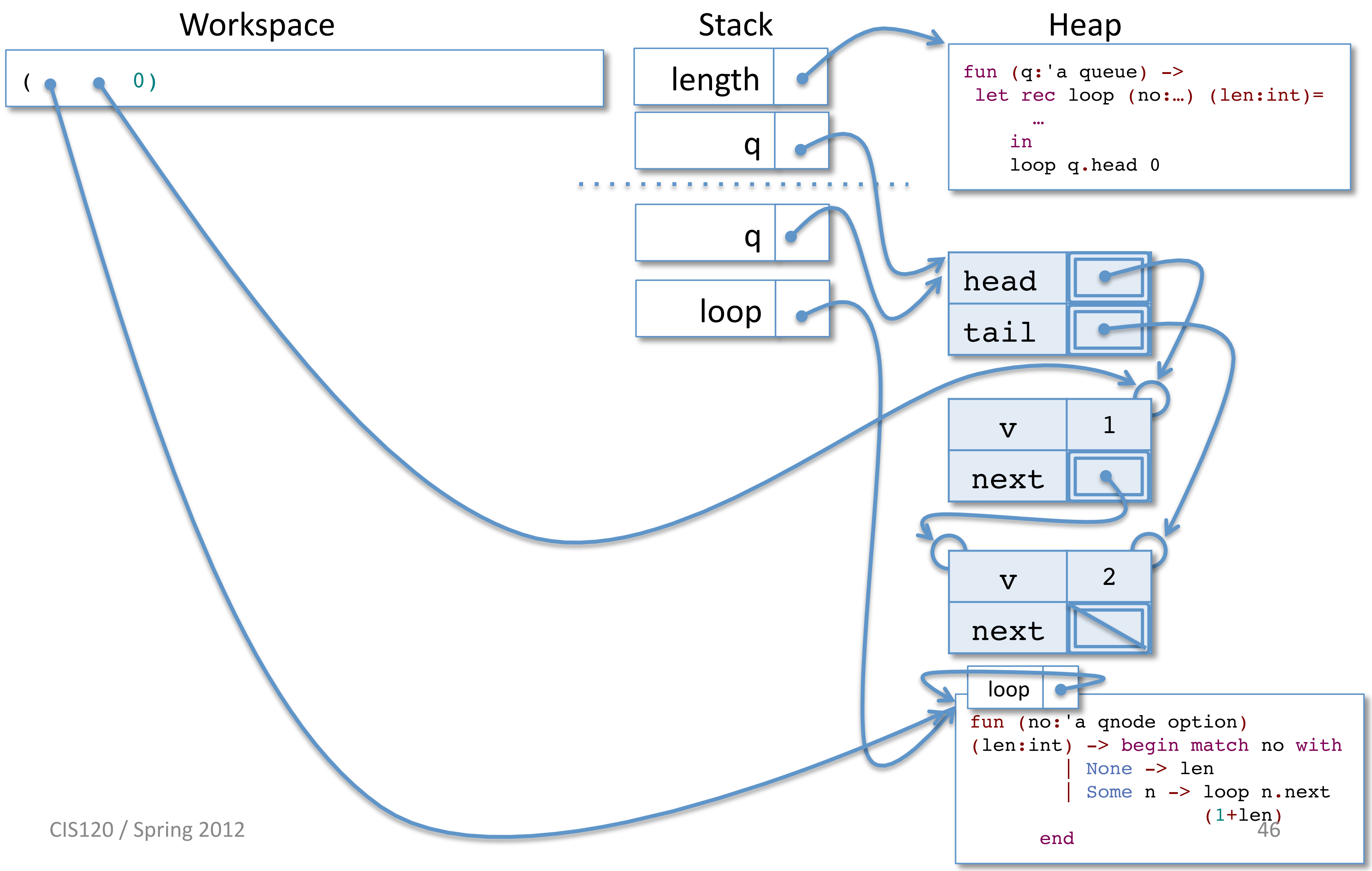


A detail we've been sweeping under the rug until now:

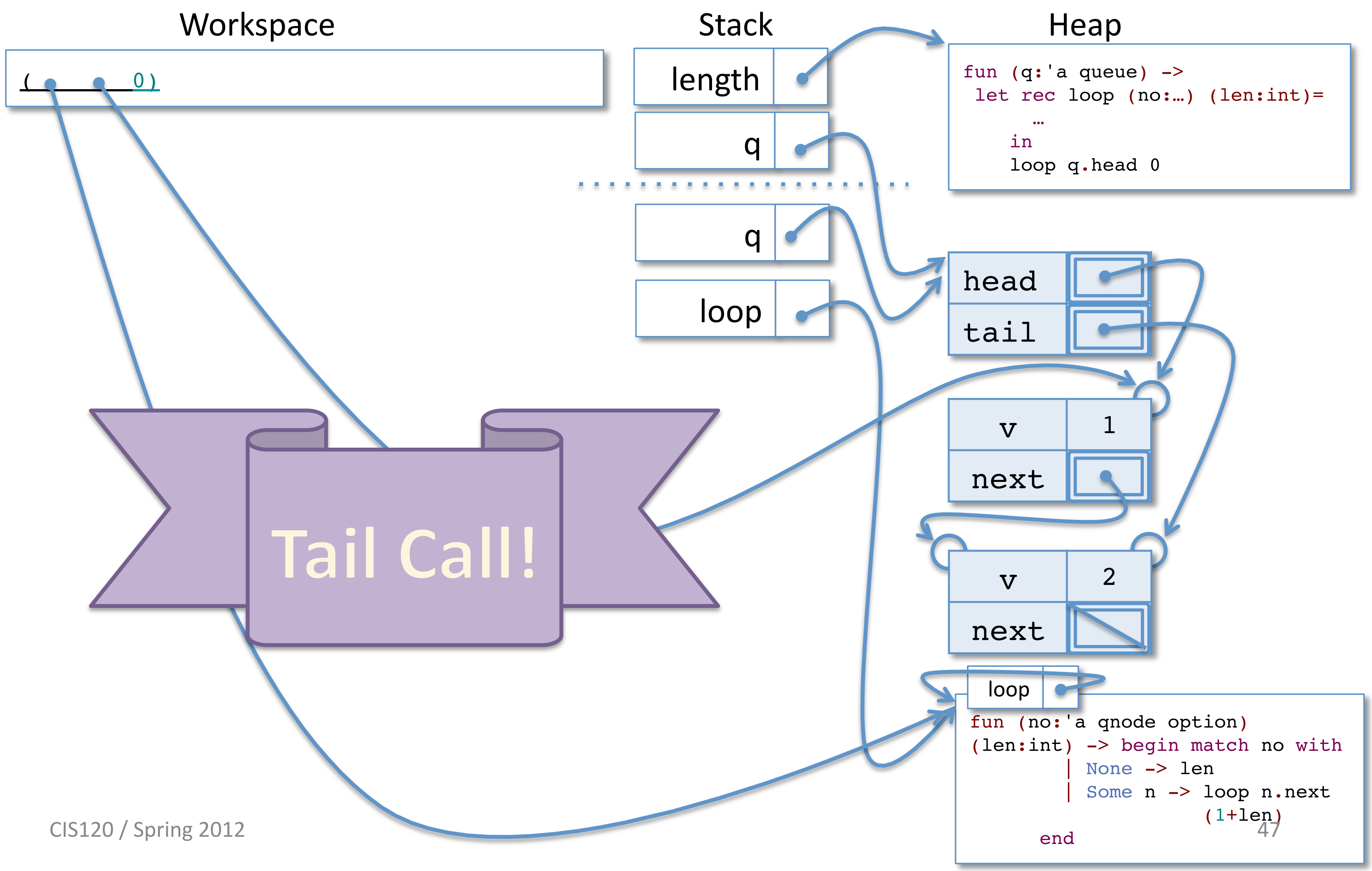
The *closure* of the local recursive function `loop` includes a binding for the loop function itself!

Why? The loop body mentions the loop identifier.

Tail Calls and Iterative length



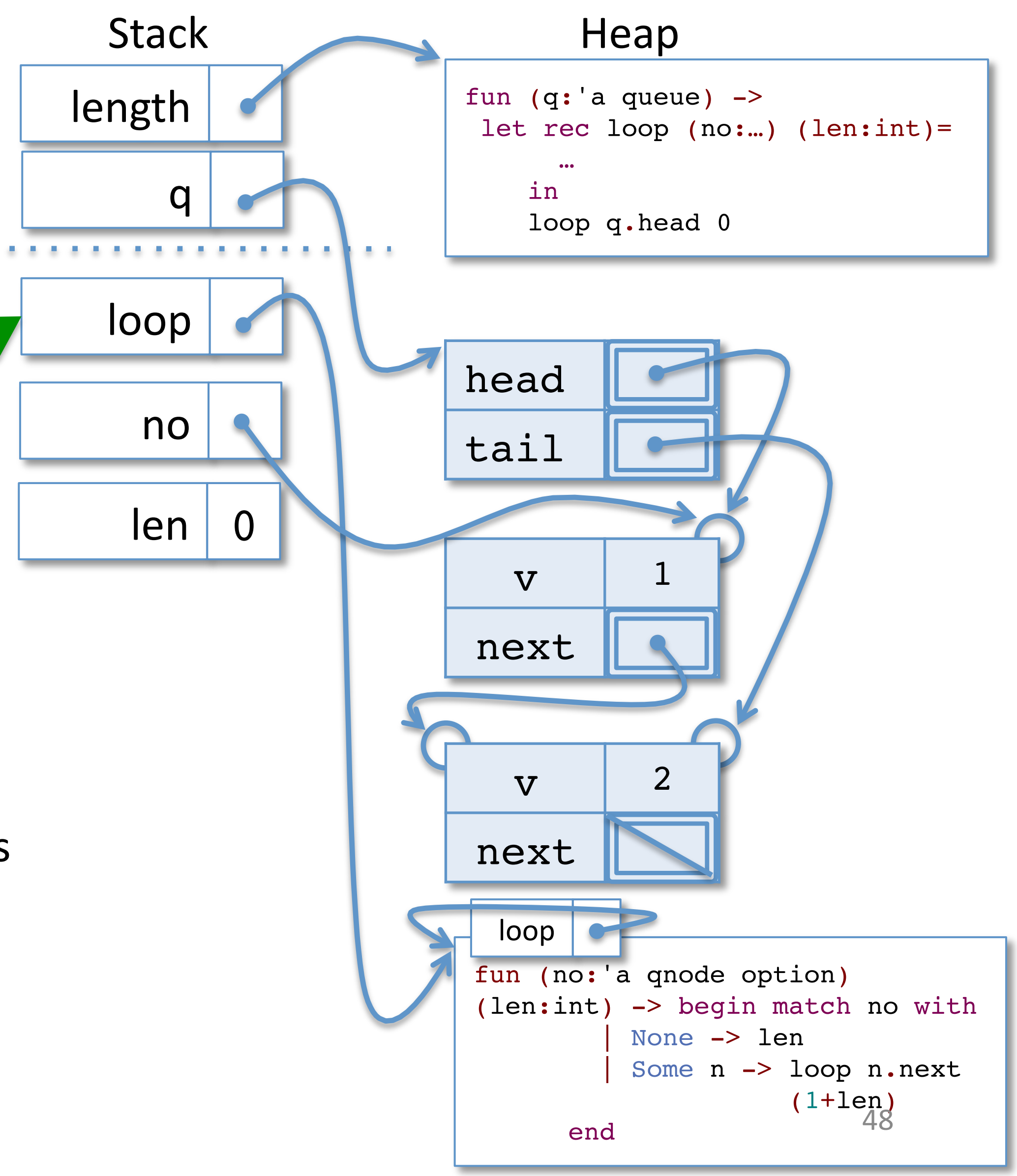
Tail Calls and Iterative length



Tail Calls and Iterative length

Workspace

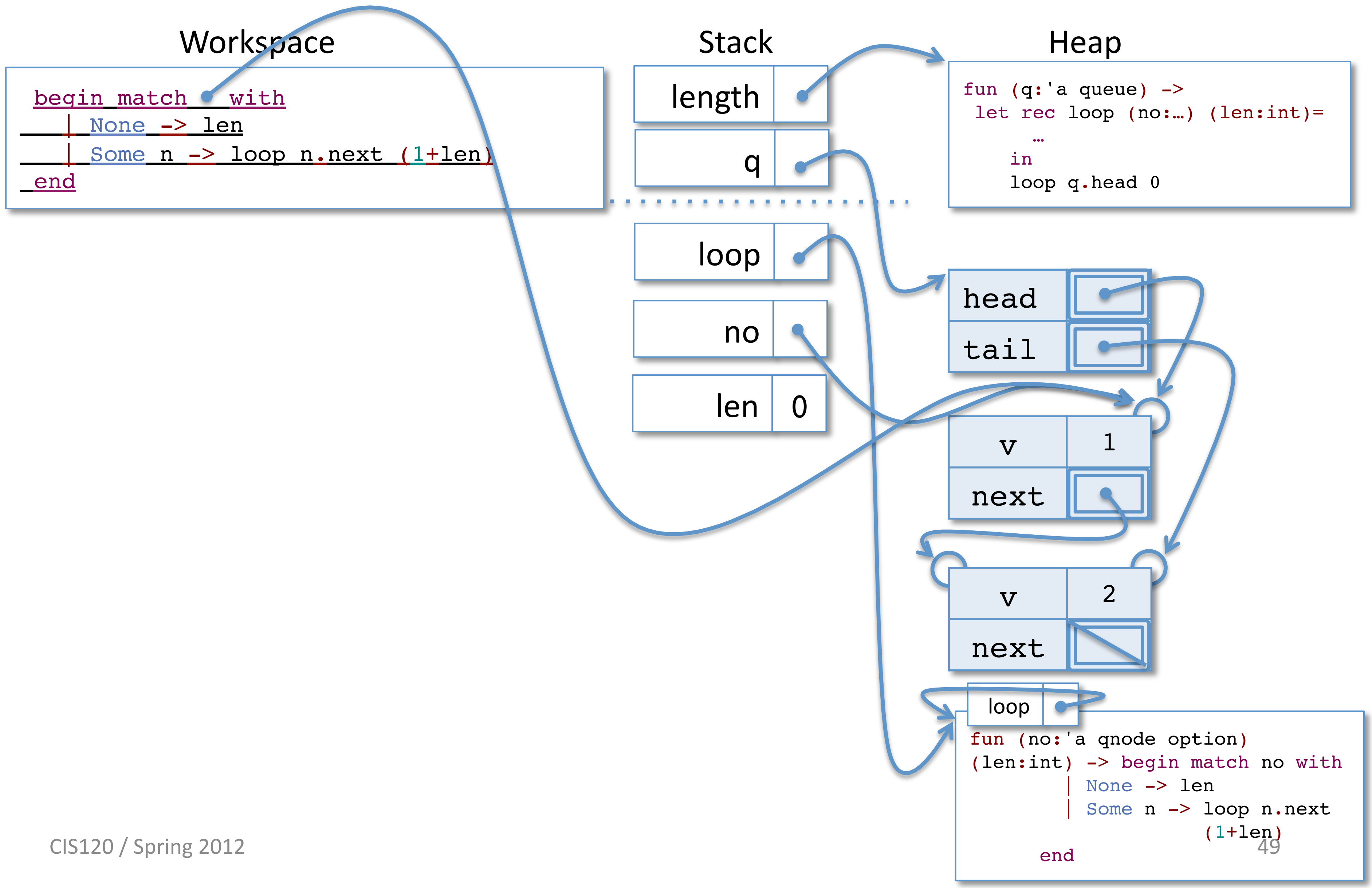
```
begin match no with
   | None -> len
   | Some n -> loop n.next (1+len)
end
```



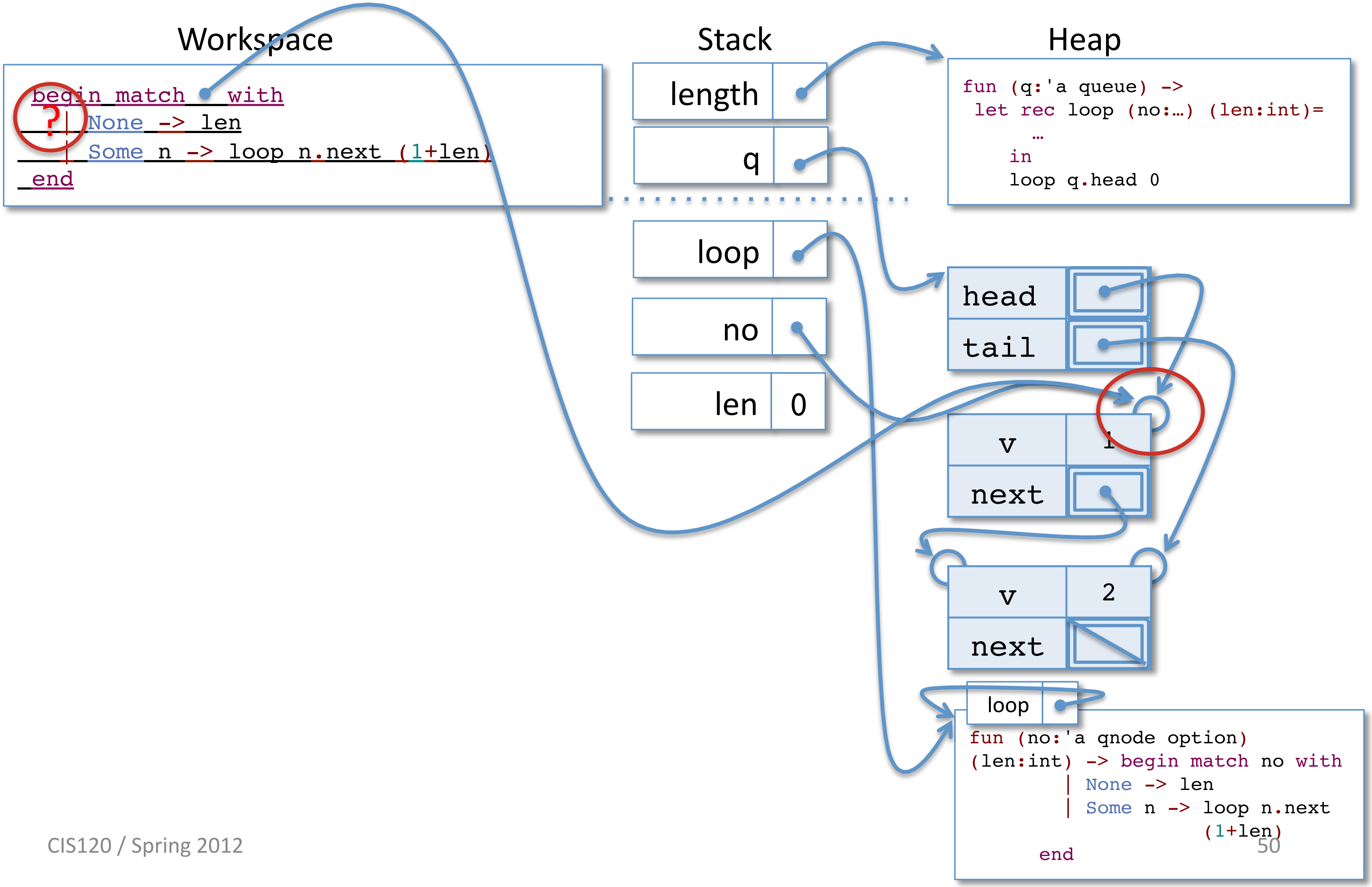
This binding comes from the closure.

Notes:
- no workspace is saved on the stack
- pop the old locals (q and loop)
- push the closure and argument bindings
- the new workspace is just the body of the function

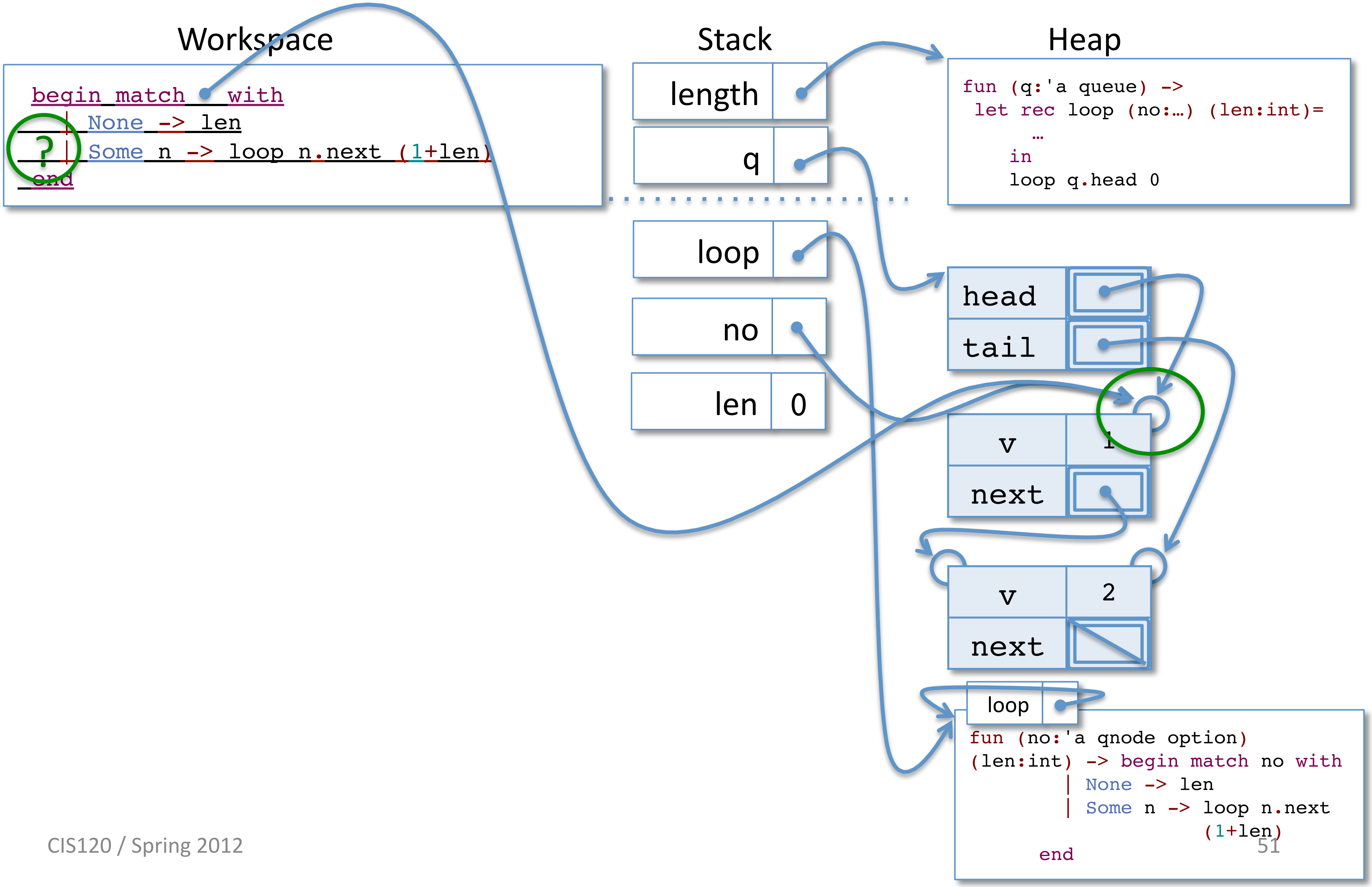
Tail Calls and Iterative length



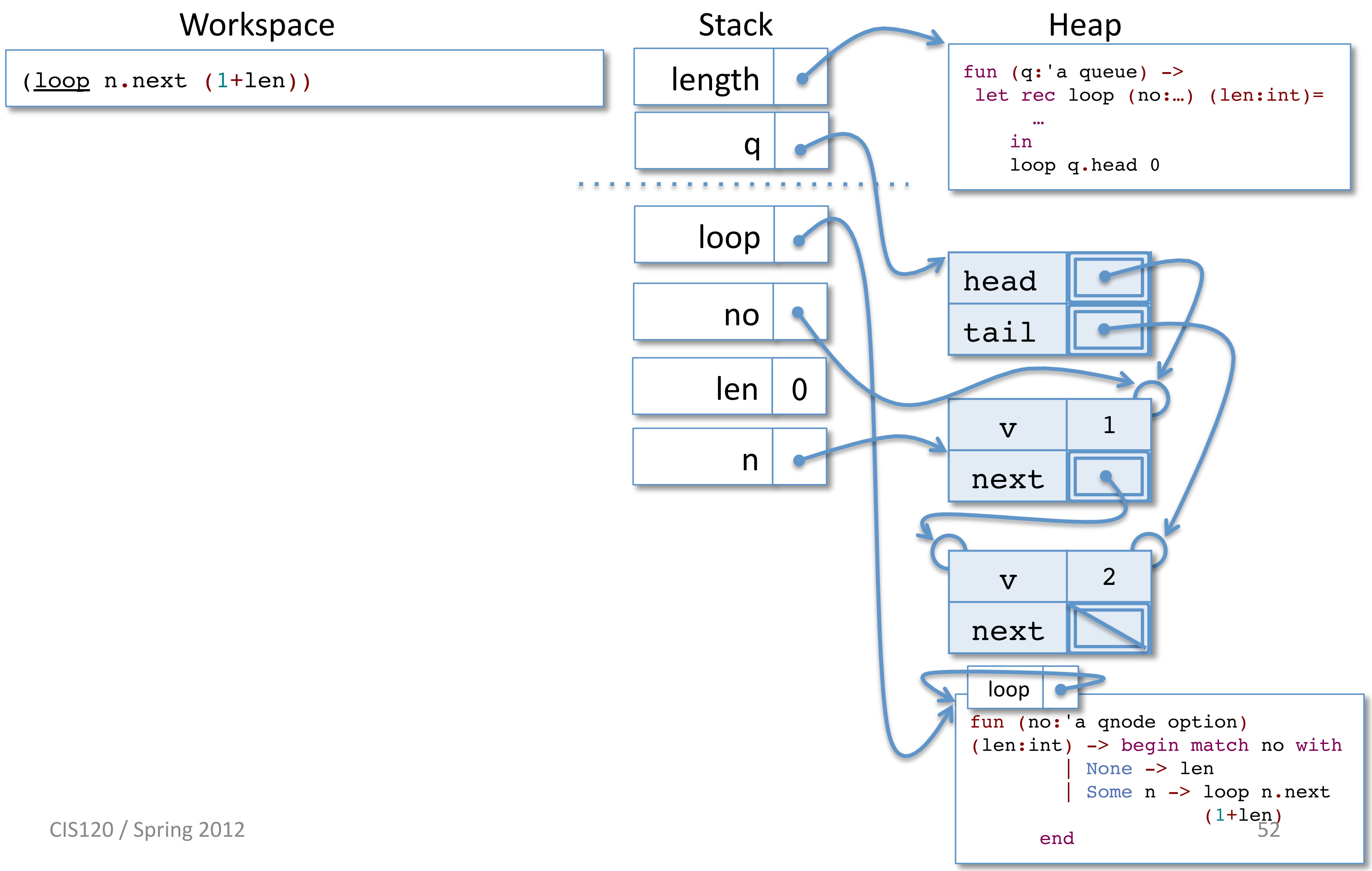
Tail Calls and Iterative length



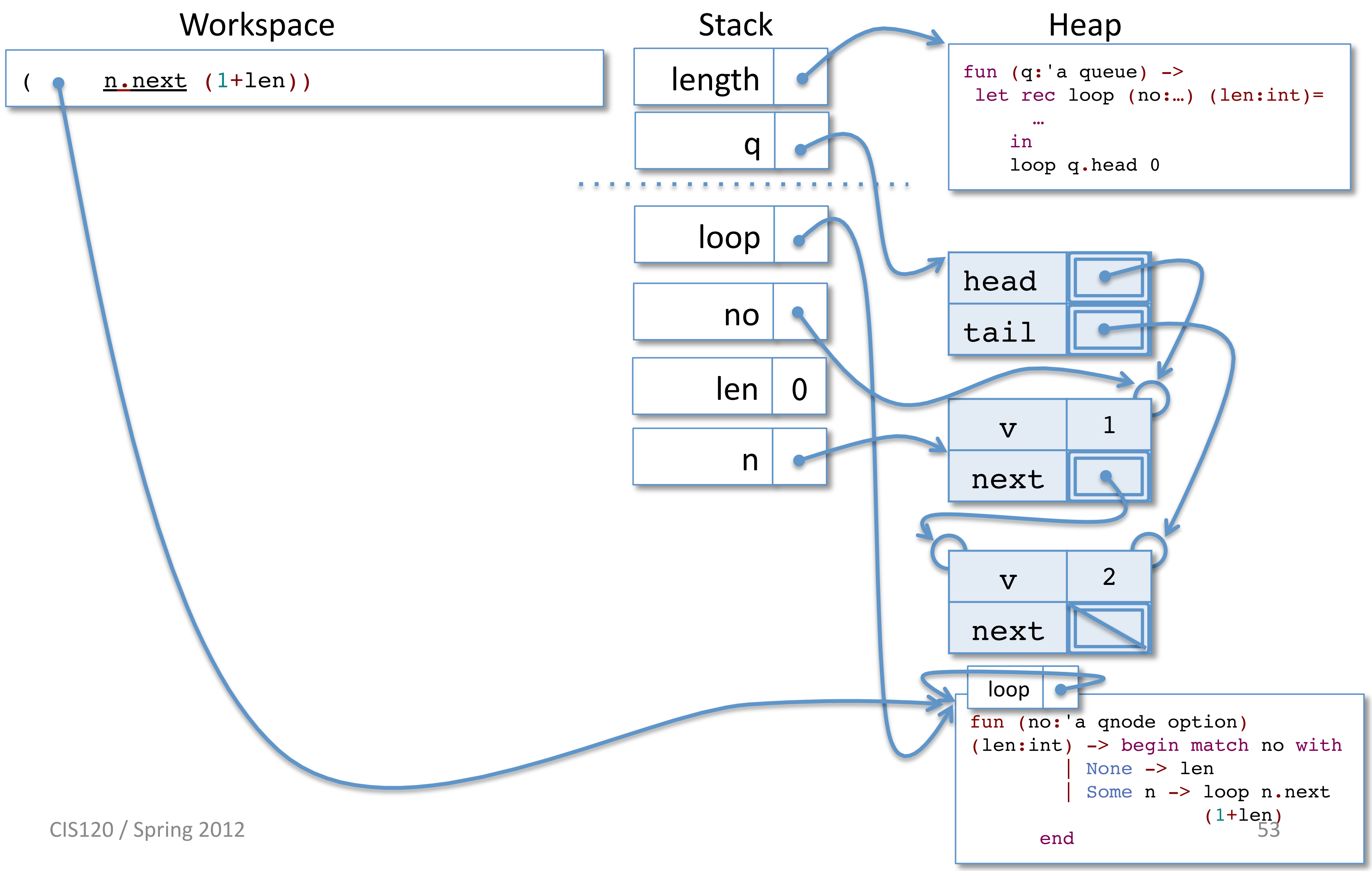
Tail Calls and Iterative length



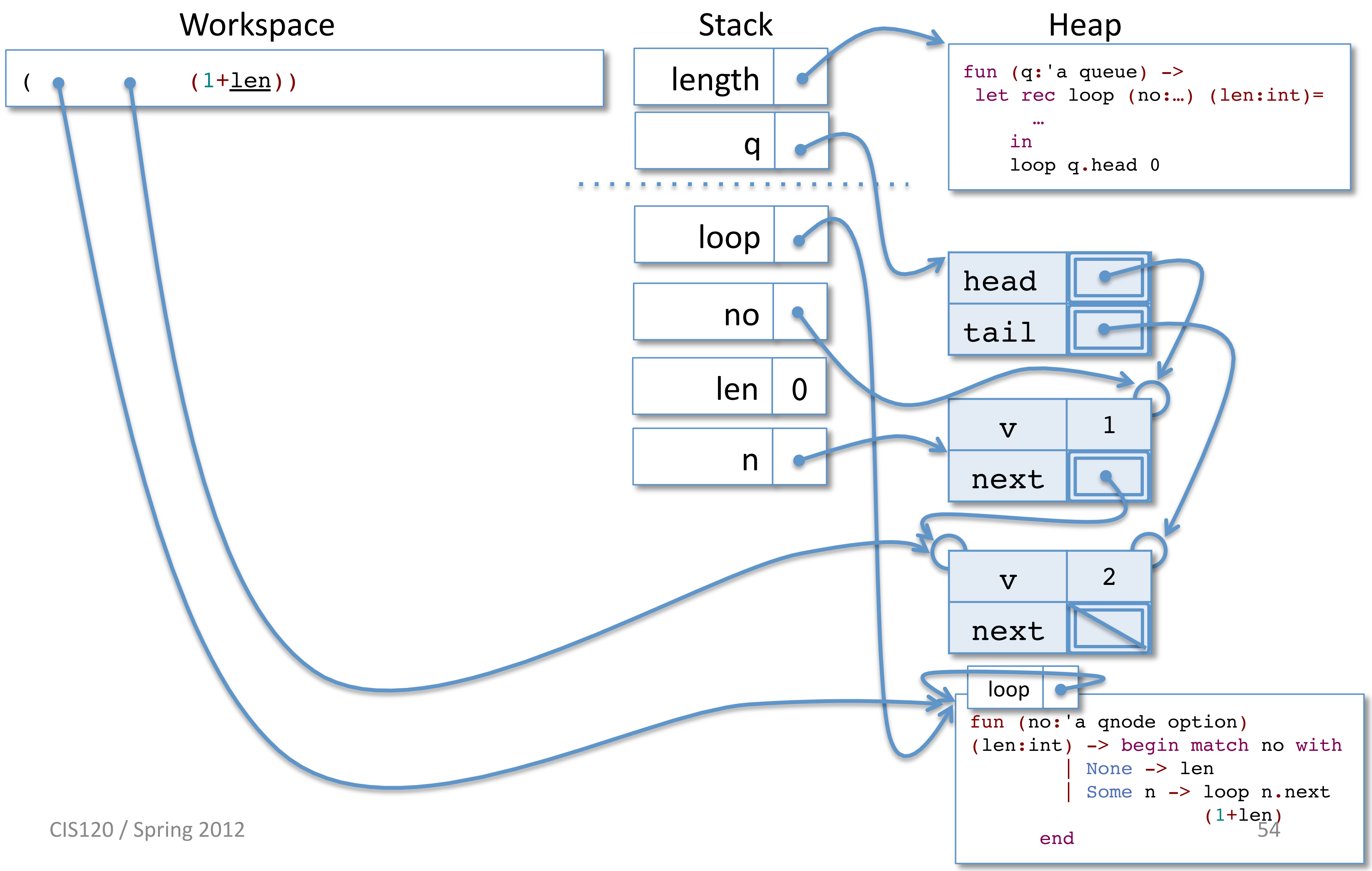
Tail Calls and Iterative length



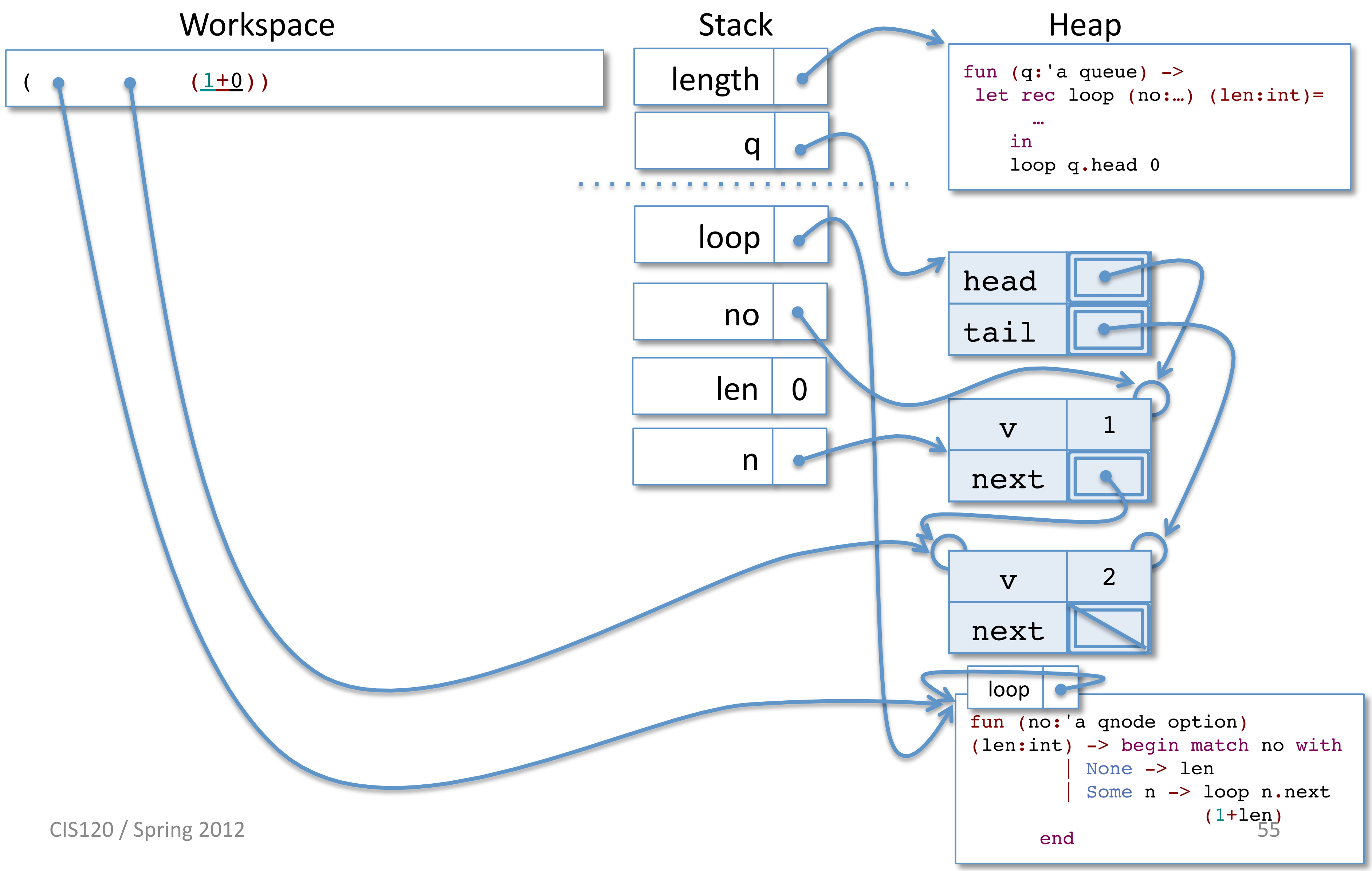
Tail Calls and Iterative length



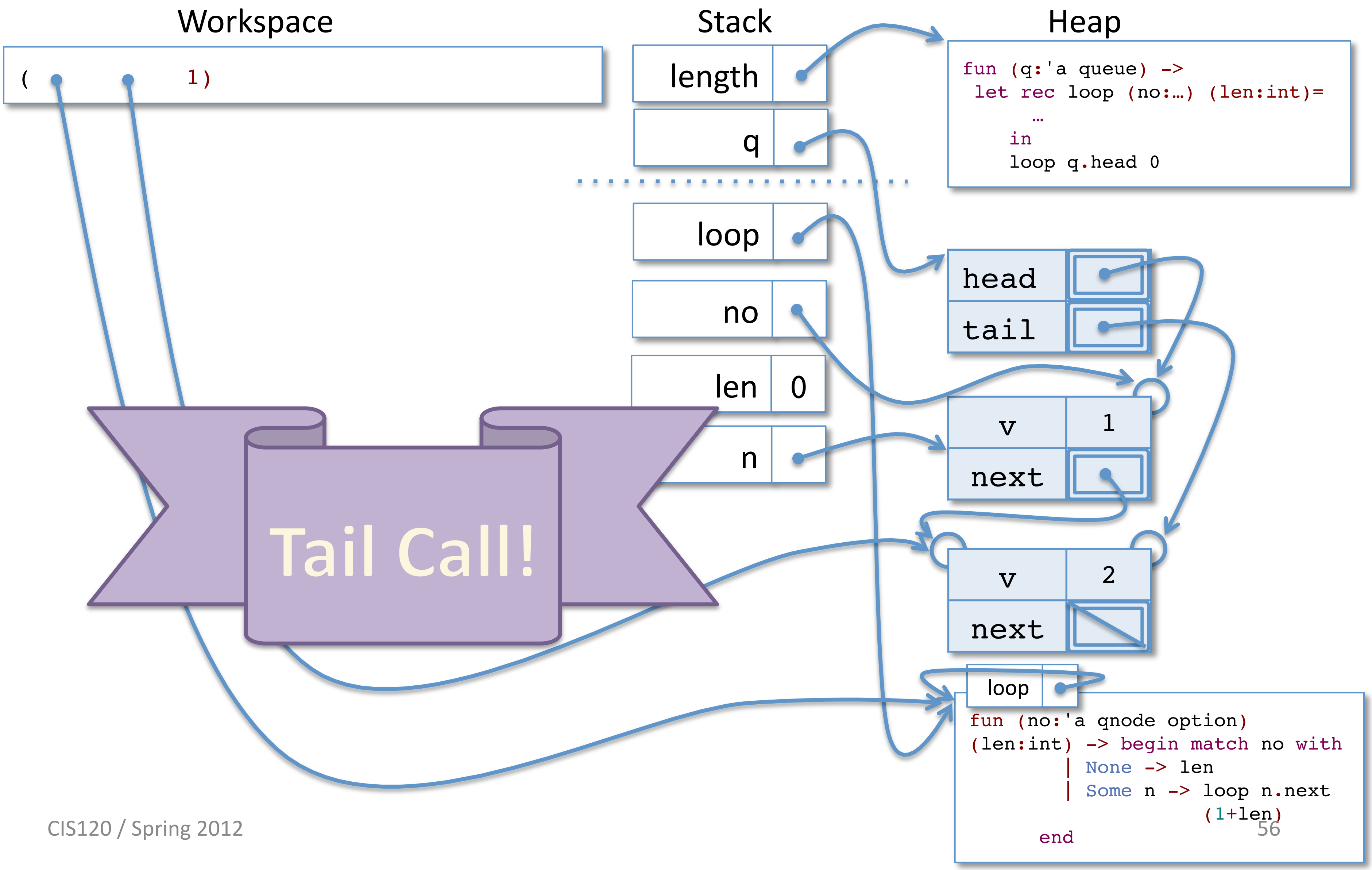
Tail Calls and Iterative length



Tail Calls and Iterative length



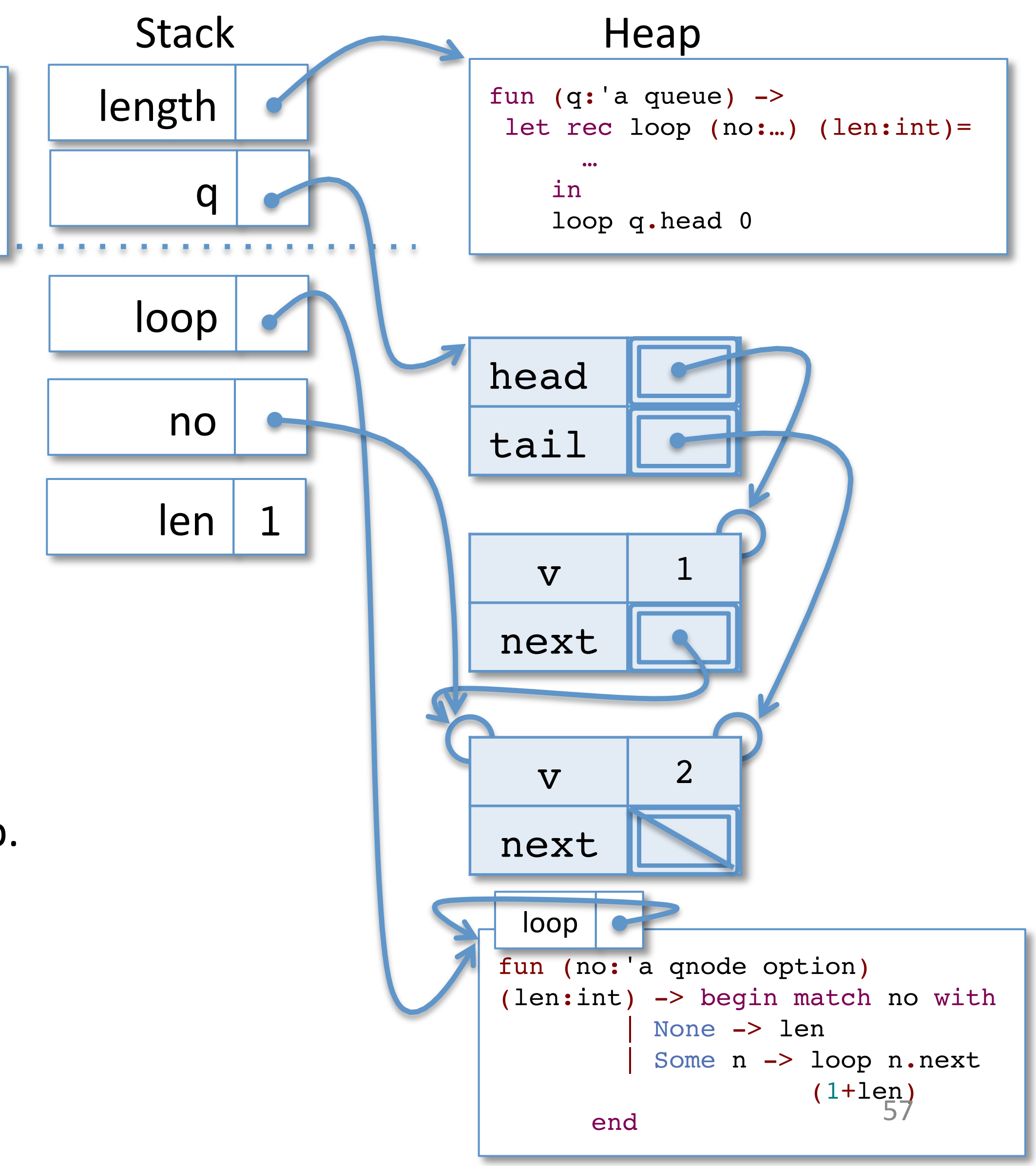
Tail Calls and Iterative length



Tail Calls and Iterative length

Workspace

```
begin match no with
   | None -> len
   | Some n -> loop n.next (1+len)
 end
```

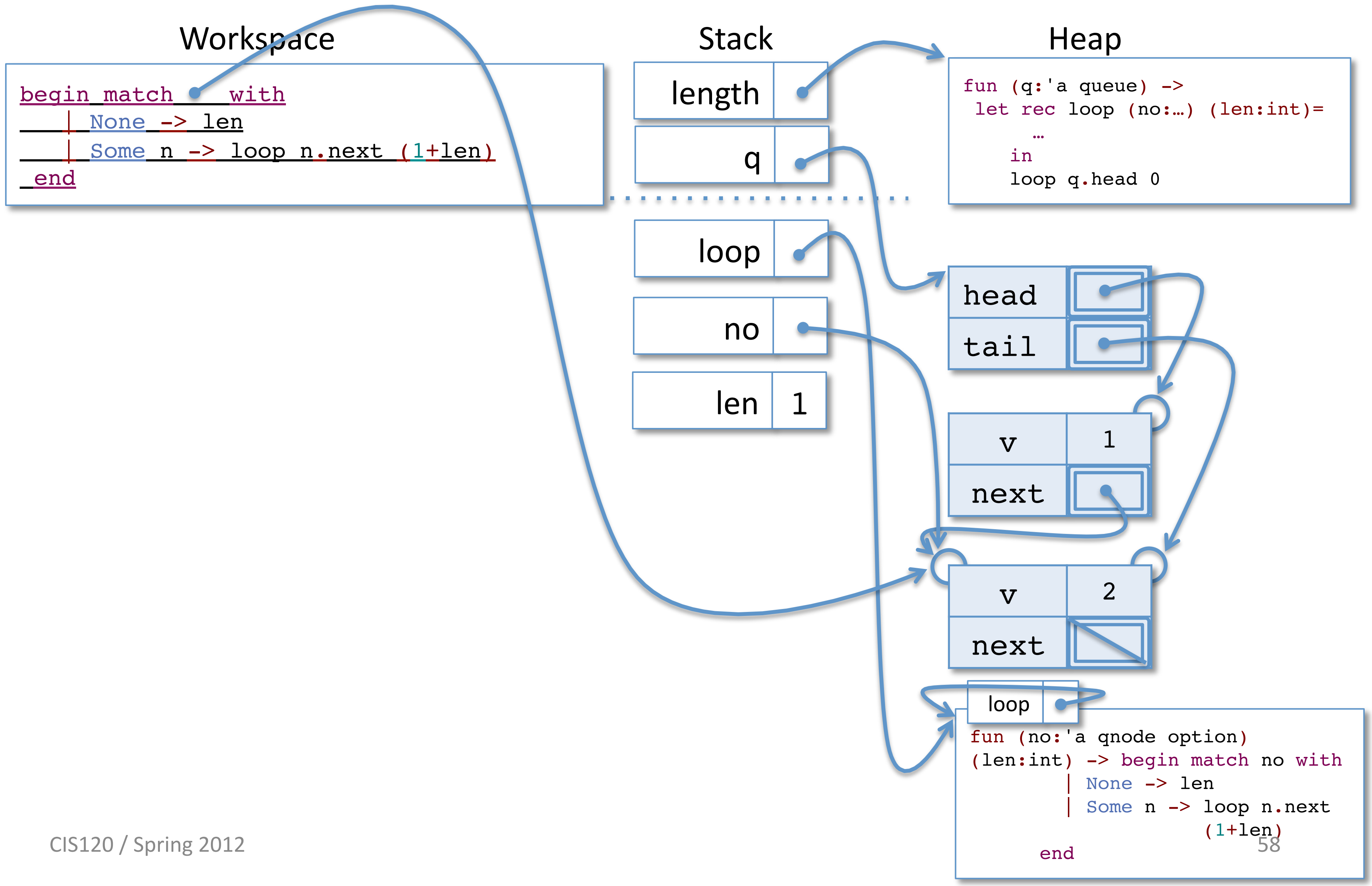


Note: we popped the old values of loop, no, len, and n when we did the tail call. Then we pushed the new values of loop, no, and len.

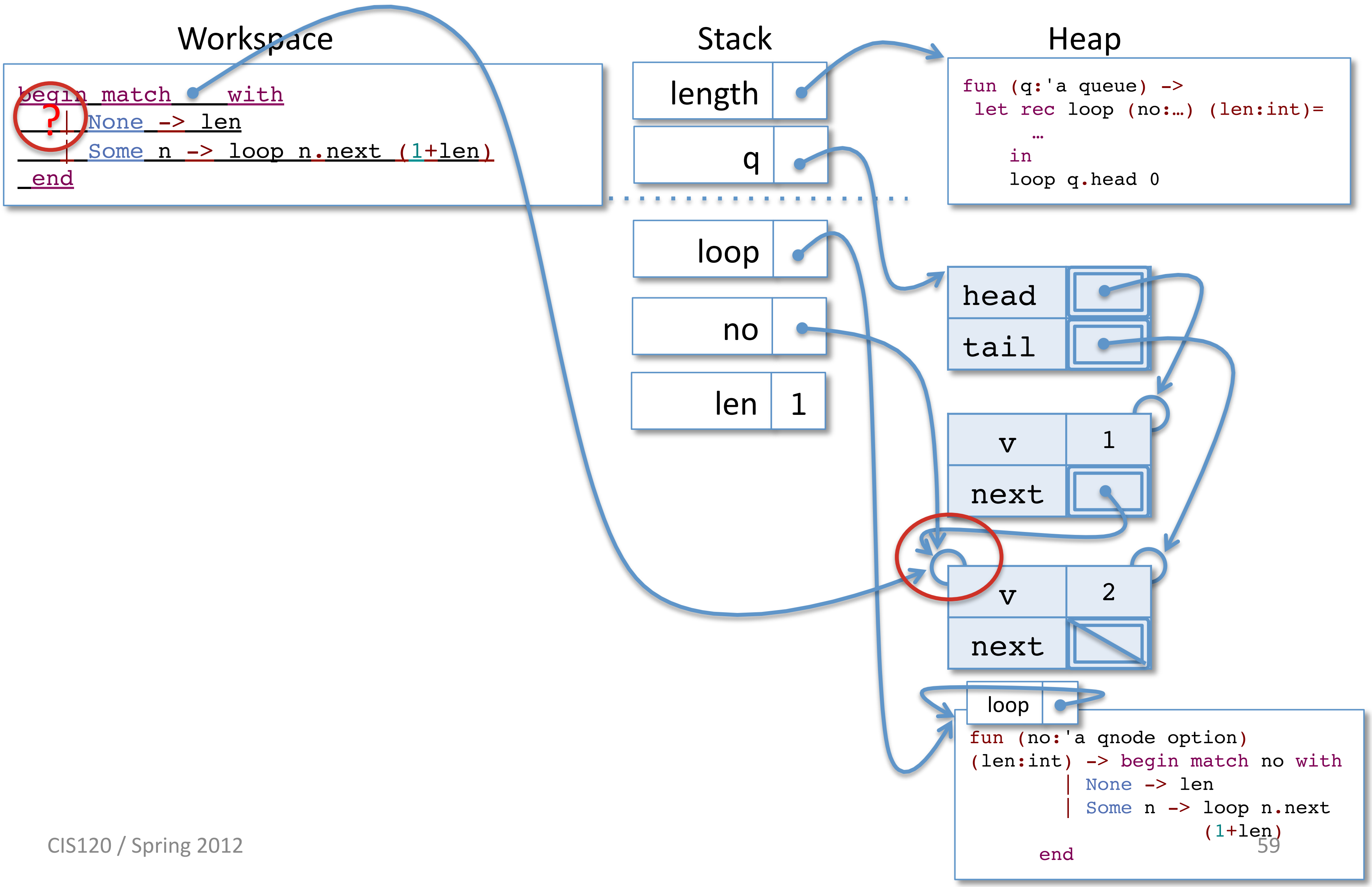
This leaves the stack in almost the same shape as when we first called loop.

Effectively, we have *updated* the stack slots for no and len.

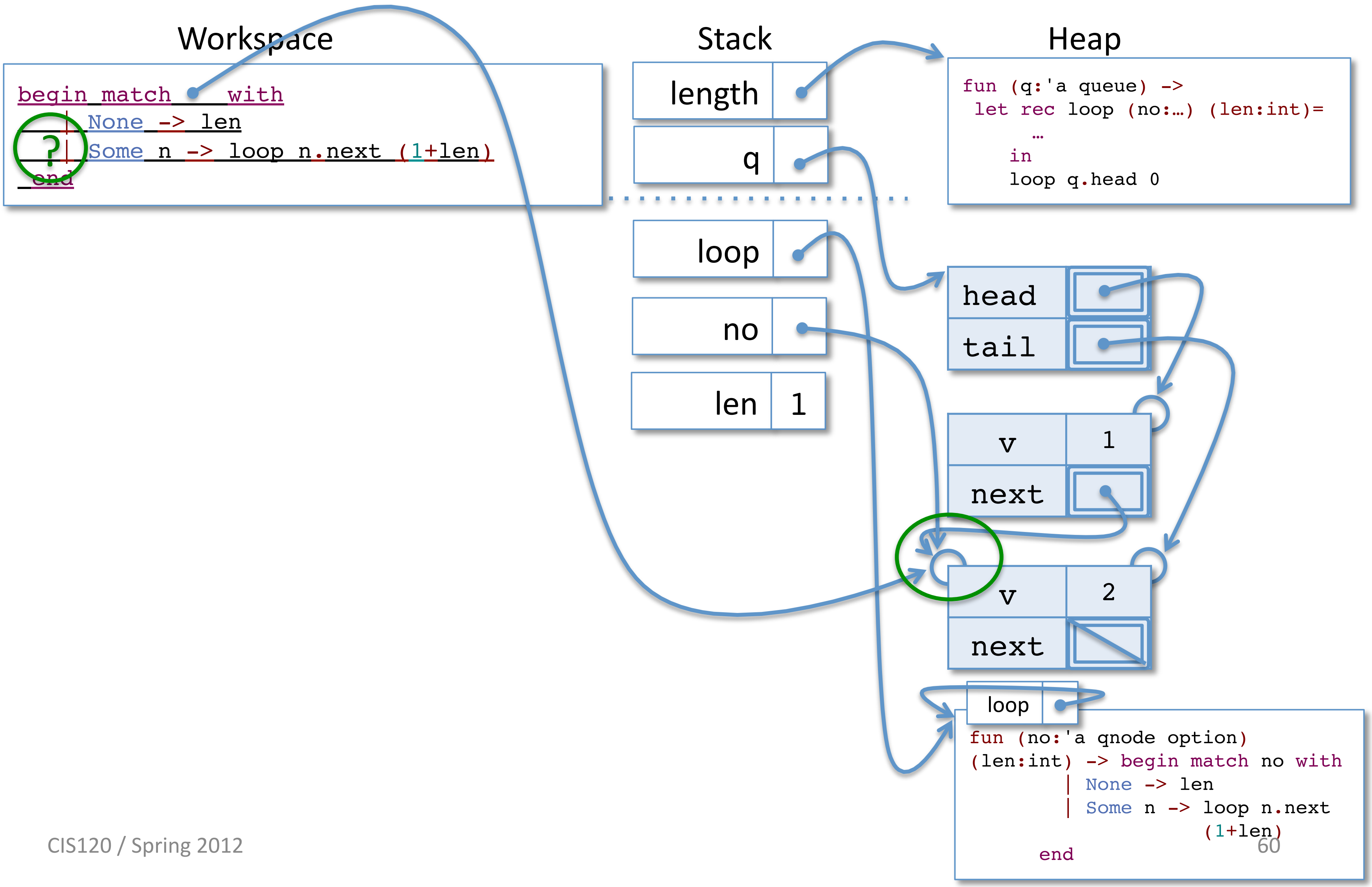
Tail Calls and Iterative length



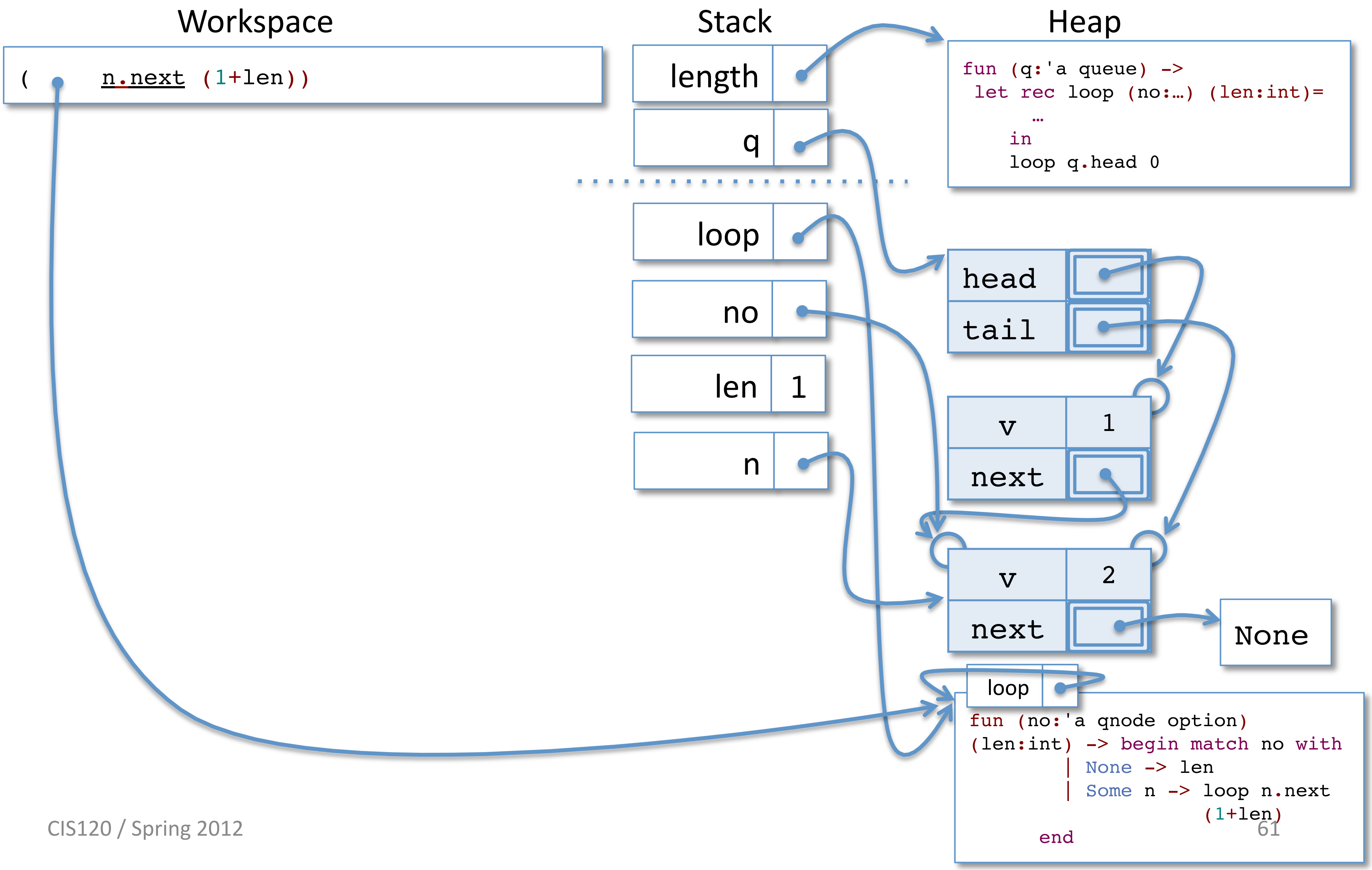
Tail Calls and Iterative length



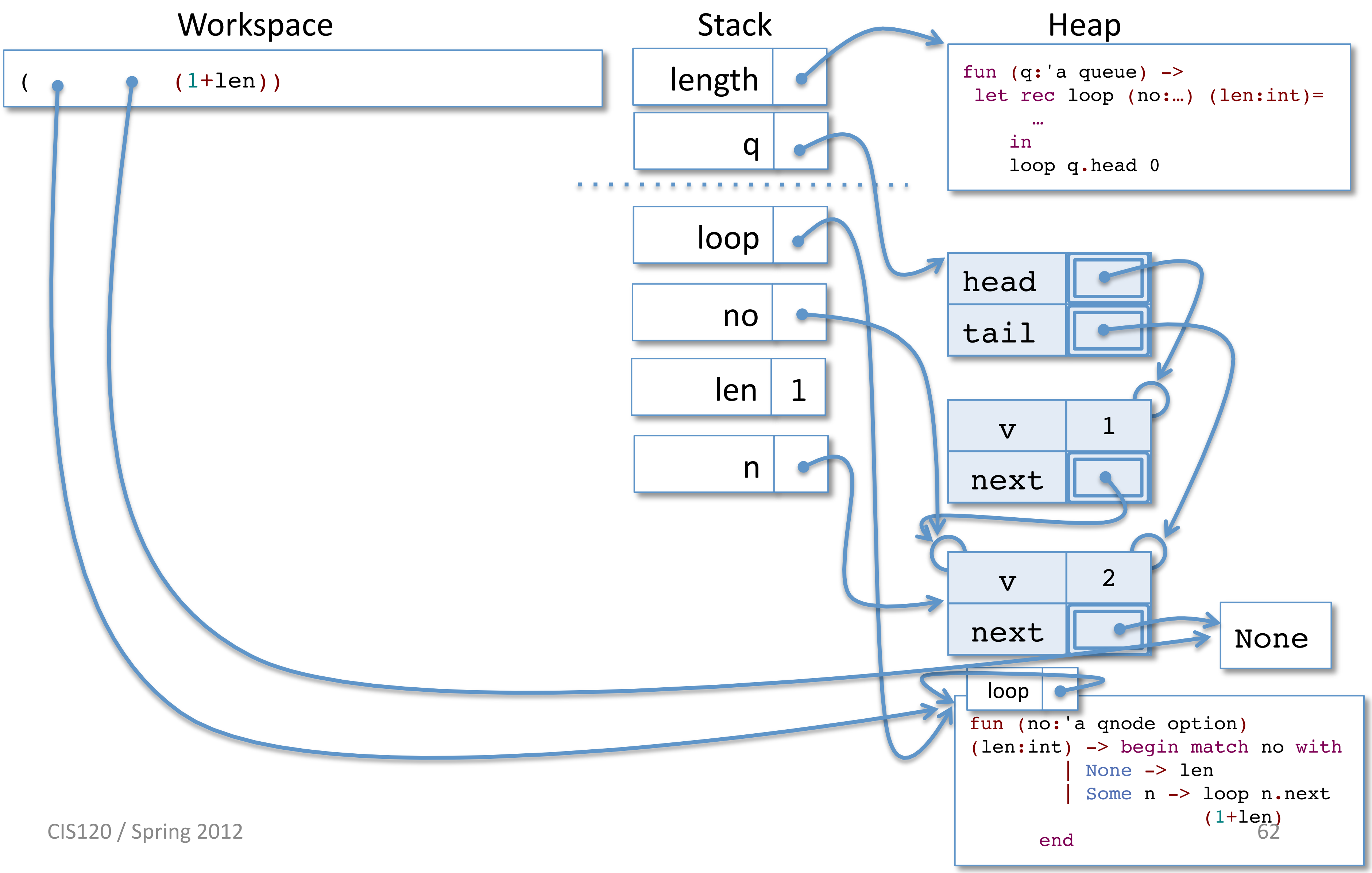
Tail Calls and Iterative length



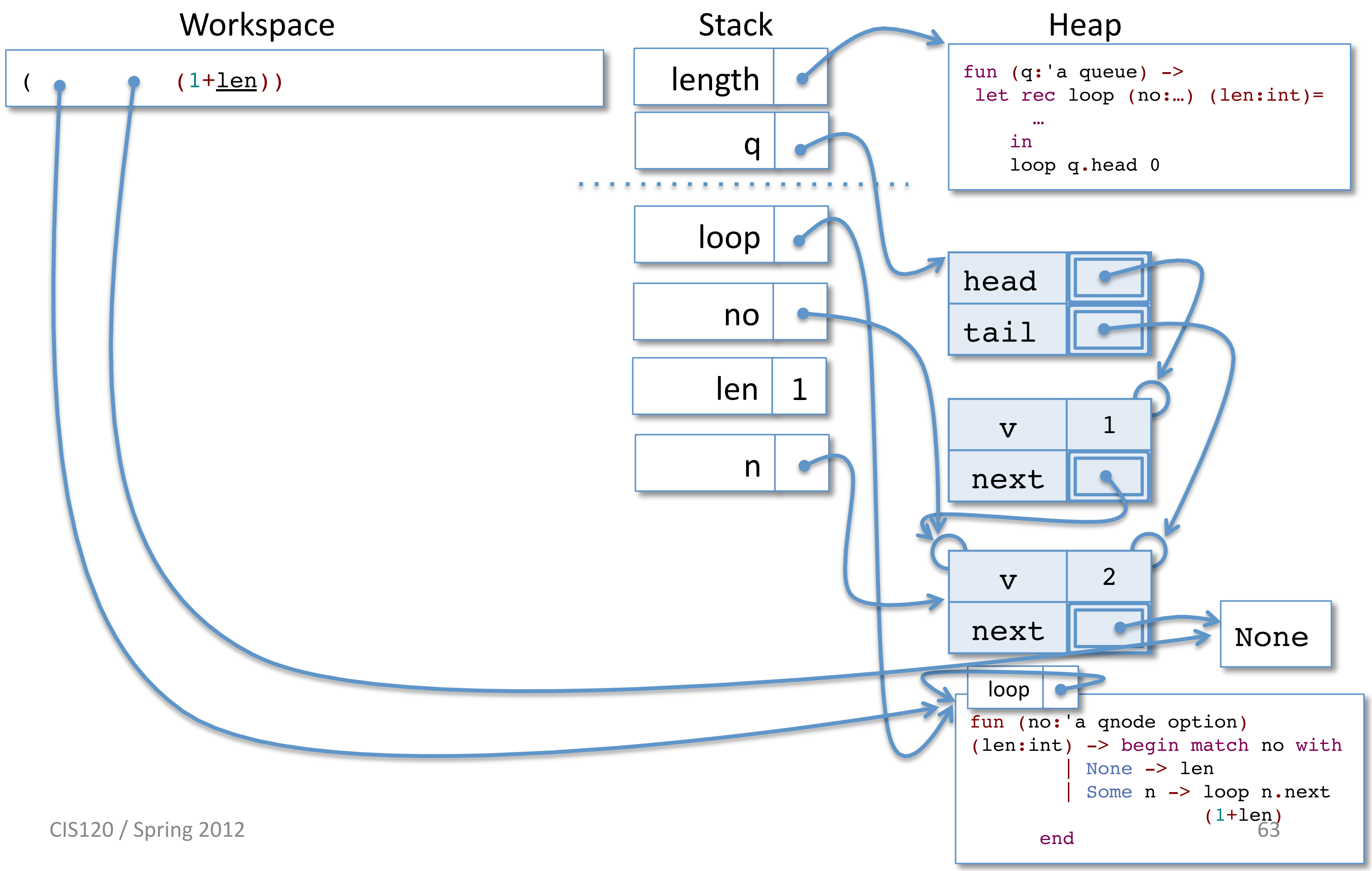
Tail Calls and Iterative length



Tail Calls and Iterative length



Tail Calls and Iterative length



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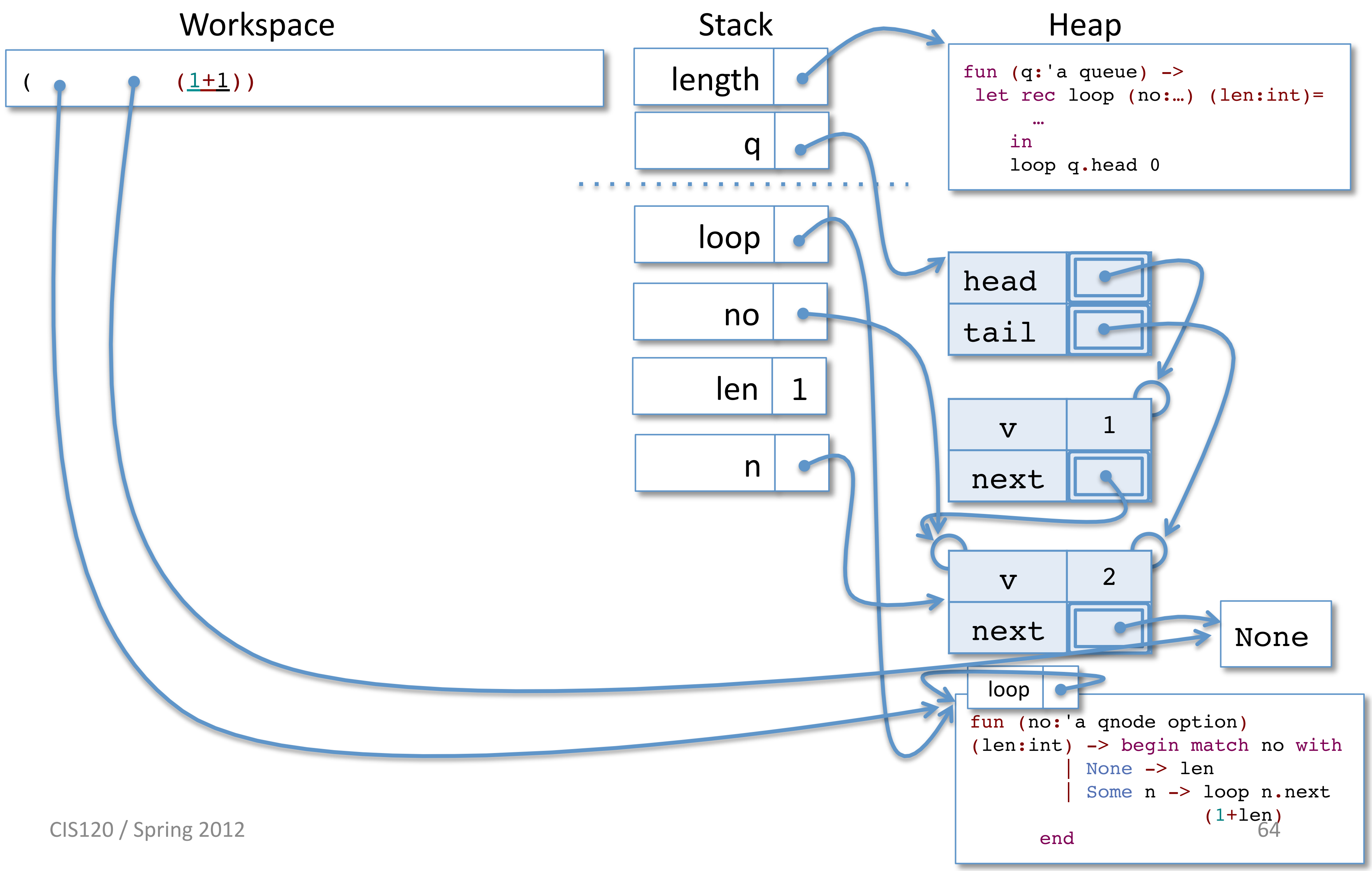
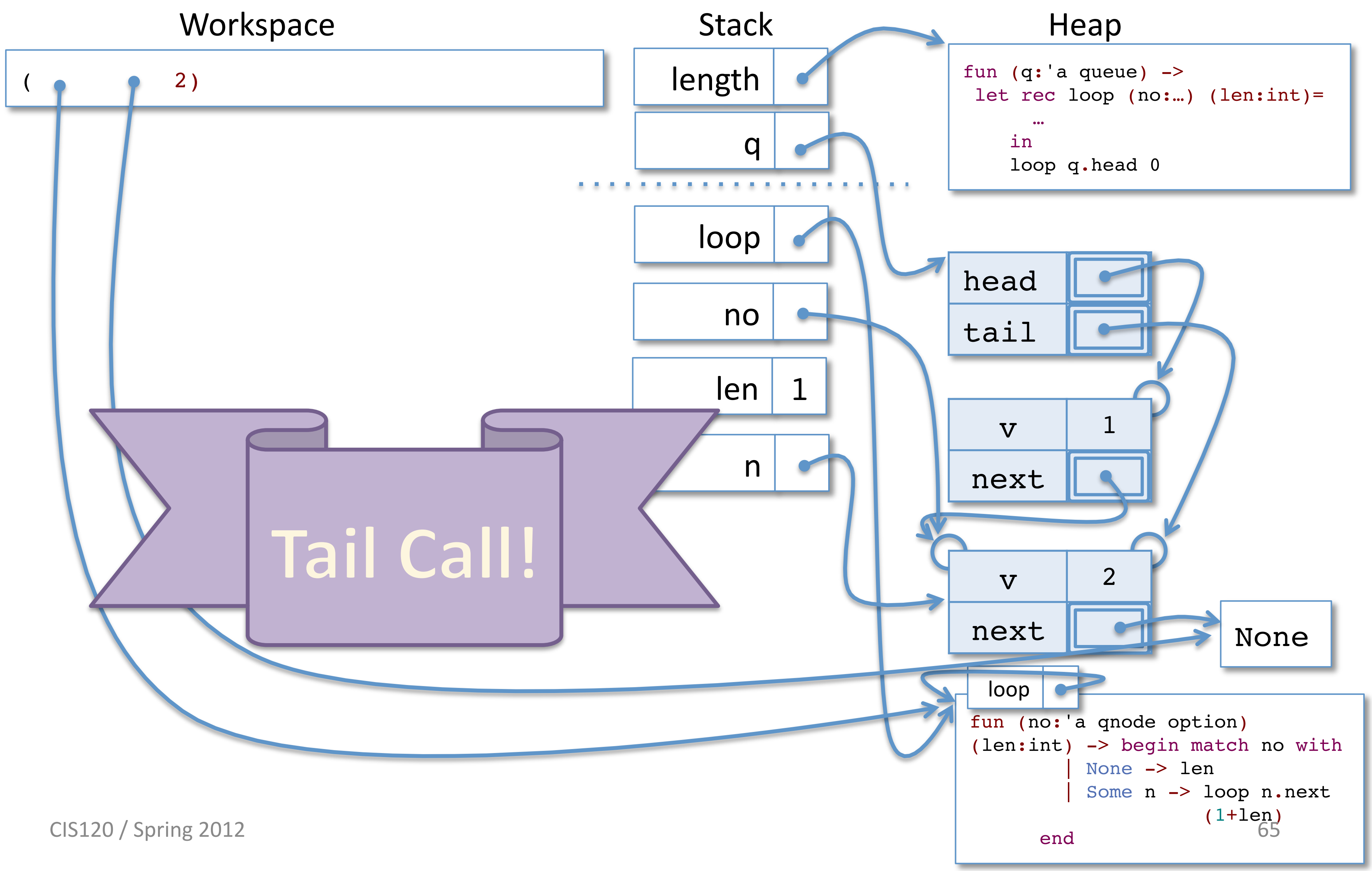


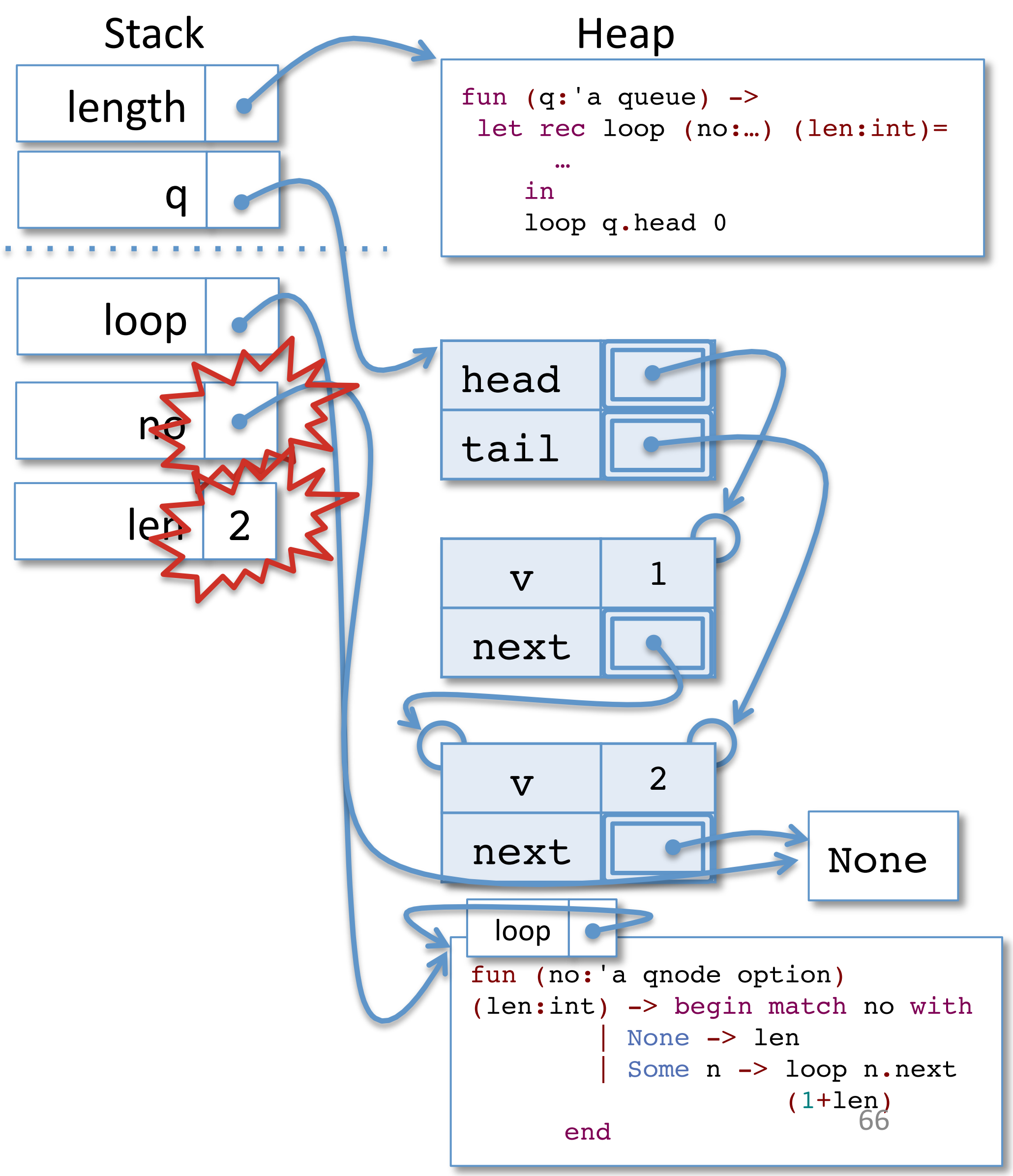
Tail Calls and Iterative length



Tail Calls and Iterative length

Workspace

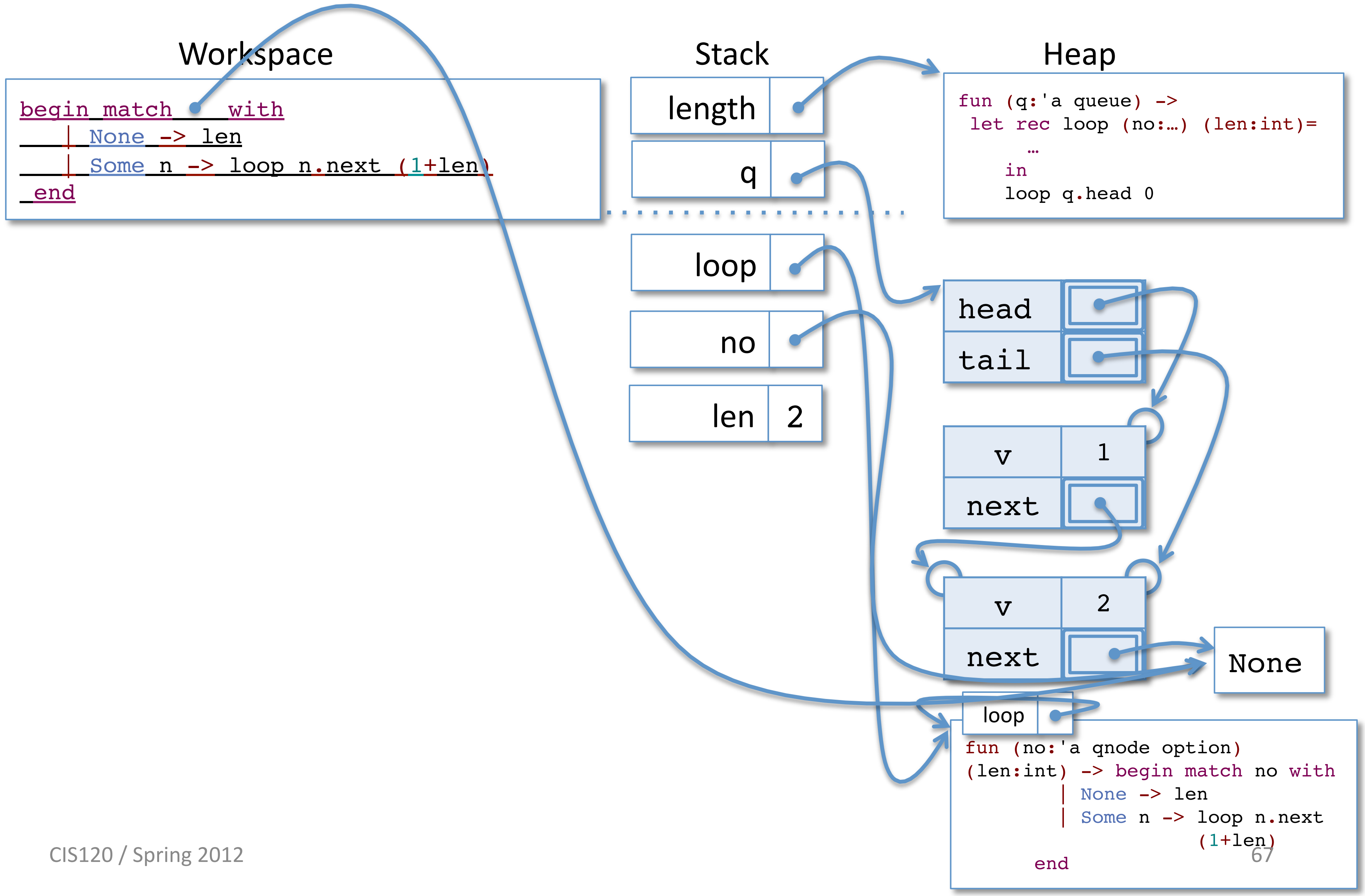
```
begin match no with
    | None -> len
    | Some n -> loop n.next (1+len)
 end
```



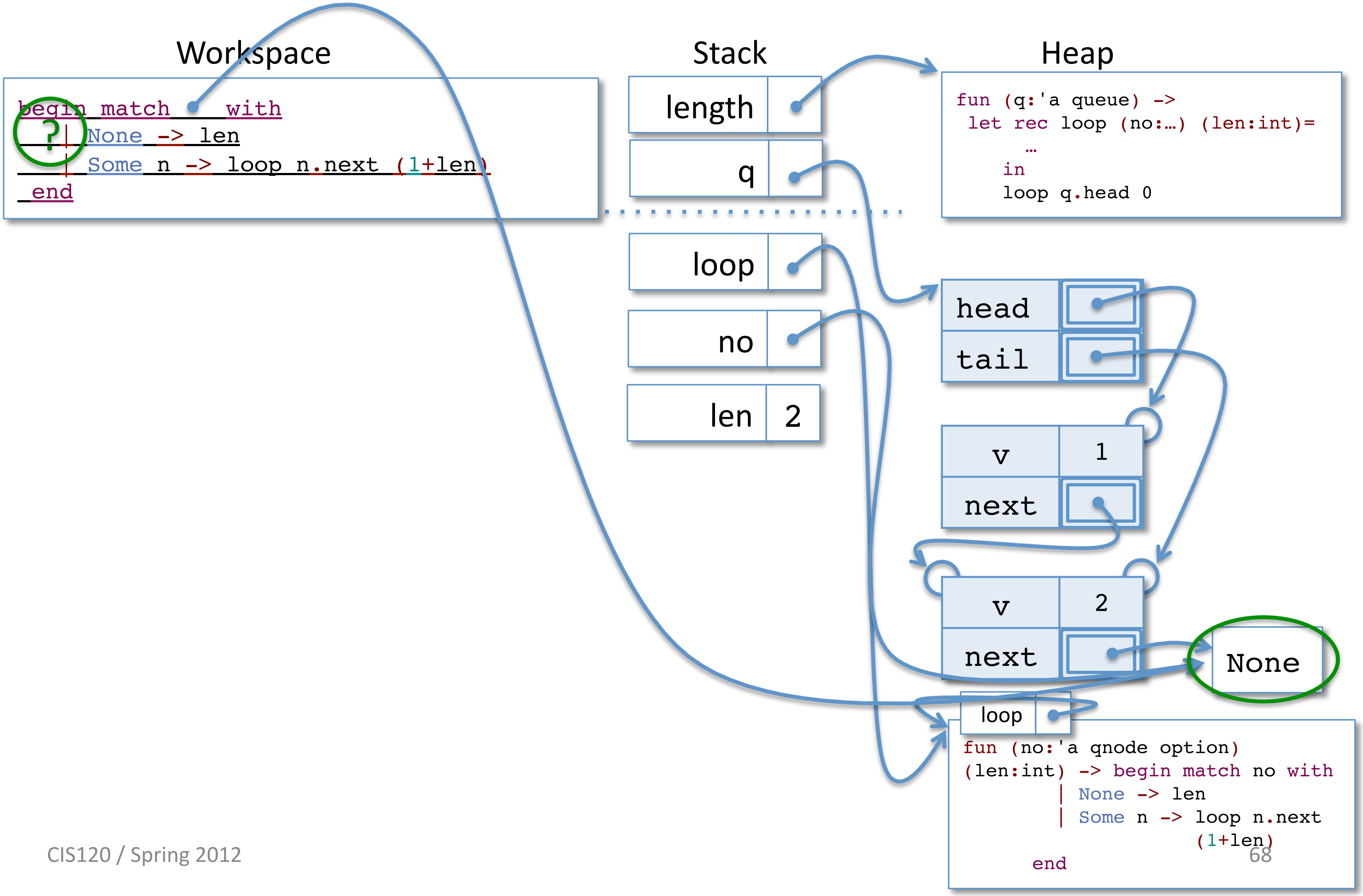
Note: Again, the tail call leaves the stack as before, but effectively updates the values of no and len.

We may as well call this in-place-update of the stack, even though technically these bindings are not mutable!

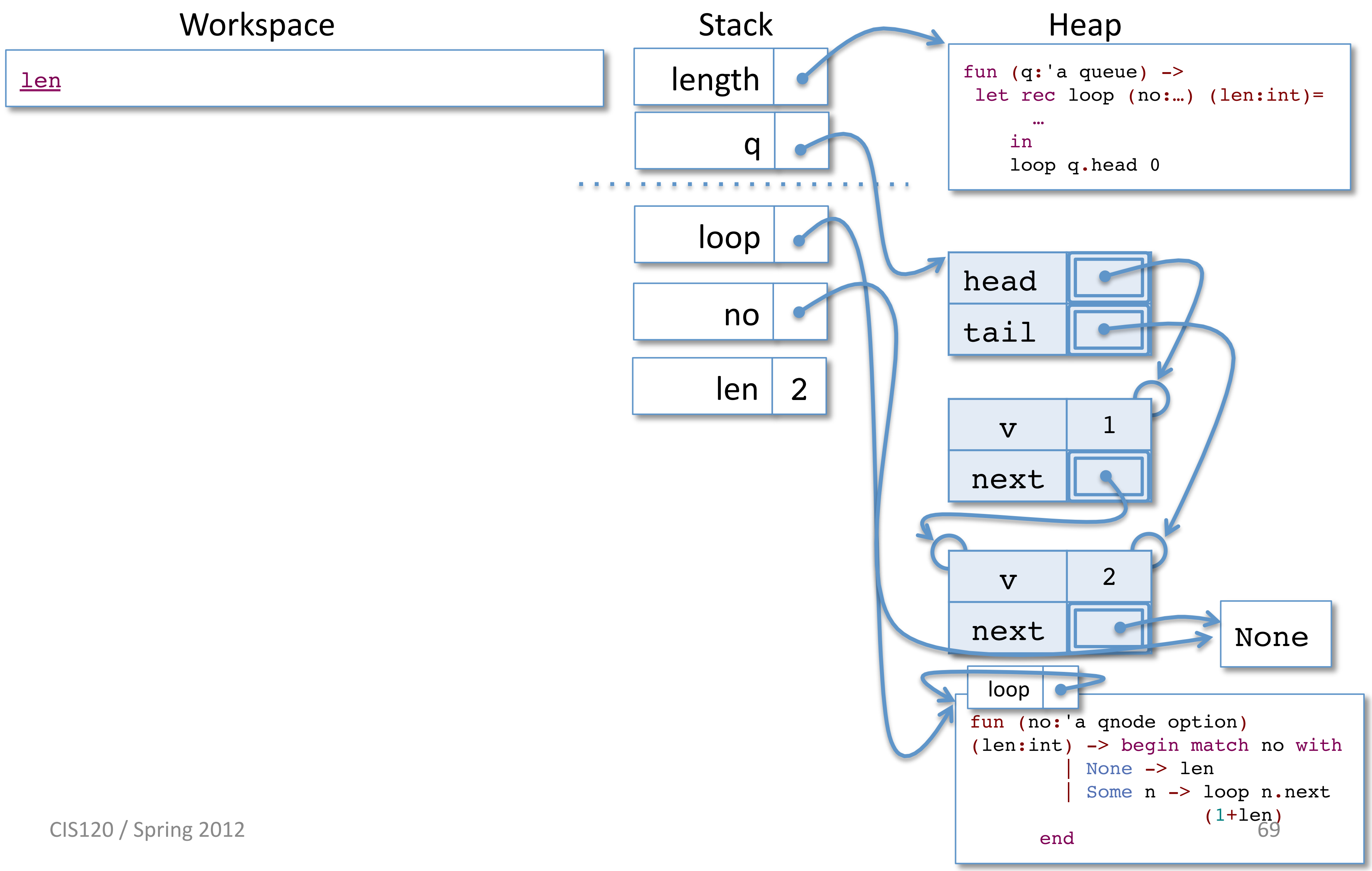
Tail Calls and Iterative length



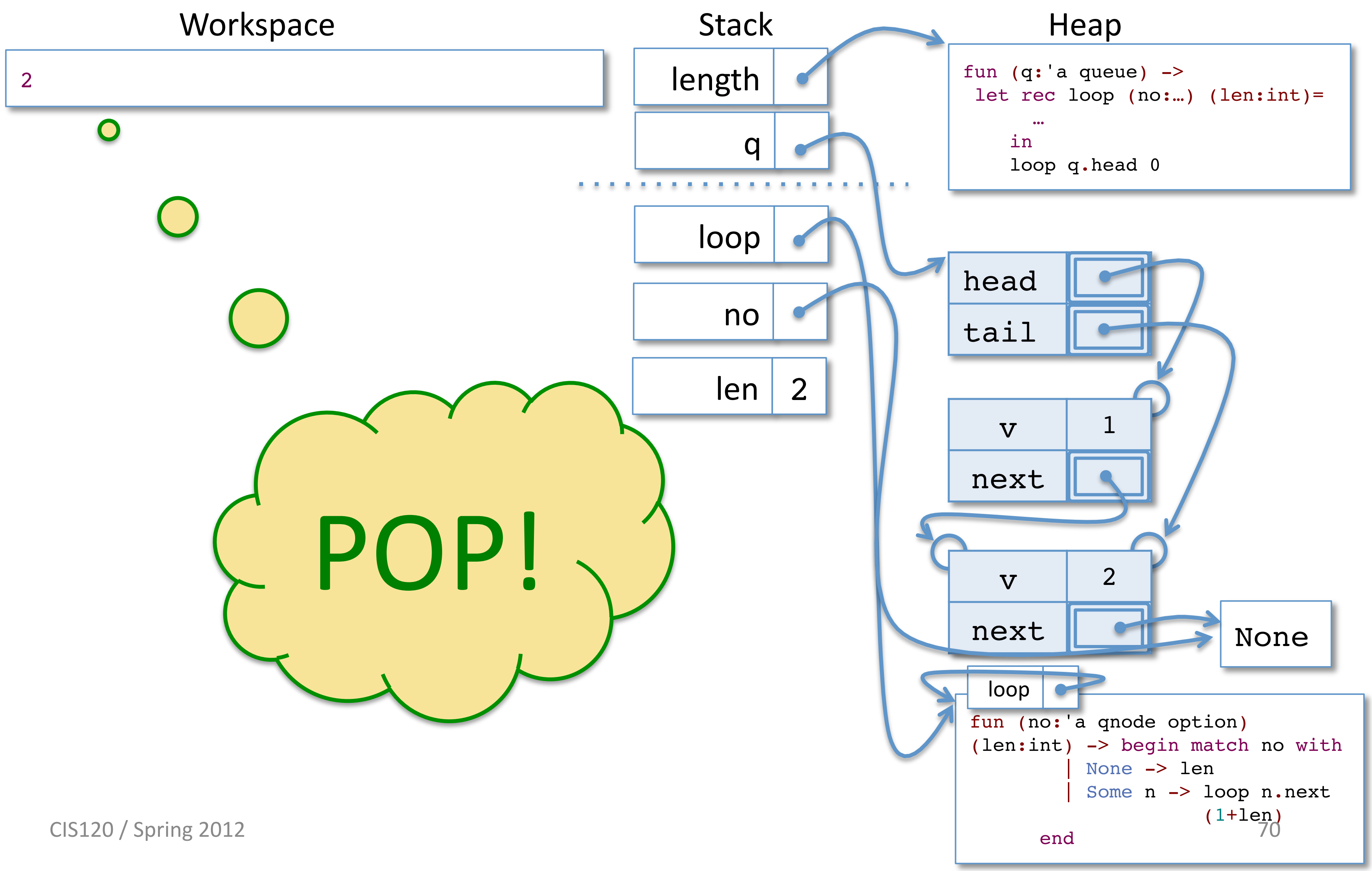
Tail Calls and Iterative length



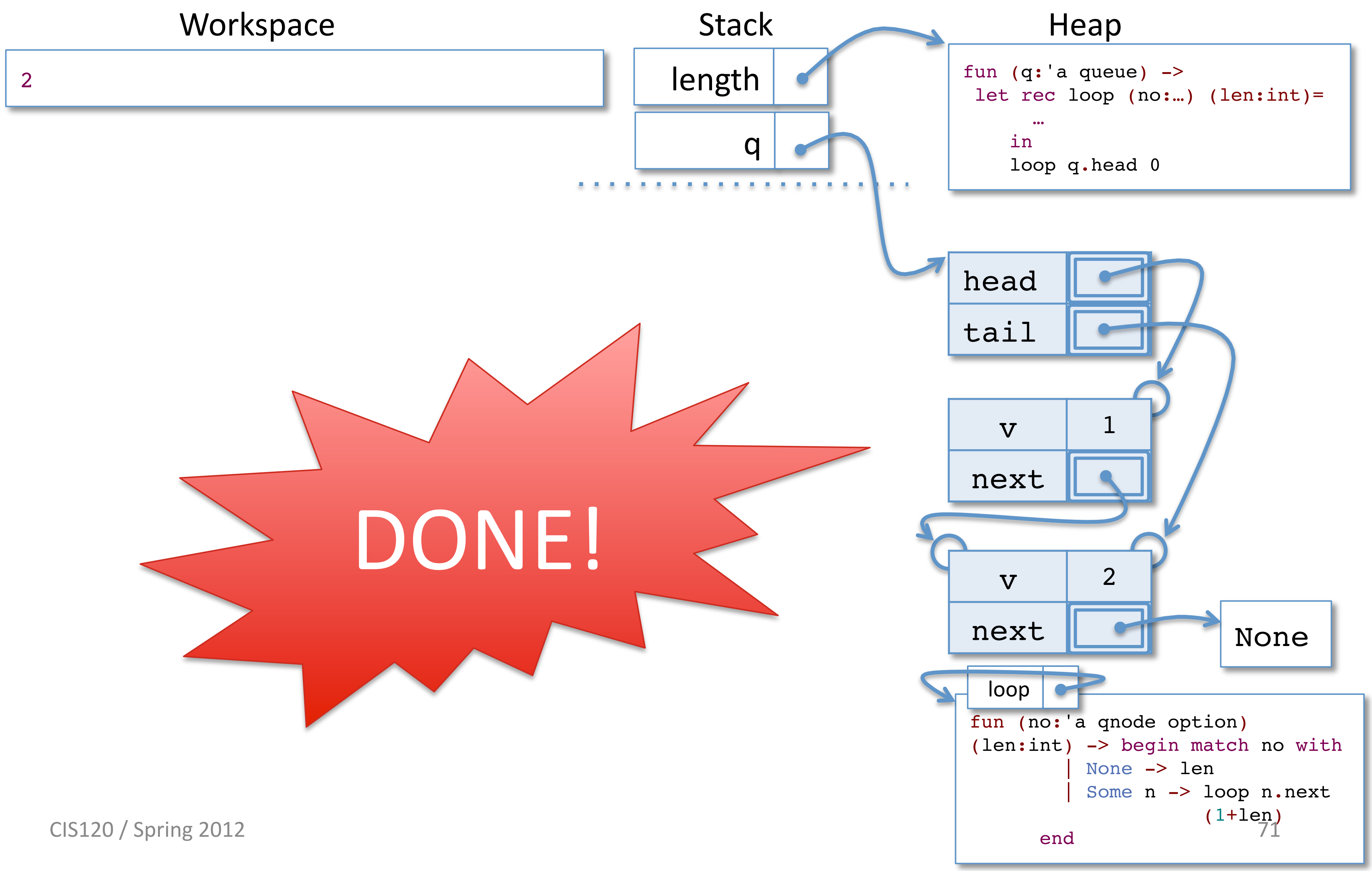
Tail Calls and Iterative length



Tail Calls and Iterative length



Tail Calls and Iterative length



Some Observations

- Tail call optimization lets the stack take only a fixed amount of space.
- The "recursive" call to loop (effectively) updates some of the stack bindings in place.
  - We can think of these bindings as the *state* being modified by each iteration of the loop.

- These two properties are the essence of iteration.
  - They are the difference between recursion and iteration
  - Most imperative programs provide iteration with "while" "for", and the related "break" and "continue" operations.
  - Tail recursion generalizes all of these.

`to_list` (using iteration)

```
(* Retrieve the list of values stored in the queue,
   ordered from head to tail. *)
let to_list (q: 'a queue) : 'a list =
  let rec loop (no: 'a qnode option) (l:'a list) : 'a list =
      begin match no with
      | None -> List.rev l
      | Some n -> loop n.next (n.v::l)
      end
  in loop q.head []
```

- Here, the state maintained across each iteration of the loop is the queue "index pointer" no and the (reversed) list of elements traversed.
- The "exit case" post processes the list be reversing it.

Infinite Loops

```
(* Accidentally go into an infinite loop… *)
let accidental_infinite_loop (q:'a queue) : int =
    let rec loop (qn:'a qnode option) (len:int) : int =
      begin match qn with
        | None -> len
        | Some n -> loop qn (len + 1)
      end
    in loop q.head 0
```

- This program will go into an infinite loop.
- Unlike a recursive program, which uses some space on each recursive call, there is no resource being exhausted, so the program will “silently diverge” and never produce an answer…

`print` (using iteration)

```
let print (q:'a queue) (string_of_element:'a -> string) : unit =
  let rec loop (no: 'a qnode option) : unit =
    begin match no with
      | None -> ()
      | Some n -> print_endline (string_of_element n.v);
                  loop n.next
    end
  in
    print_endline "--- queue contents ---";
    loop q.head;
    print_endline "--- end of queue -----"
```

- Here, the only state needed is the queue "index pointer".

`get_tail` (using iteration)

```
(* get the tail (if any) from a queue *)
let rec get_tail (q: 'a queue) : 'a qnode option =
    let rec loop (qn: 'a qnode) (seen: 'a qnode list)
        : 'a qnode option =
      begin match qn.next with
        | None -> Some qn
        | Some n ->
            if contains_alias n seen then None
            else loop n (qn::seen)
      end
    in loop q.head []
```

- This function does *not* assume that q has no cycles.
- It returns Some n if n is a "tail" reachable from q.head
- It returns None if there is a cycle in the queue found by following next pointers.
- The state is an index pointer and a list of all the nodes seen.
  - contains_alias is a helper function that checks to see whether n has an alias in the list