Programming Languages and Techniques (CIS120)

Lecture 7

Jan 25, 2013

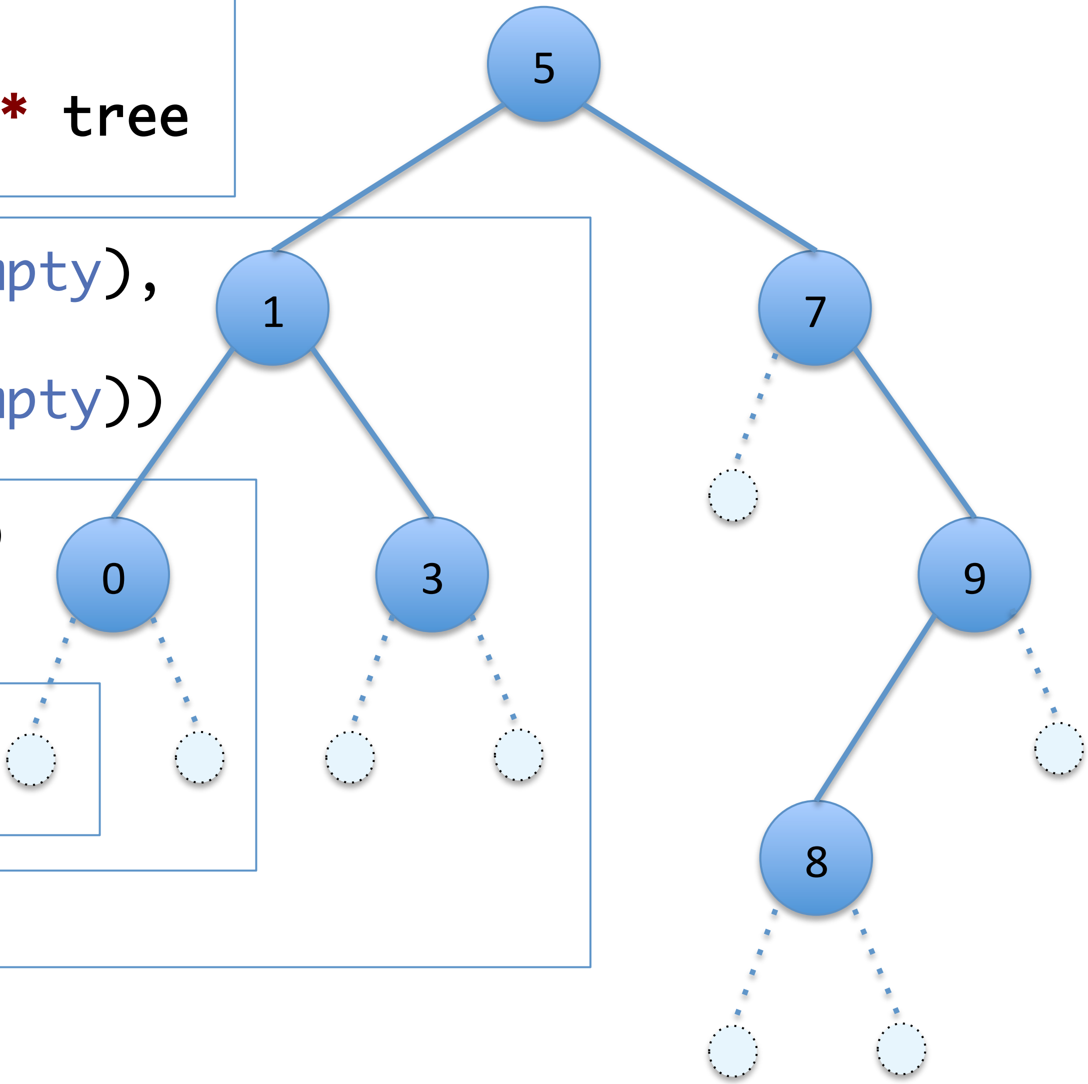
Binary Search Trees

Announcements

- Homework 2 is due *Tuesday*, Jan 29th, at 11:59:59pm
- Did the CIS 120 web page get hacked this week?

Representing trees

```
type tree =
| Empty
| Node of tree * int * tree
```

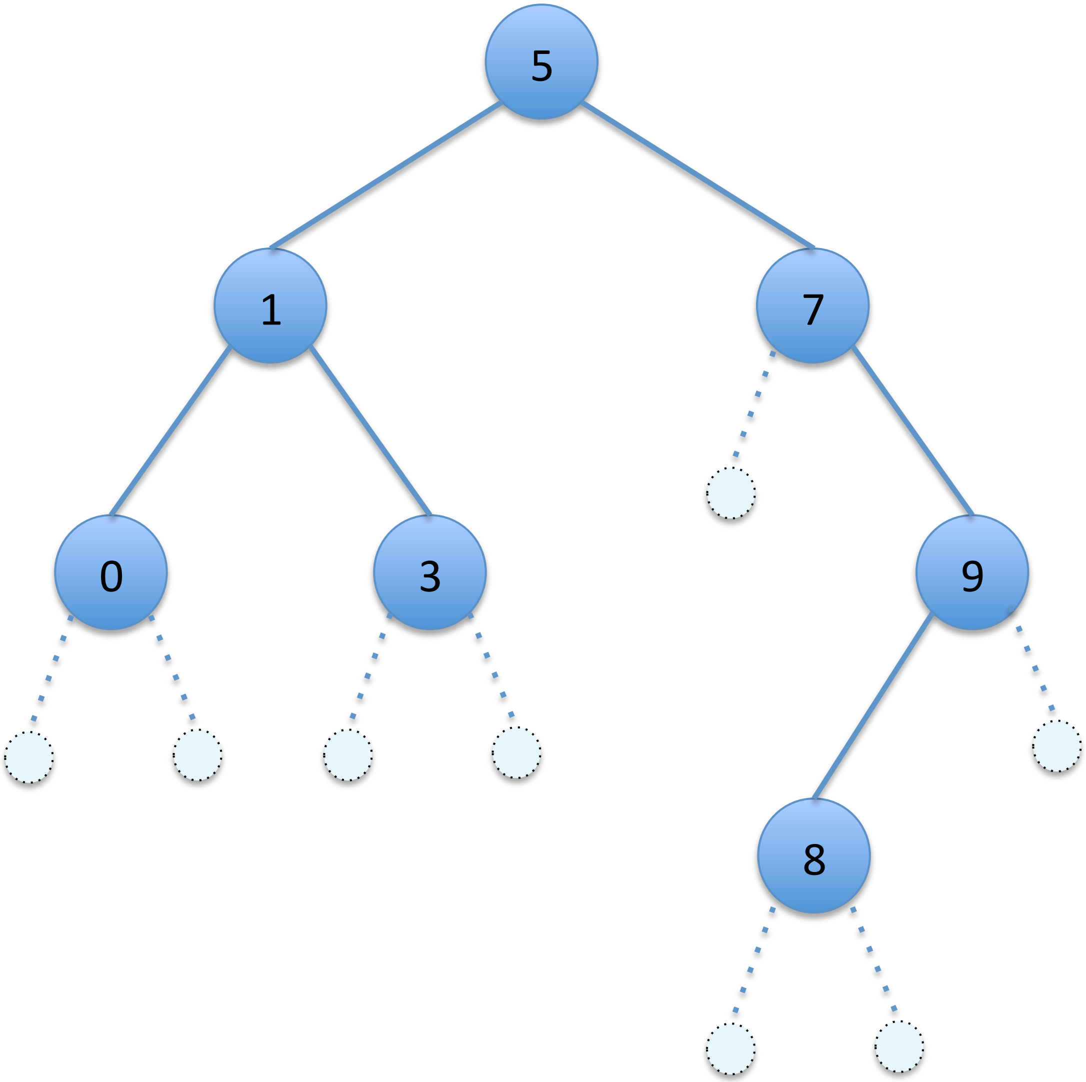


```
Node (Node (Empty, 0, Empty),
      1,
      Node (Empty, 3, Empty))
```

```
Node (Empty, 0, Empty)
```

```
Empty
```

Demo: bst.ml



Trees as Containers

- Like lists, trees aggregate ordered data
- Like lists, we can determine whether the data structure *contains* a particular element

- CHALLENGE: can we use the tree structure to make this process faster?

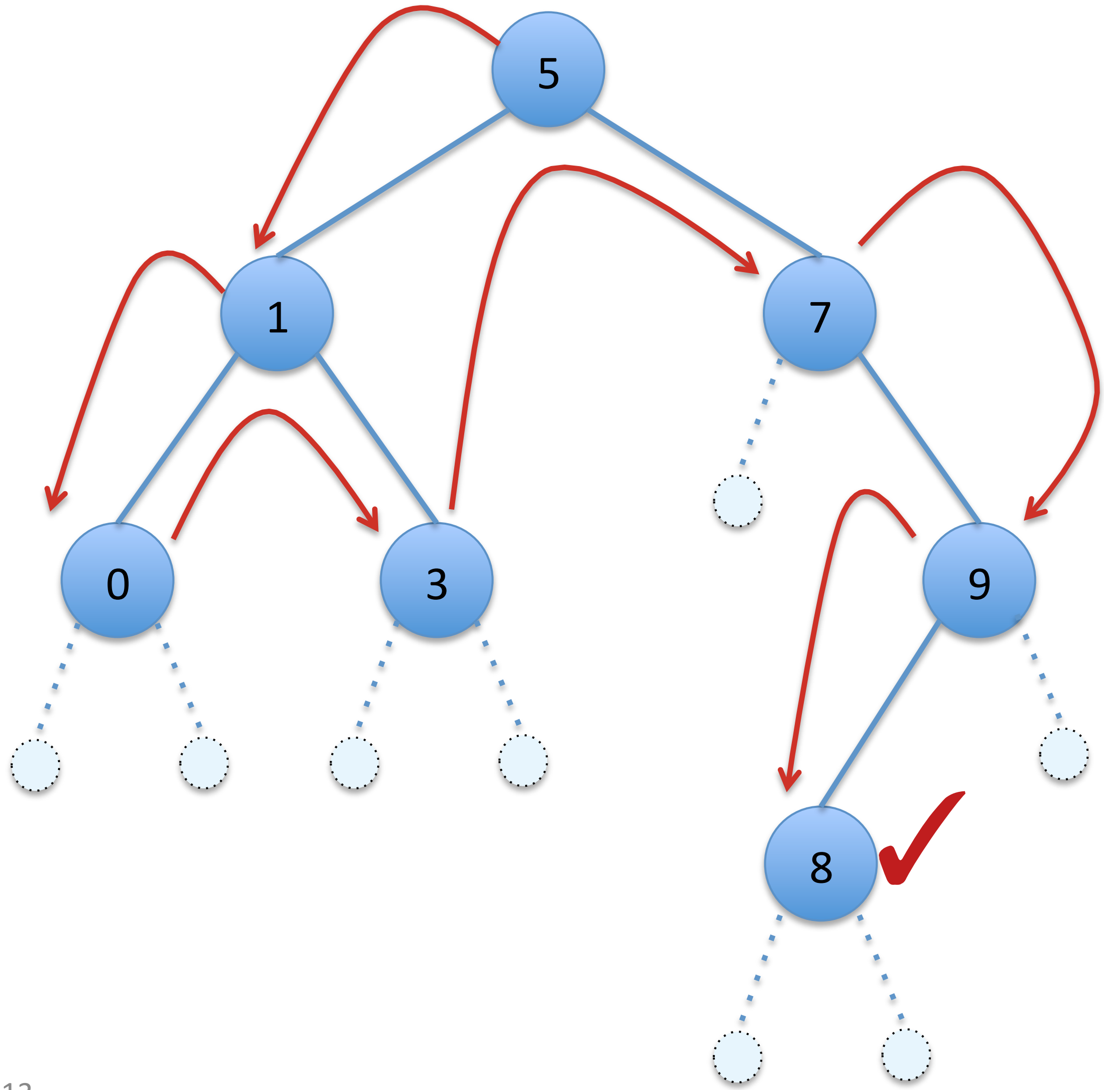
Searching for Data in a Tree

- Recall the contains function:

```
let rec contains (t:tree) (n:int) : bool =
  begin match t with
  | Empty -> false
  | Node(lt,x,rt) -> x = n ||
                       (contains lt n) || (contains rt n)
  end
```

- It searches through the tree, looking for n
  - In this case, the search is a *pre-order* traversal of the tree
  - Other traversal strategies would work equally well
- In the worst case, it might search through the entire tree
- Can we do better?

Search during `(contains t 8)`

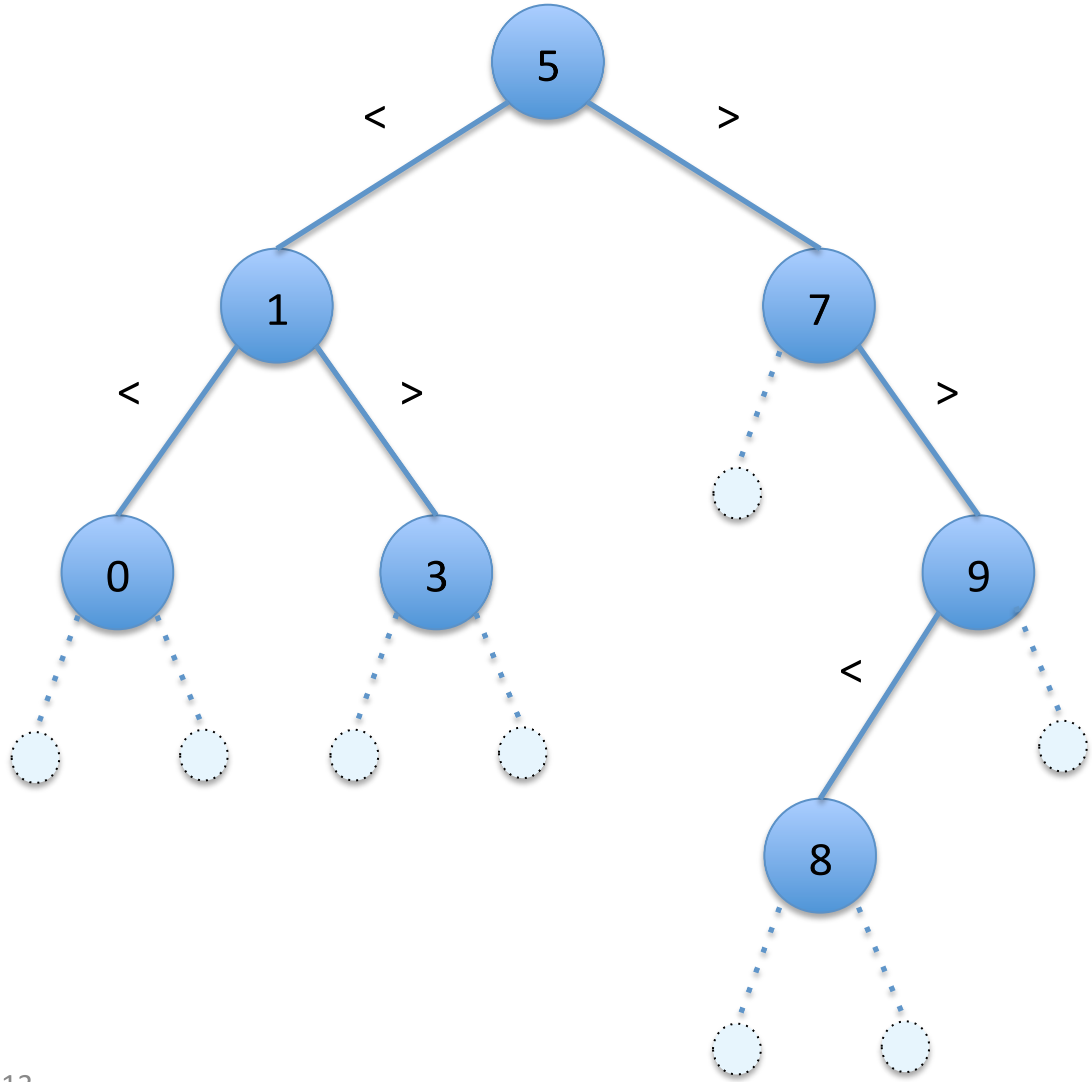


Binary Search Trees (BST)

- Key insight: *Ordered* data can be searched more quickly than unordered data.
  - This is why telephone books are arranged alphabetically
  - But requires the ability to focus on *half* of the current data
- A BST is a binary tree with additional *invariants:*

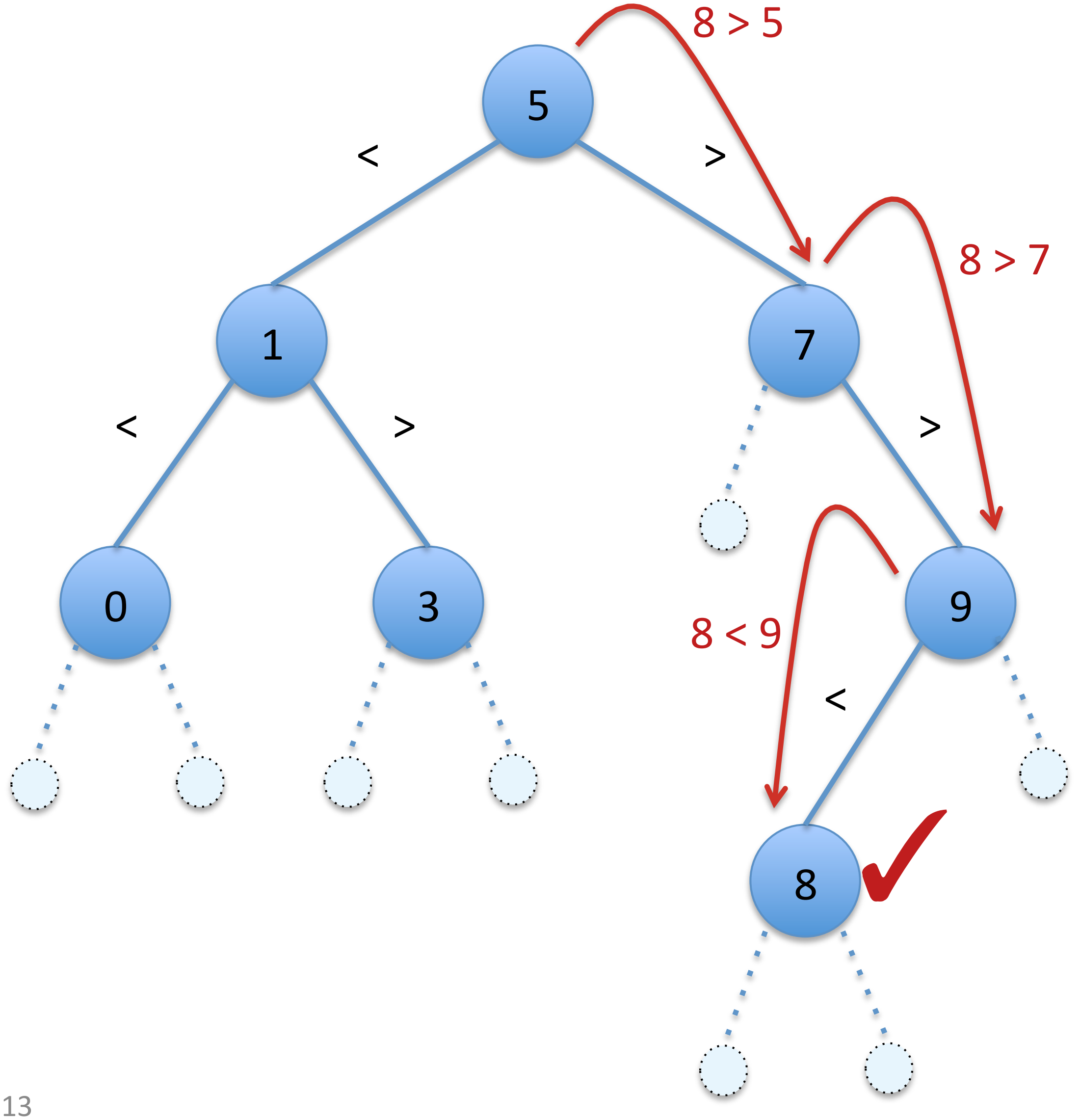
- `Empty` is a BST
- `Node(lt,x,rt)` is a BST if
  - `lt` and `rt` are both BSTs
  - all nodes of `lt` are `<` `x`
  - all nodes of `rt` are `>` `x`

An Example Binary Search Tree



Note that the BST invariants hold for this tree.

Search in a BST: `(lookup t 8)`



Searching a BST

```
(* Assumes that t is a BST *)
let rec lookup (t:tree) (n:int) : bool =
  begin match t with
  | Empty -> false
  | Node(lt,x,rt) ->
      if x = n then true
       else if n < x then (lookup lt n)
       else (lookup rt n)
  end
```

- The BST invariants guide the search.
- Note that lookup may fail (i.e. return an incorrect answer) if the input is *not* a BST.

How to we construct a BST?

- Option 1:
  - Write a function to check whether an arbitrary tree satisfies the BST invariant.
  - Call the check whenever we need to know about a given tree.
- Option 2:
  - Create functions that *preserve* the BST invariant
  - Starting from some trivial BST (e.g. `Empty`), we can apply such functions to get other BSTs
  - Examples: `insert` and `delete`

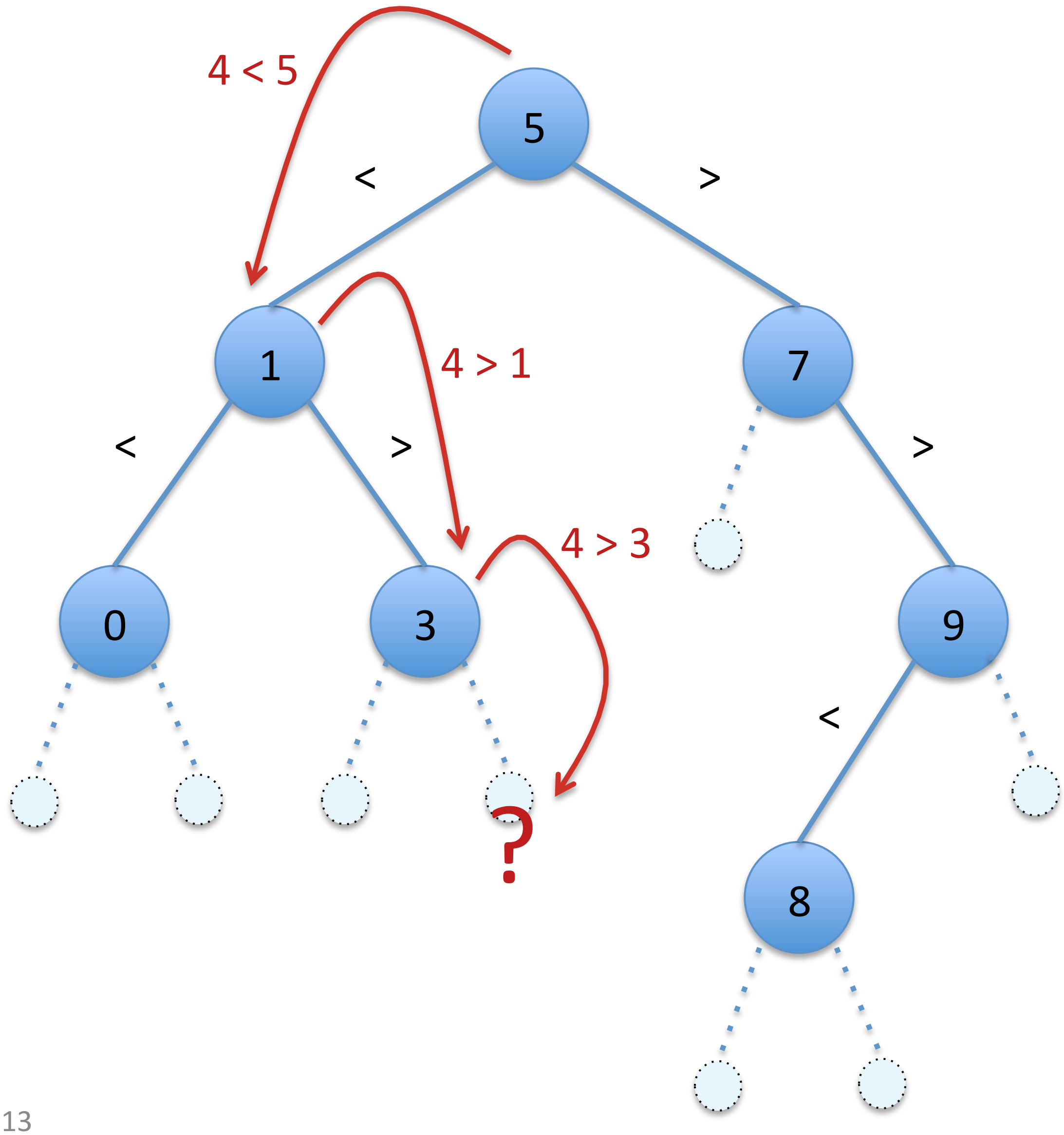
Checking the BST Invariants

```
(* Check whether all nodes of t are < n *)
let rec tree_less (t:tree) (n:int) : bool =
  begin match t with
  | Empty -> true
  | Node(lt,x,rt) ->
      x < n && (tree_less lt n) && (tree_less rt n)
  end
```

```
(* Determines whether t is a BST *)
let rec is_bst (t:tree) : bool =
  begin match t with
  | Empty -> true
  | Node(lt,x,rt) ->
      is_bst lt && is_bst rt &&
      (tree_less lt x) && (tree_gtr rt x)
  end
```

*Definition of tree_gtr omitted (it's similar to tree_less)

Inserting a new node: `(insert t 4)`



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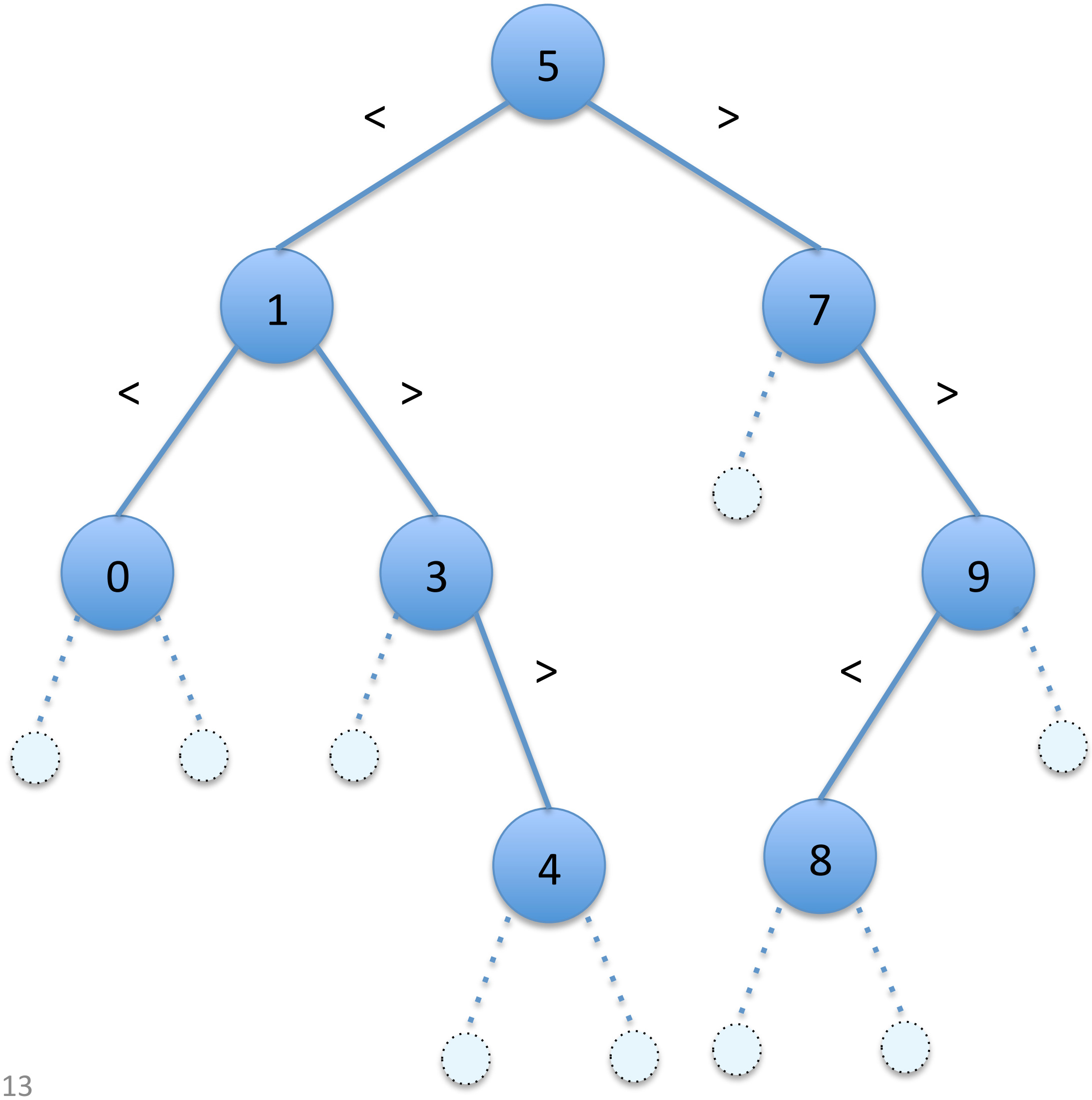

Inserting Into a BST

```
(* Inserts n into the BST t *)
let rec insert (t:tree) (n:int) : tree =
  begin match t with
  | Empty -> Node(Empty,n,Empty)
  | Node(lt,x,rt) ->
       if x = n then t
        else if n < x then Node(insert lt n, x, rt)
        else Node(lt, x, insert rt n)
  end
```

- Note the similarity to searching the tree.
- Assuming that t is a BST, the result is also a BST. Why?