Programming Languages and Techniques (CIS120)

Lecture 15

February 17, 2016

Mutable Queues

Lecture notes: Chapter 16

```
type point = {mutable x:int; mutable y:int}
```

What answer does the following expression produce?

```
let p1 = {x=0; y=0} in
let p2 = p1 in
p1.x <- 17;
p2.x <- 42;
p1.x
```

1. 17
2. 42
3. 0
4. runtime error

Answer: 42

Allocate a Record

Workspace

```
let p1 : point = {x=1; y=1;}
let p2 : point = p1
let ans : int =
    p2.x <- 17; p1.x
```

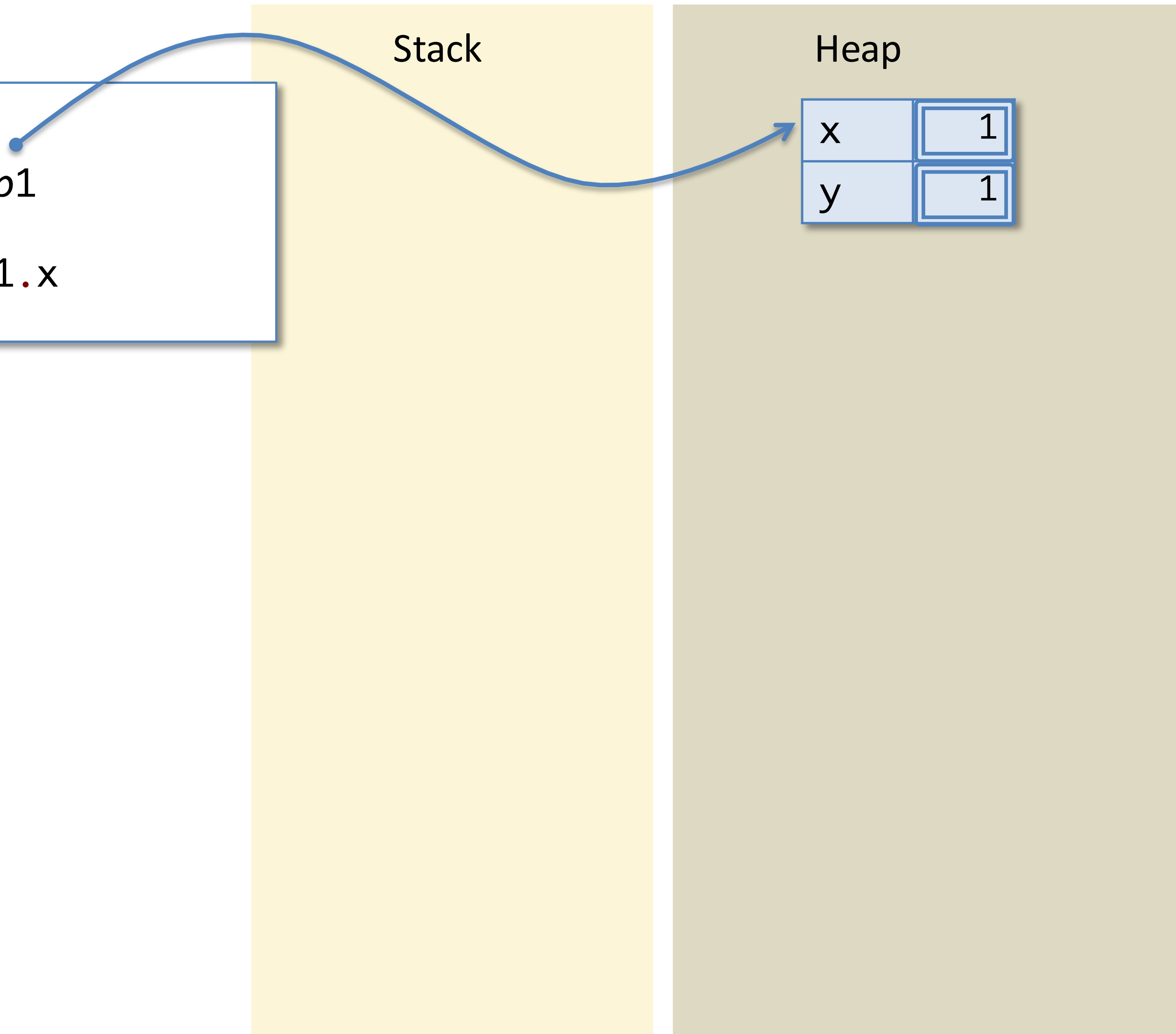
Stack

Heap

Allocate a Record

Workspace

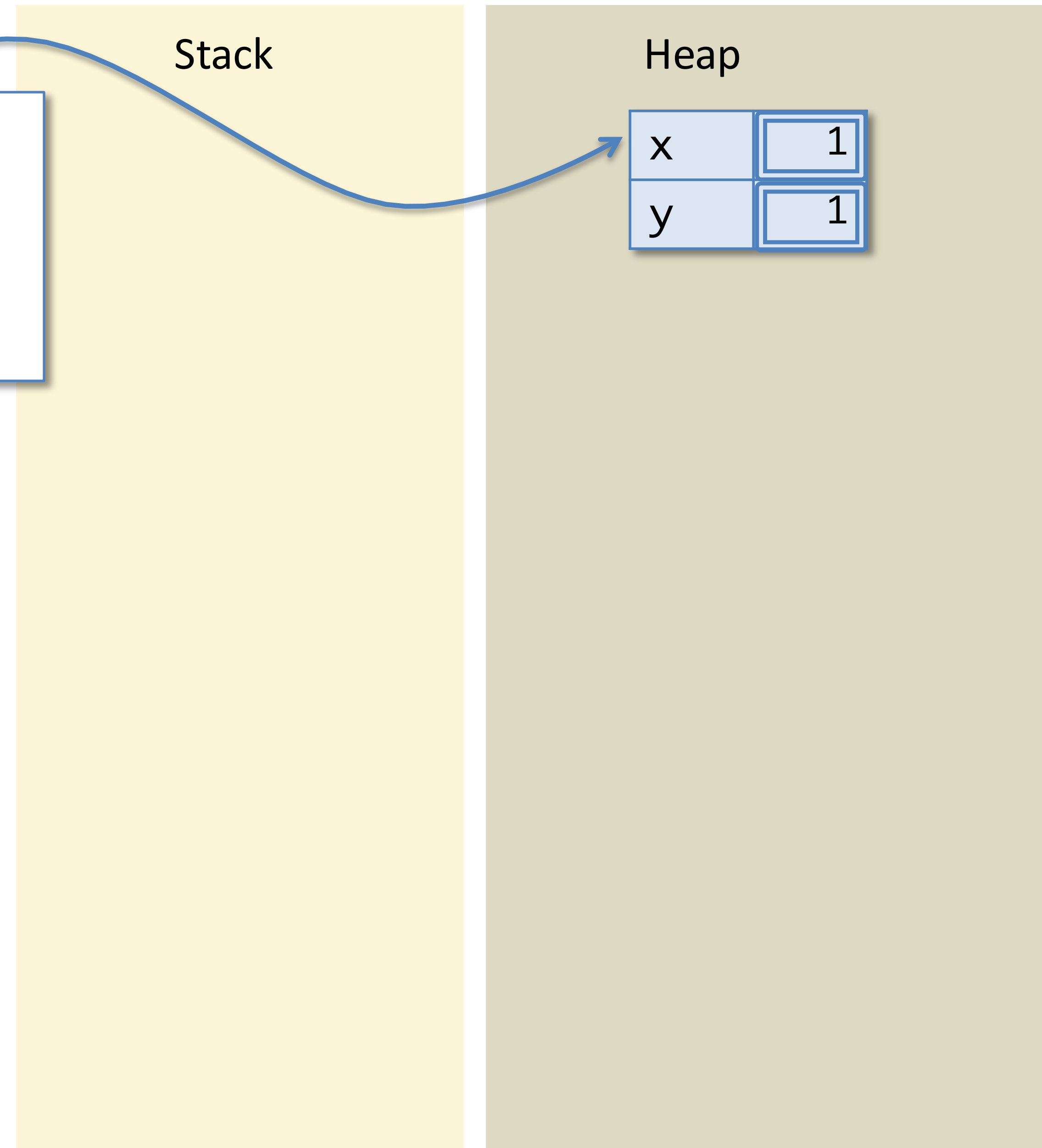
```
let p1 : point =
let p2 : point = p1
let ans : int =
    p2.x <- 17; p1.x
```



Let Expression

Workspace

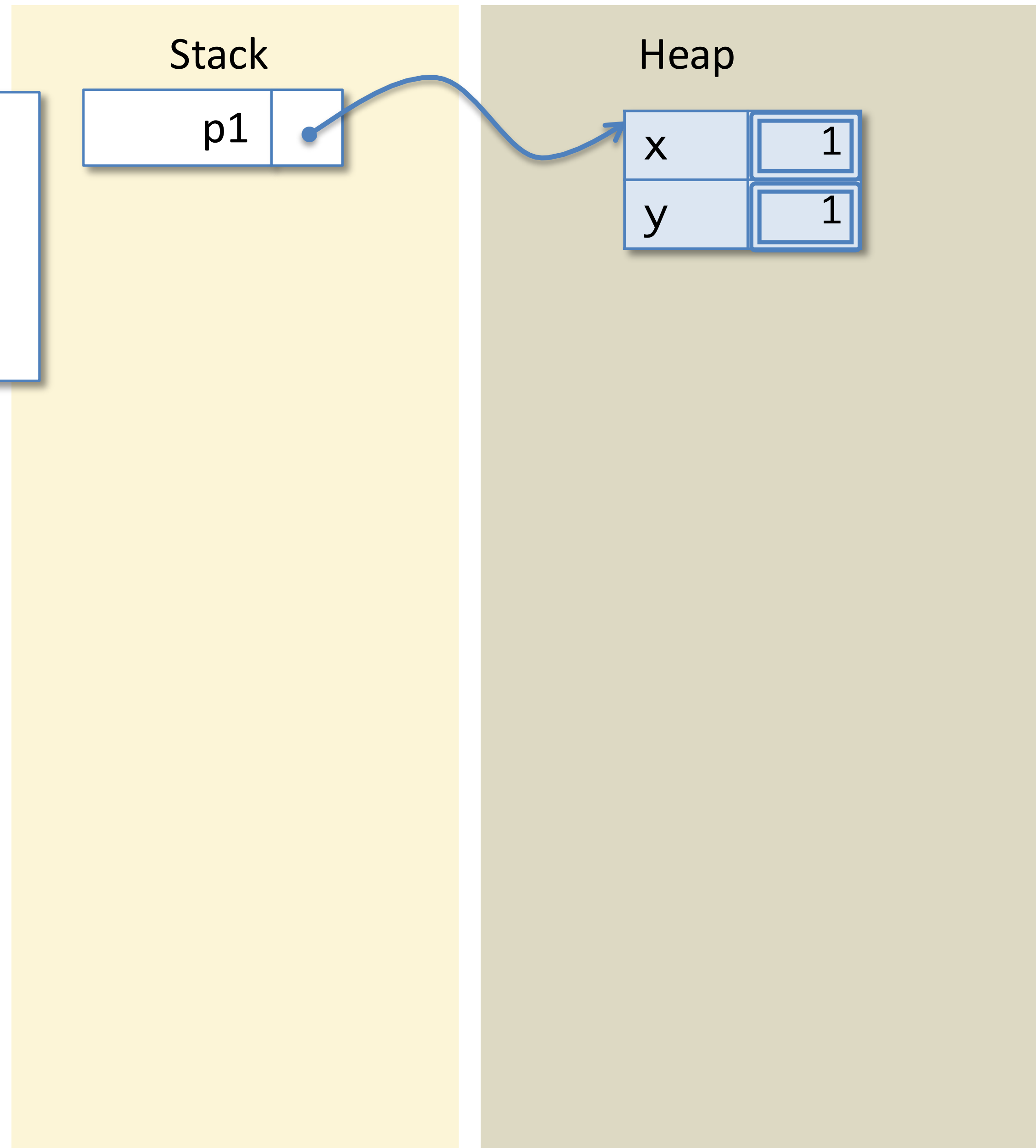
```
let p1 : point =  .
let p2 : point = p1
let ans : int =
    p2.x <- 17; p1.x
```



Push p1

Workspace

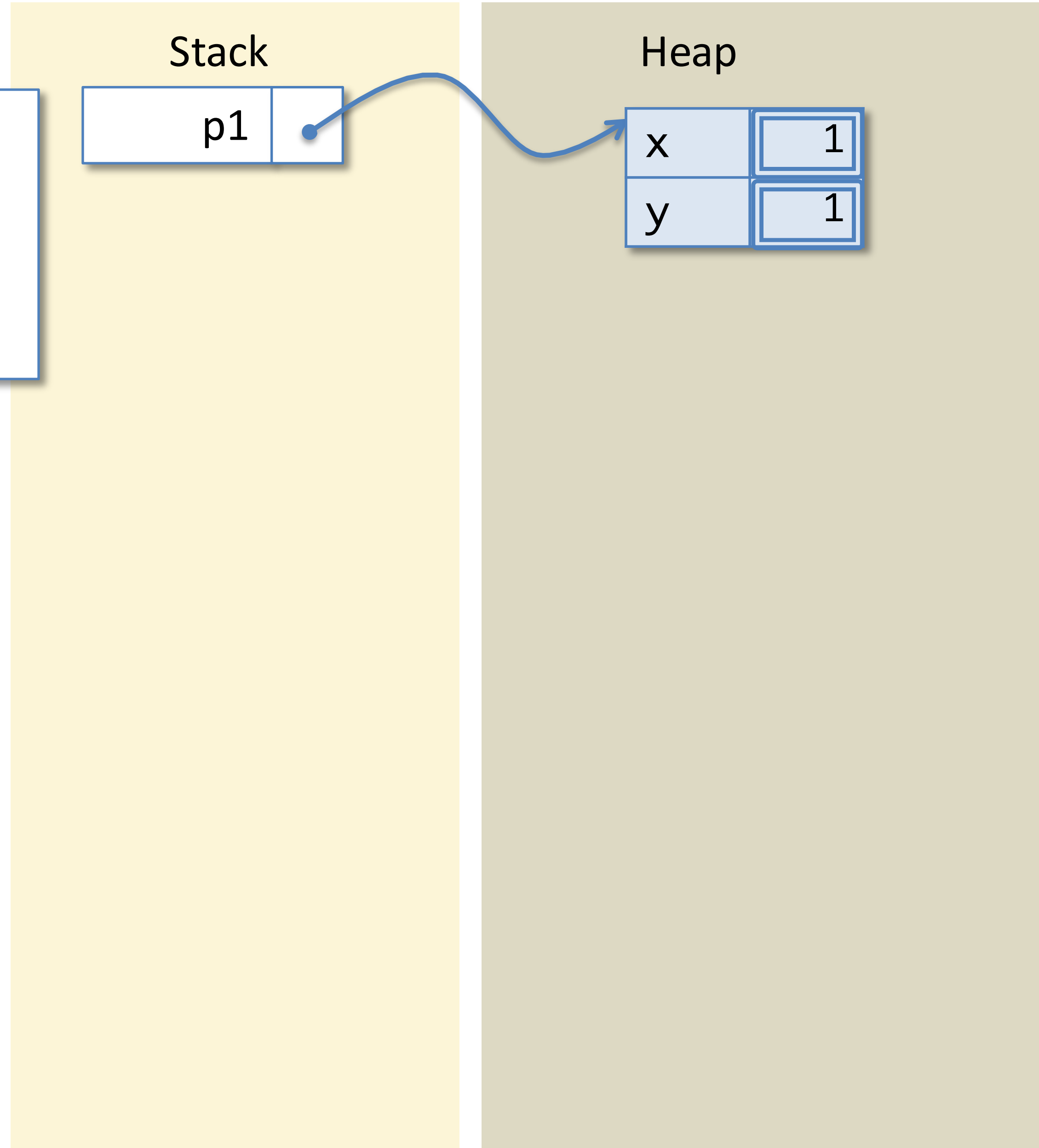
```
let p2 : point = p1
let ans : int =
    p2.x <- 17; p1.x
```



Look Up ‘p1’

Workspace

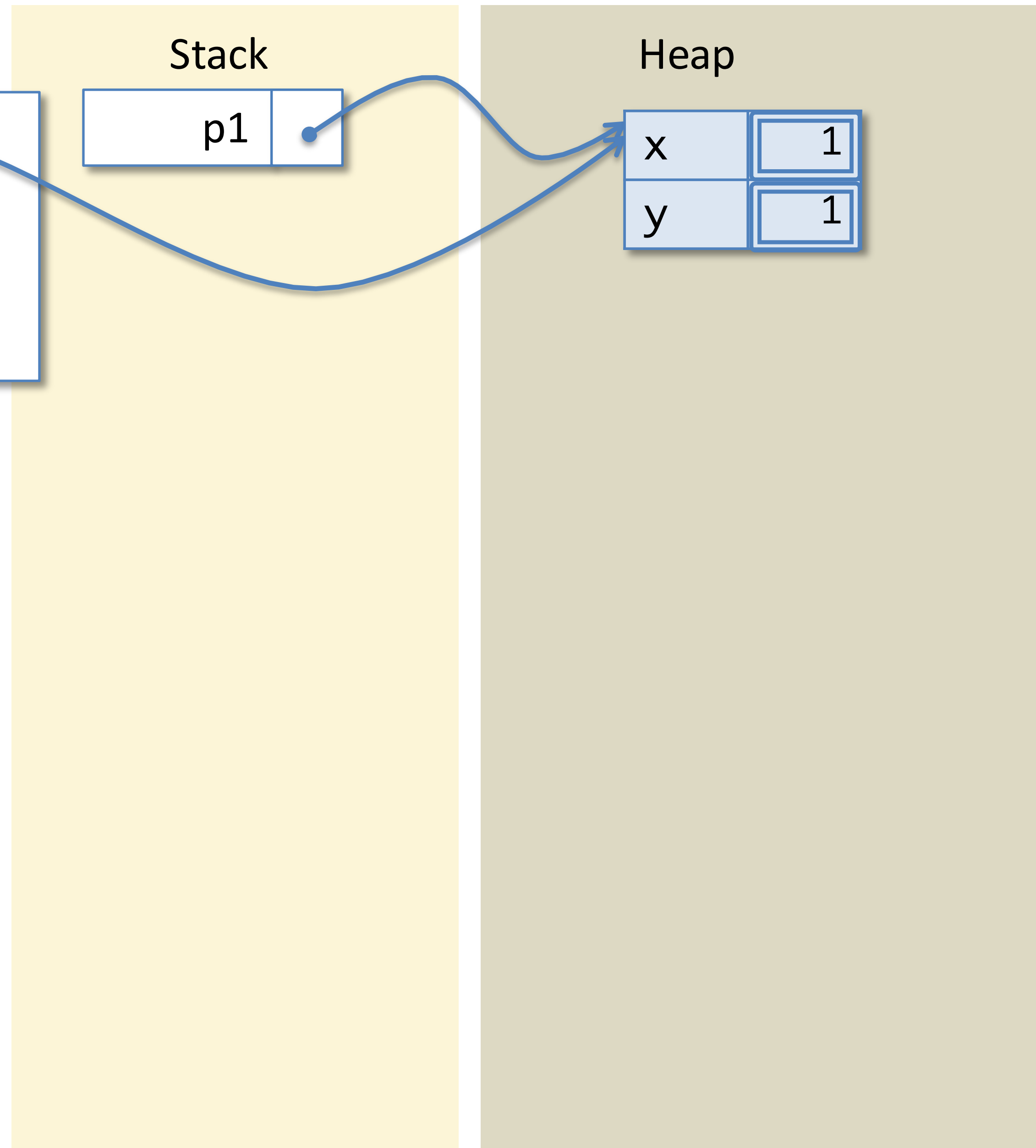
```
let p2 : point = p1
let ans : int =
    p2.x <- 17; p1.x
```



Look Up ‘p1’

Workspace

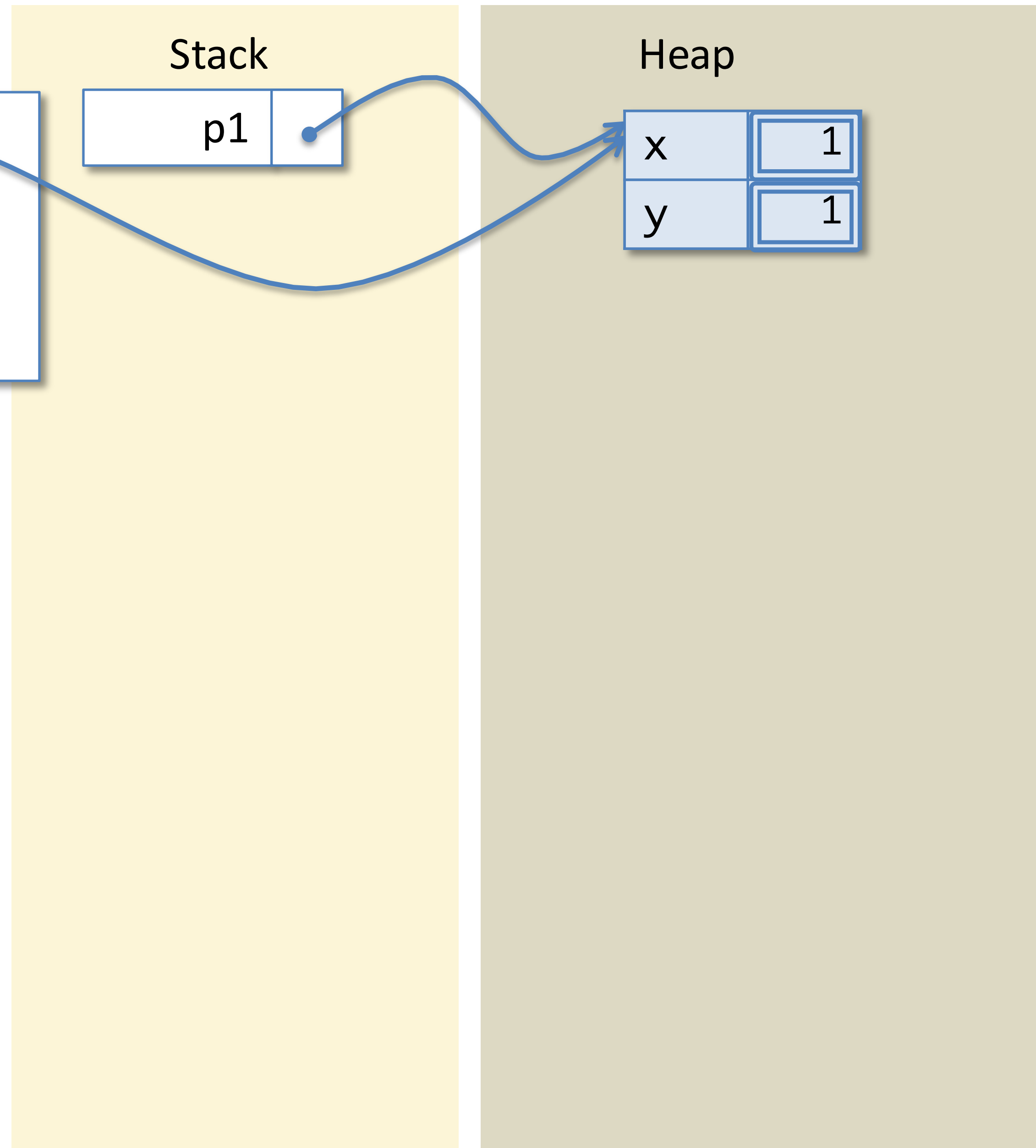
```
let p2 : point = 
let ans : int =
    p2.x <- 17; p1.x
```



Let Expression

Workspace

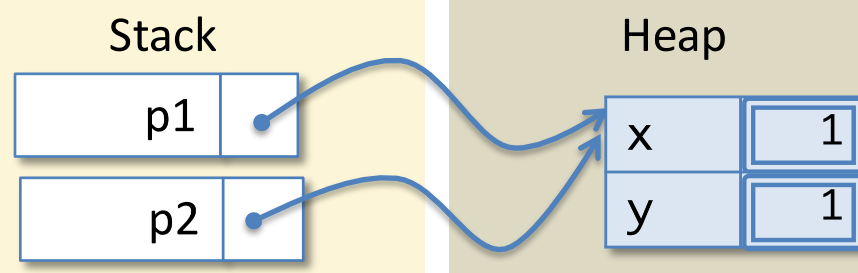
```
let p2 : point =  .
let ans : int =
    p2.x <- 17; p1.x
```



Push p2

Workspace

```
let ans : int =
    p2.x <- 17; p1.x
```

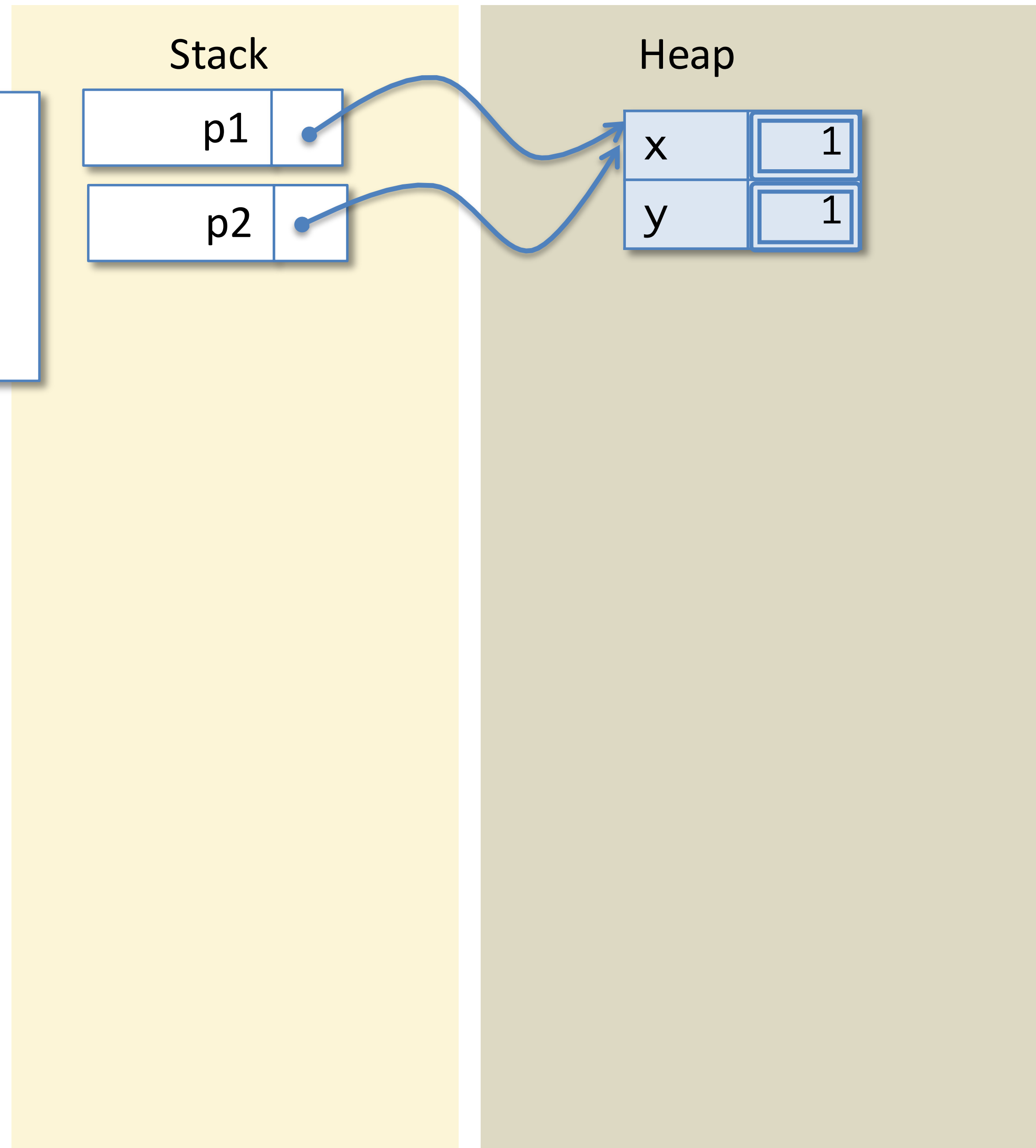


Note: p1 and p2 are references to the *same* heap record.
They are *aliases* – two different names for the *same thing*.

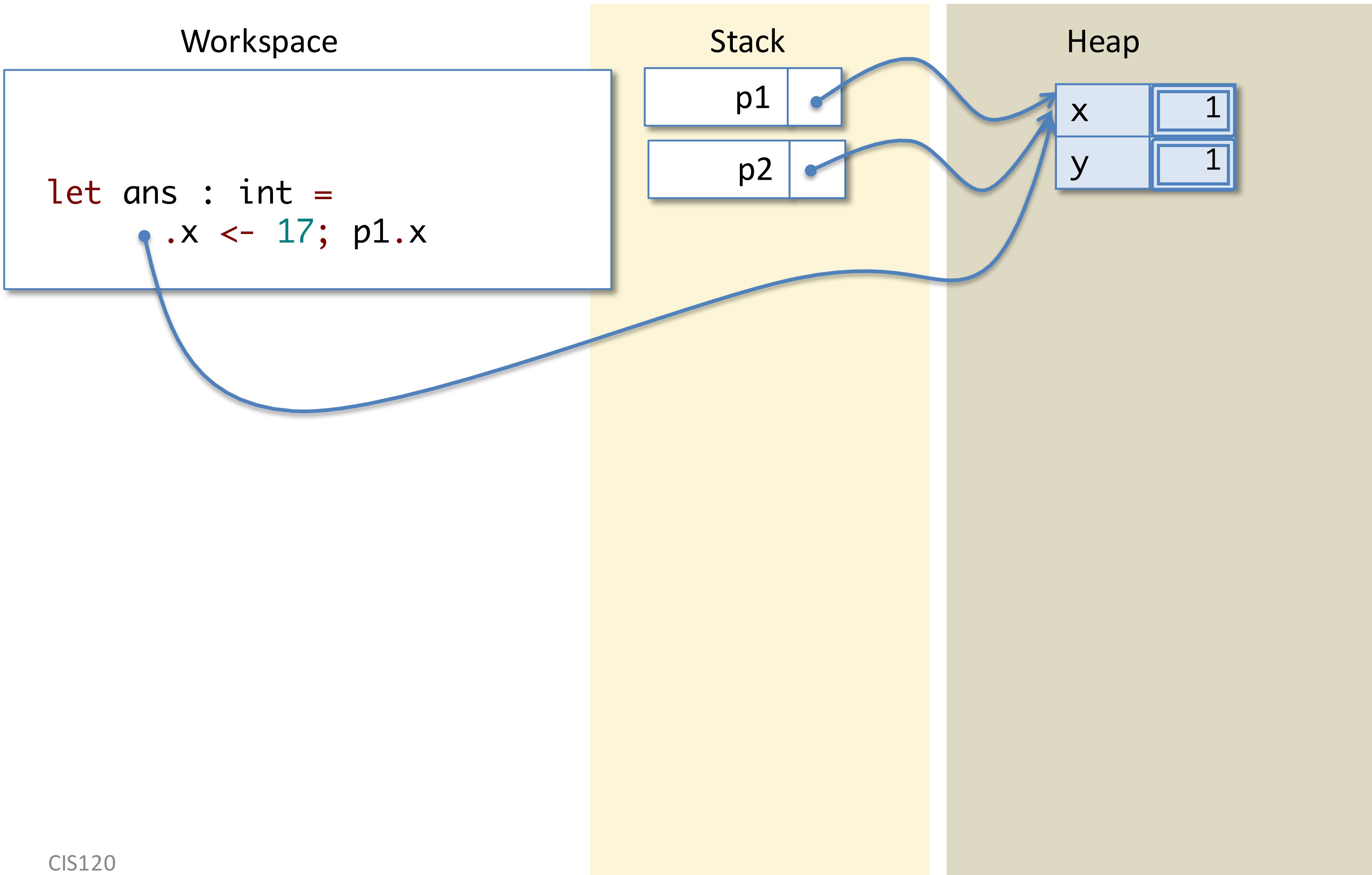
Look Up 'p2'

Workspace

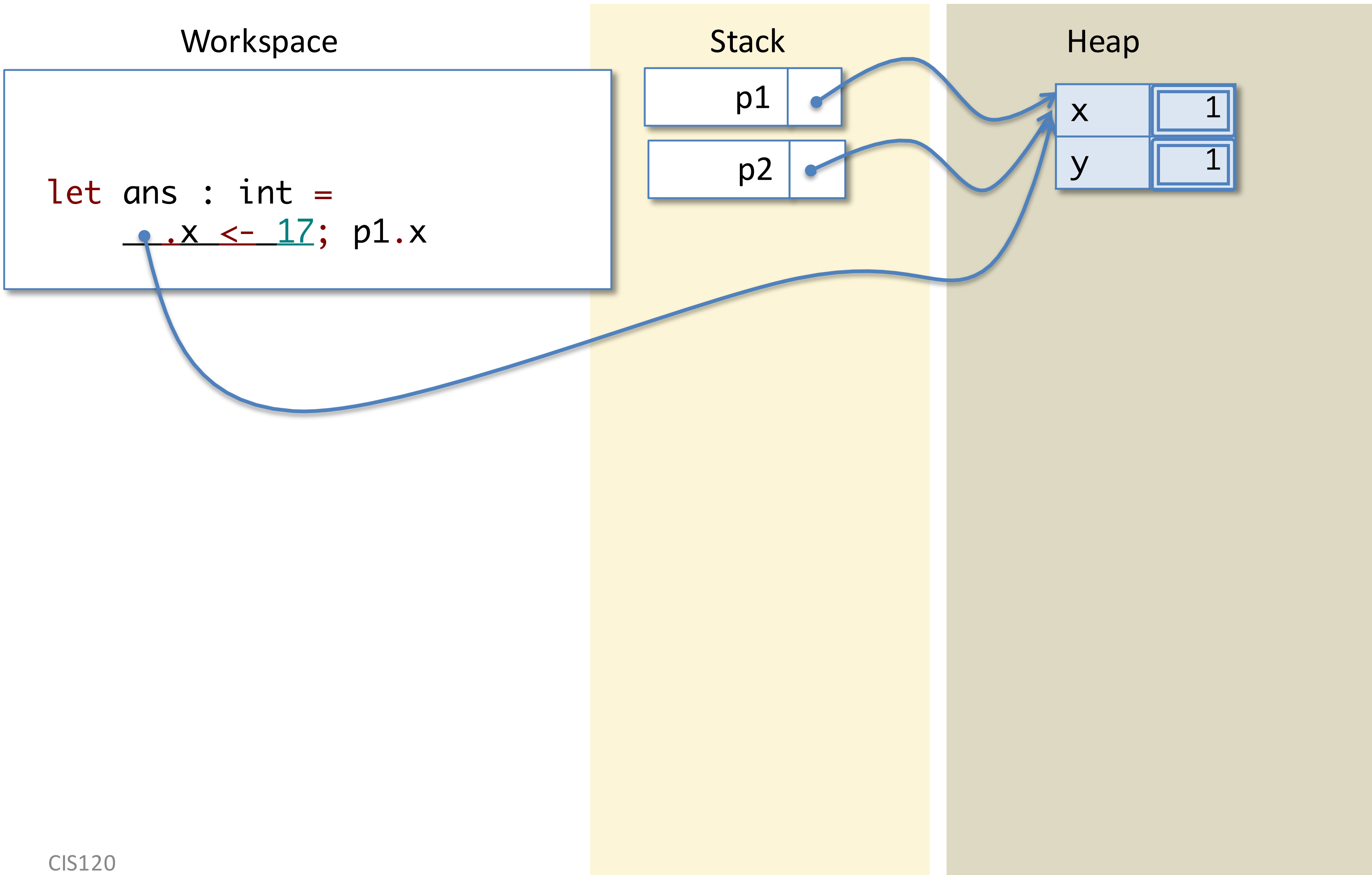
```
let ans : int =
    p2.x <- 17; p1.x
```



Look Up ‘p2’



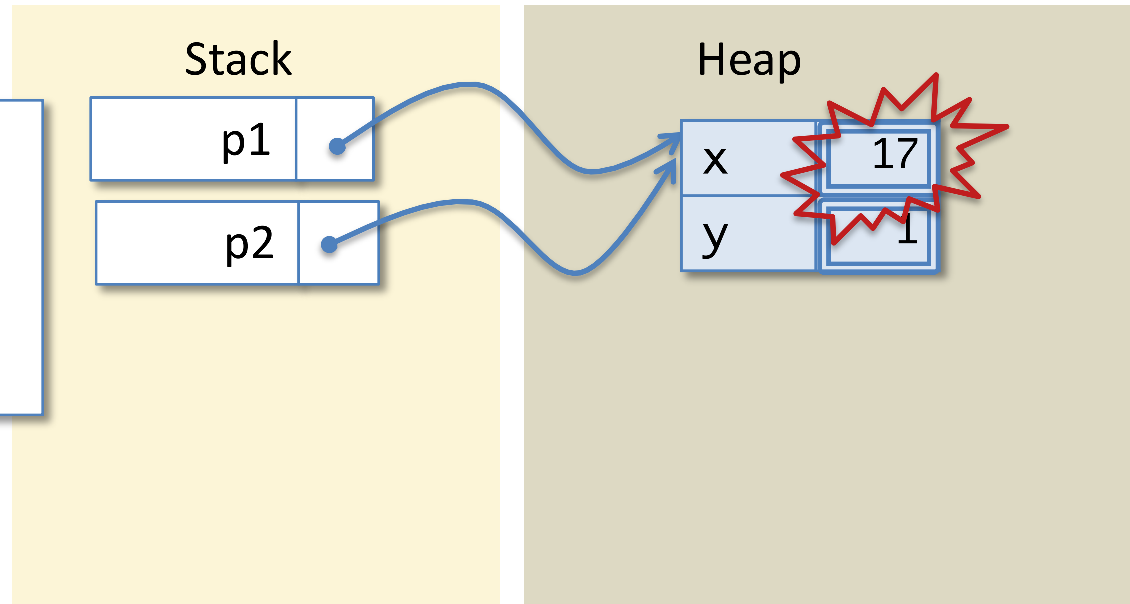
Assign to x field



Assign to x field

Workspace

```
let ans : int =
    (); p1.x
```

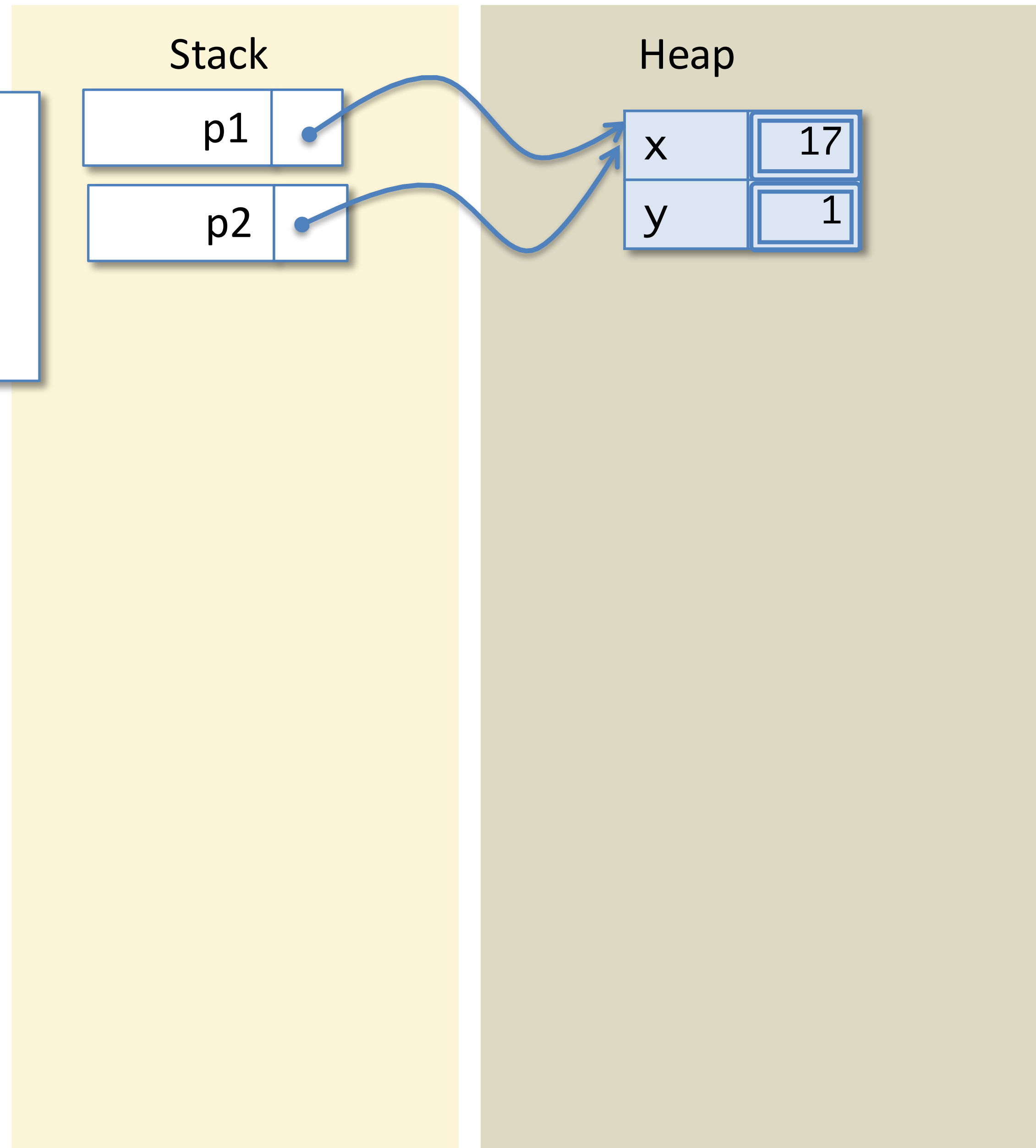


This is the step in which the 'imperative' update occurs. The mutable field x has been modified in place to contain the value 17.

Sequence ';' Discards Unit

Workspace

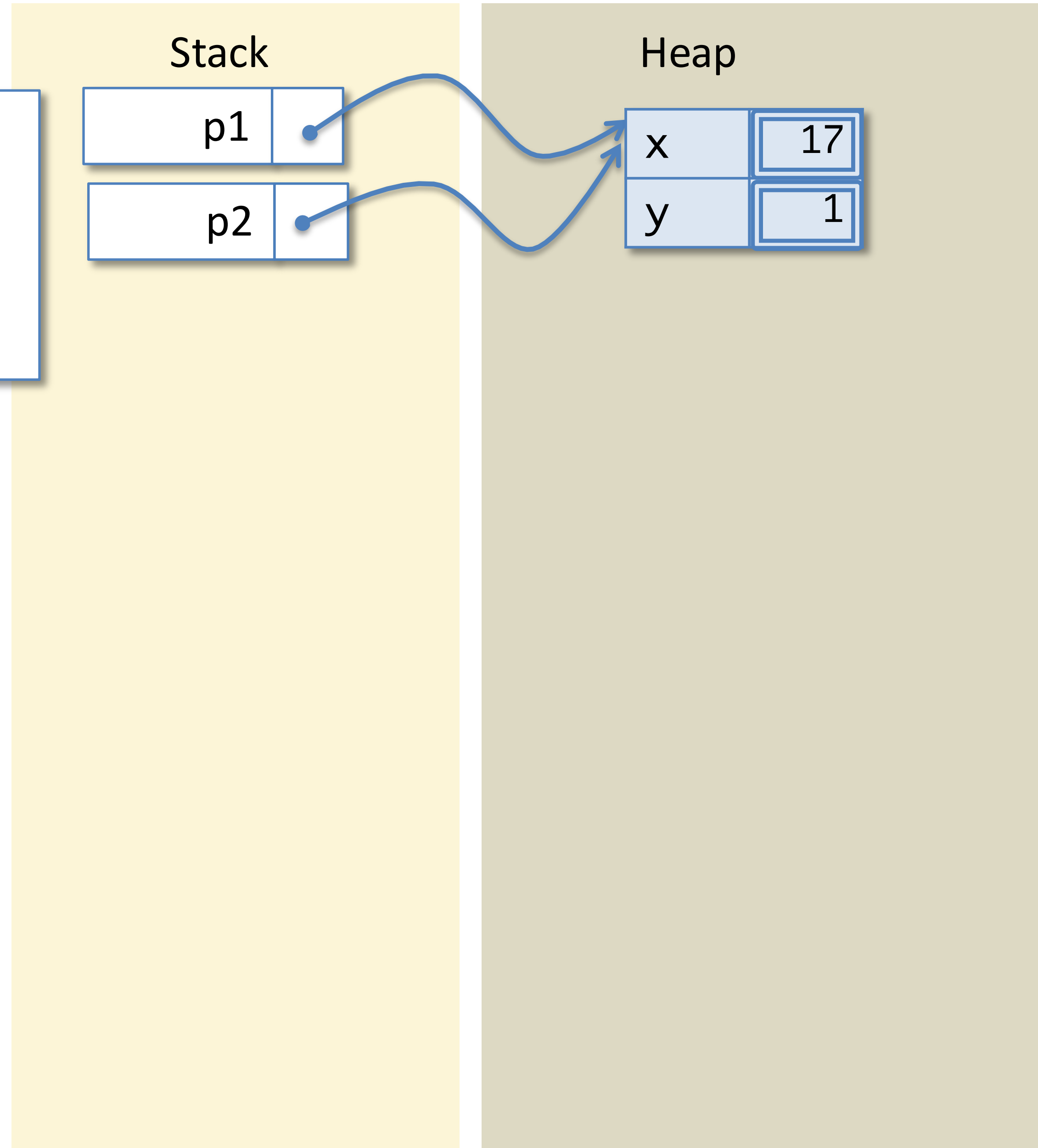
```
let ans : int =
    (); p1.x
```



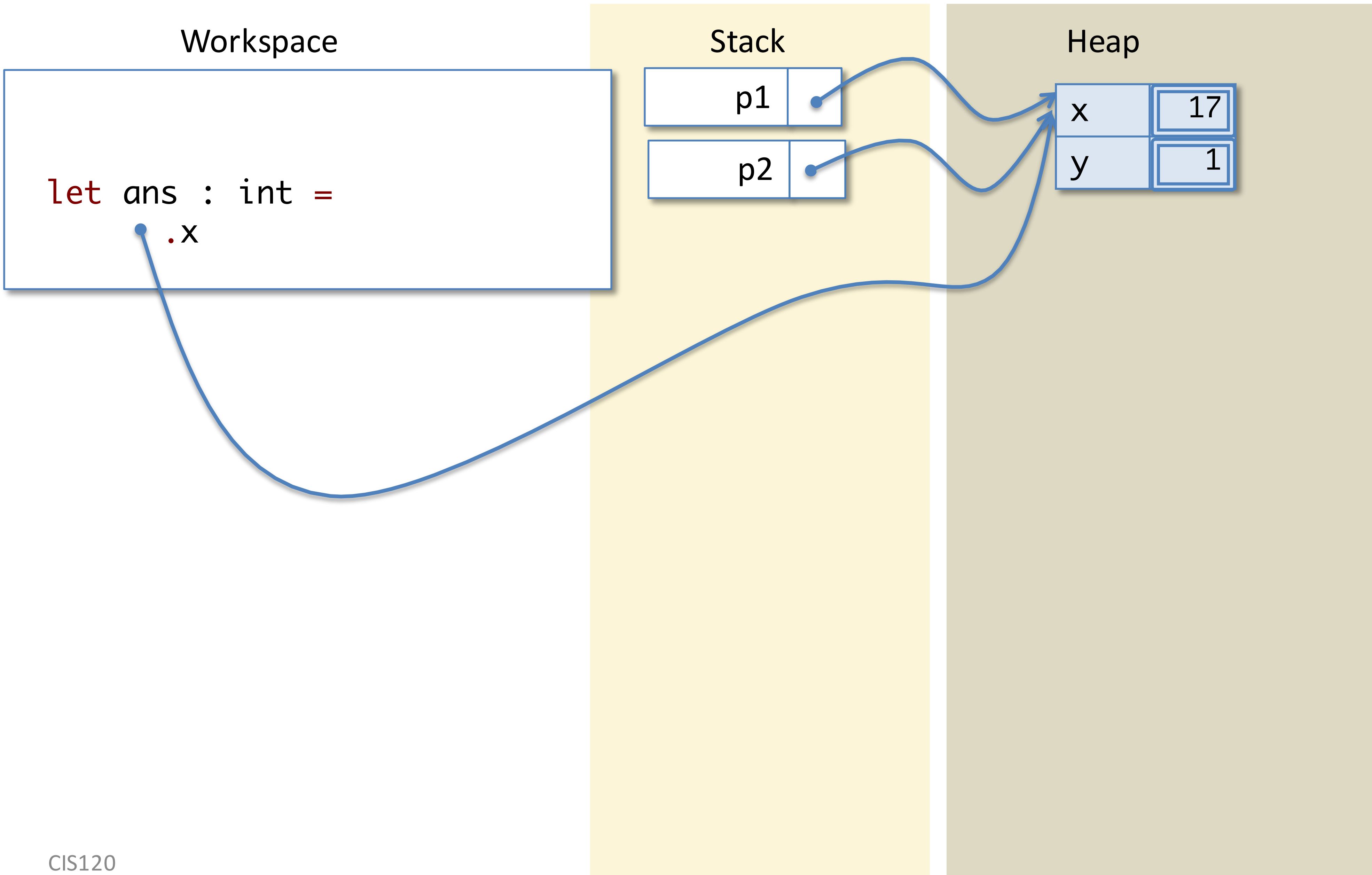
Look Up ‘p1’

Workspace

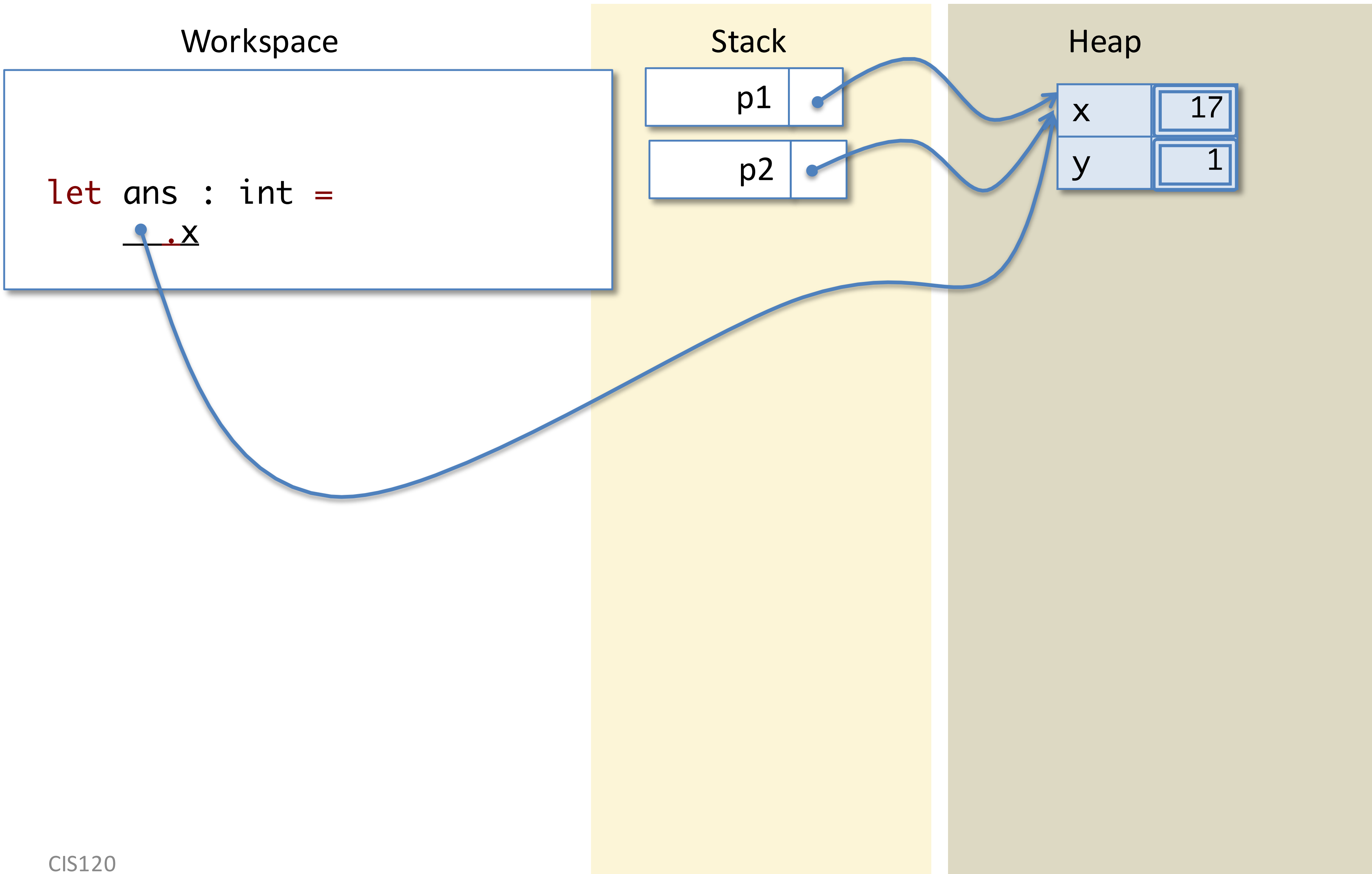
```
let ans : int =
    p1.x
```



Look Up ‘p1’



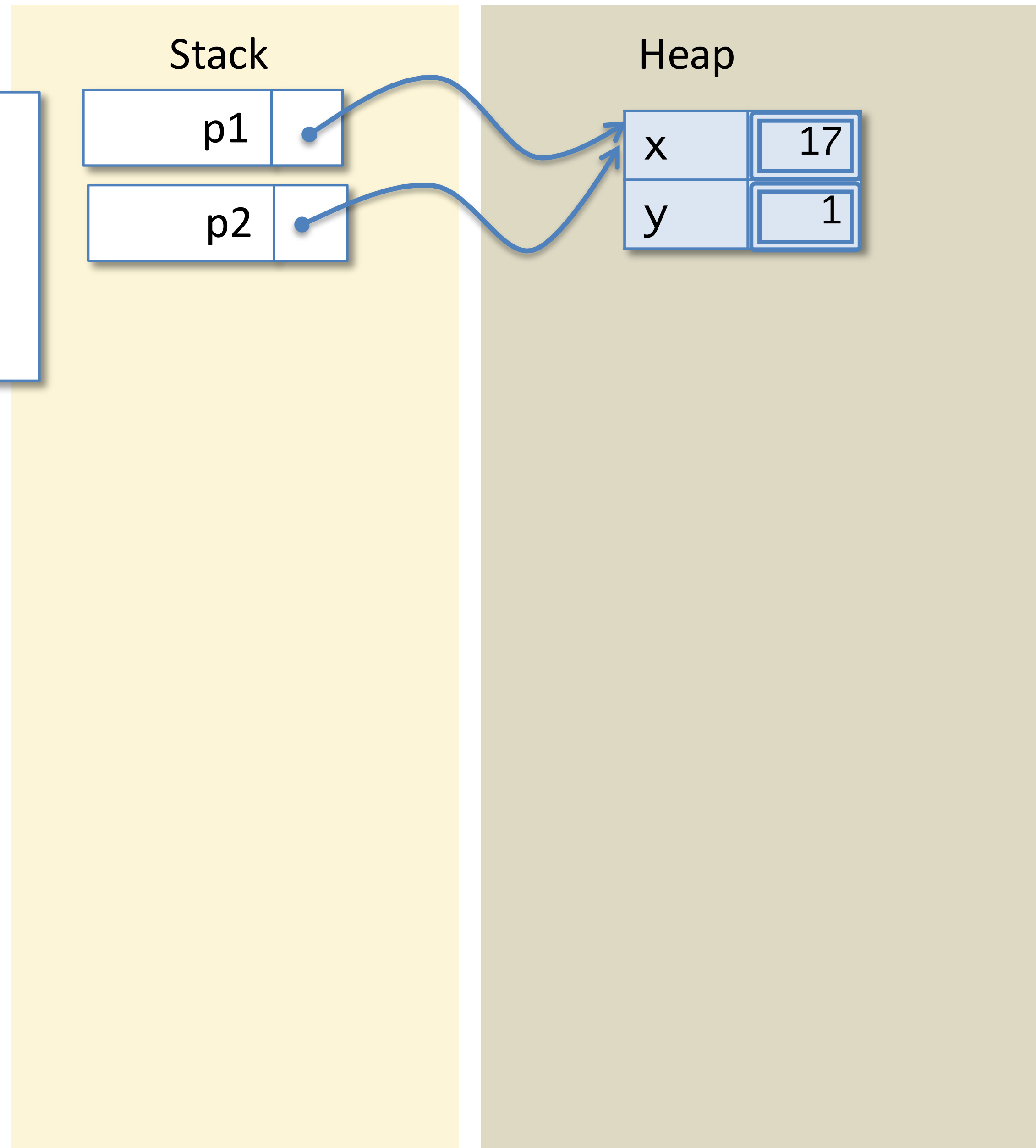
Project the ‘x’ field



Project the ‘x’ field

Workspace

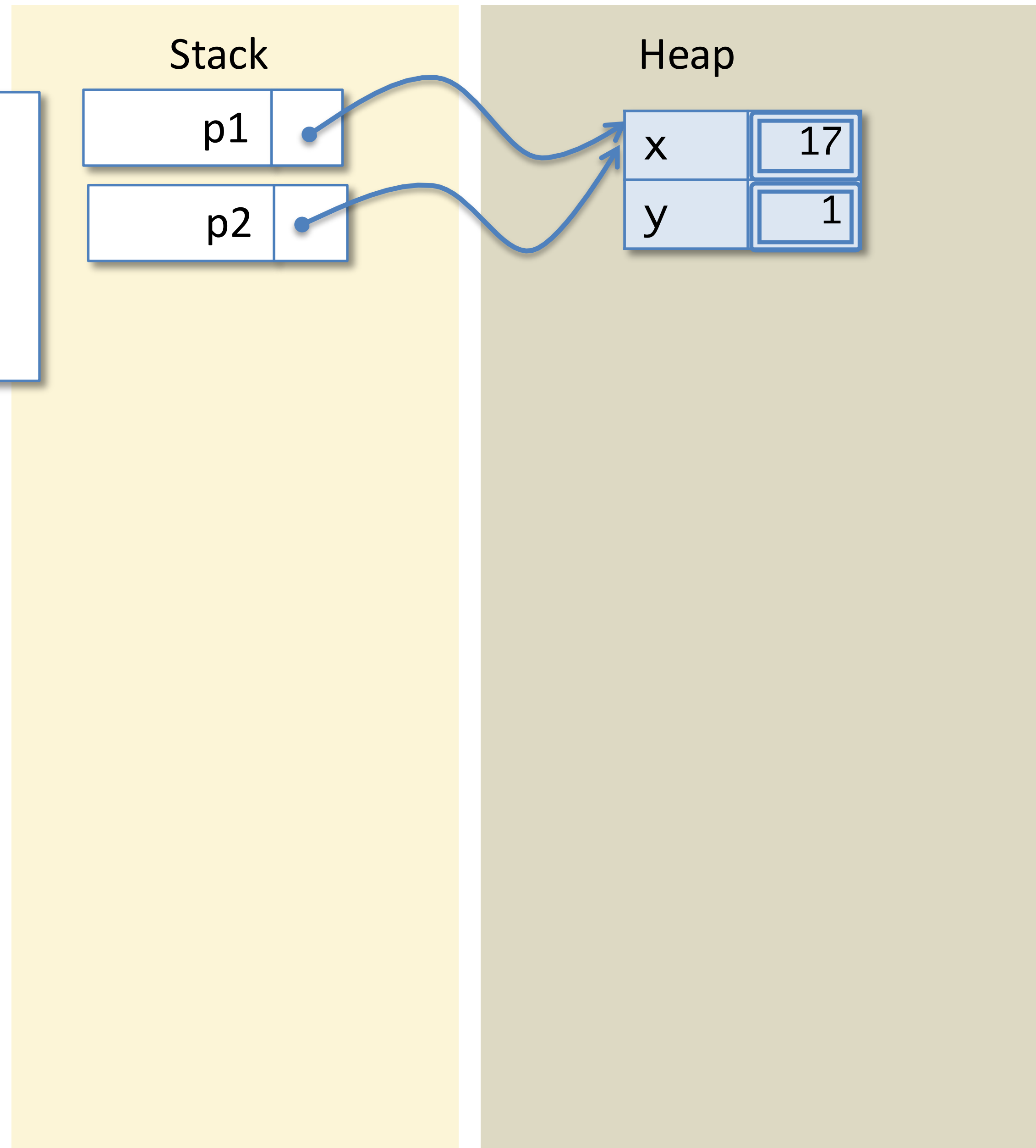
```
let ans : int =
    17
```



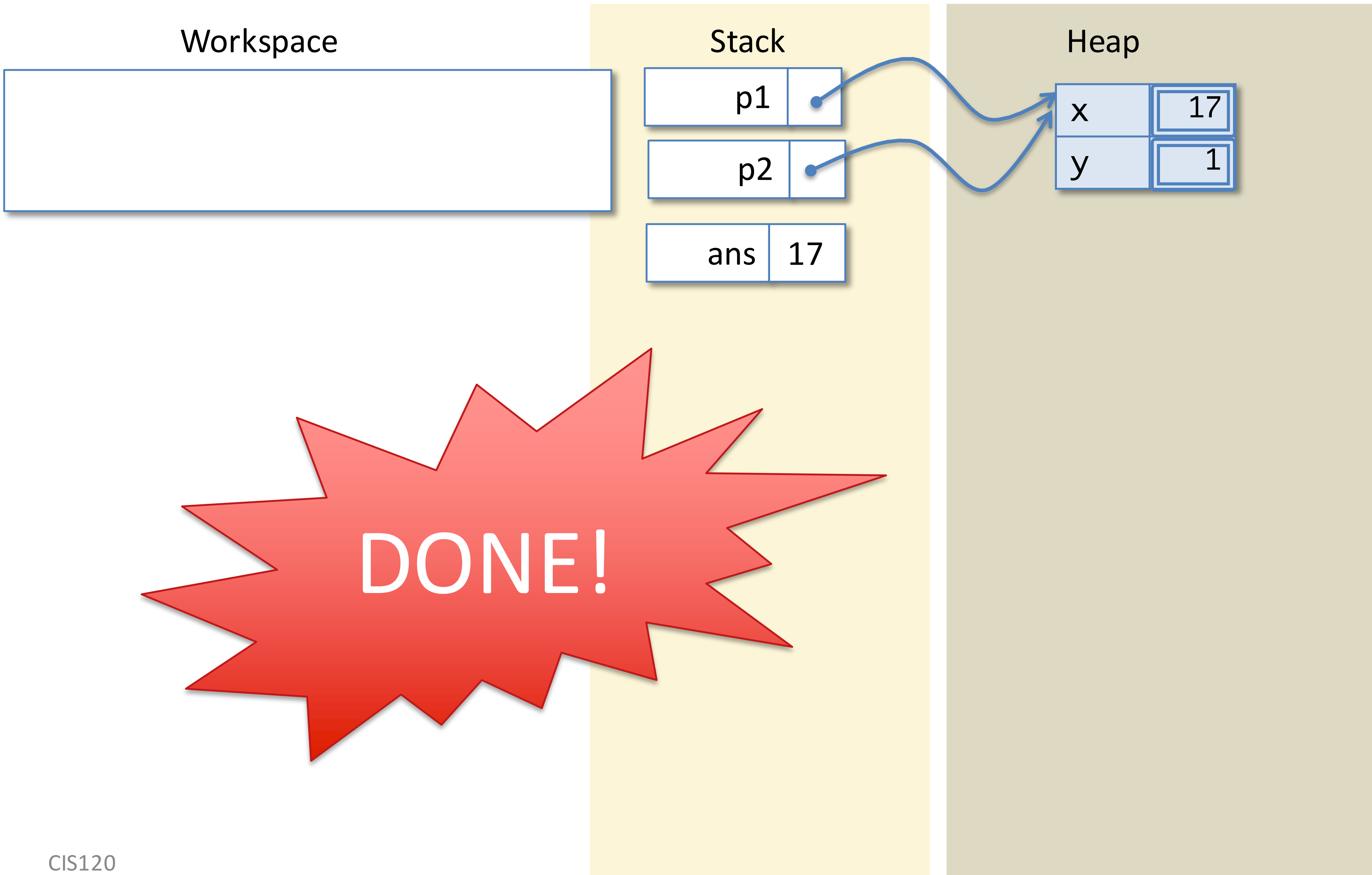
Let Expression

Workspace

```
let ans : int =
      17
```

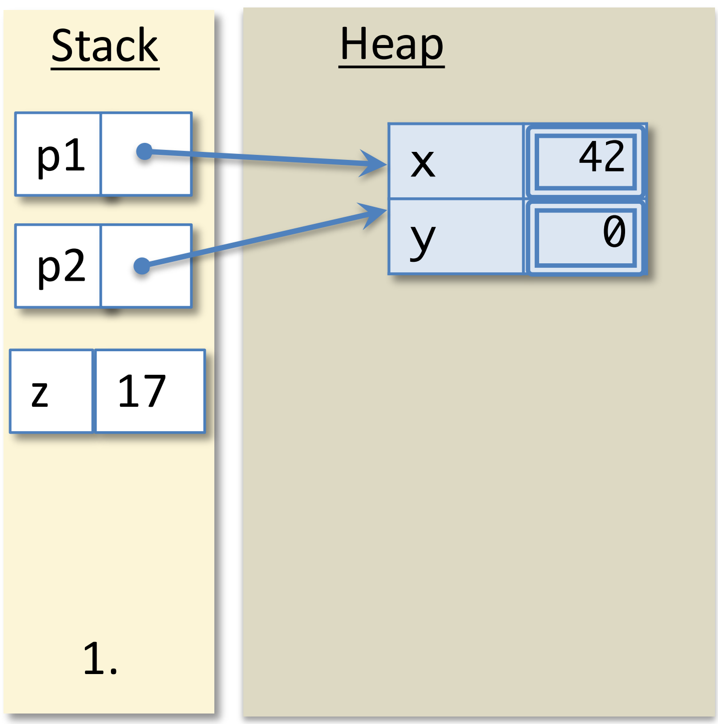


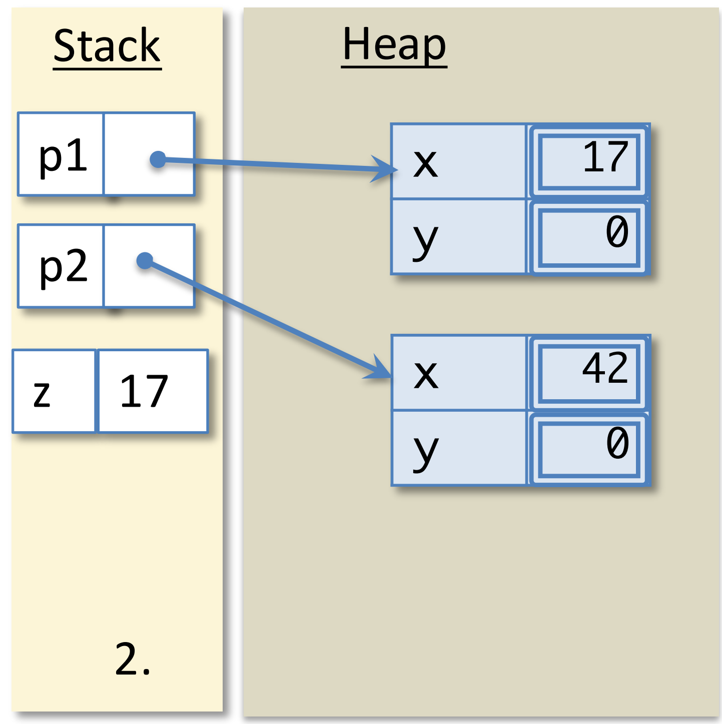
Push ans



What do the Stack and Heap look like after simplifying the following code on the workspace?

```
let p1 = {x=0; y=0} in
let p2 = p1 in
p1.x <- 17;
let z = p1.x in
p2.x <- 42;
p1.x
```





Answer: 1

Reference and Equality

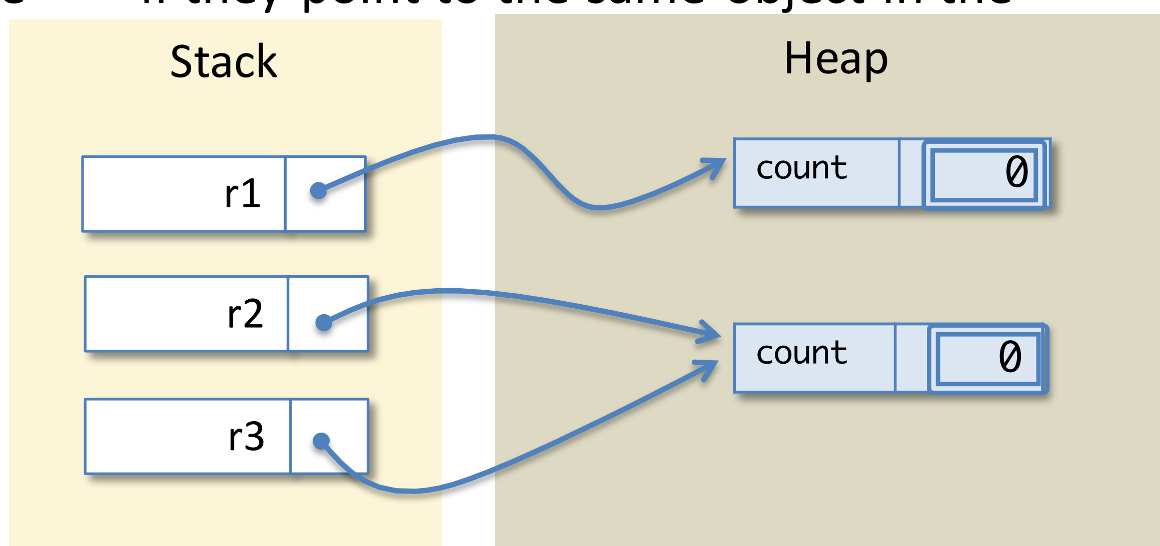
= vs. ==

Reference Equality

- Suppose we have two counters. How do we know whether they share the same internal state?
  - `type counter = { mutable count : int }`
  - We could increment one and see whether the other's value changes.
  - But we could also just test whether the references alias directly.

- Ocaml uses '==' to mean *reference* equality:
  - two reference values are '==' if they point to the same object in the heap; so:

```
r2 == r3

not (r1 == r2)

r1 = r2
```



Structural vs. Reference Equality

- *Structural (in)equality*: `v1 = v2`   `v1 <> v2`
  - recursively traverses over the *structure* of the data, comparing the two values' components for structural equality
  - function values are never structurally equivalent to anything
  - structural equality can go into an infinite loop (on cyclic structures)
  - appropriate for comparing *immutable* datatypes

- *Reference (in)equality*: `v1 == v2`   `v1 != v2`
  - Only looks at where the two references point in the heap
  - function values are only equal to themselves
  - equates strictly fewer things than structural equality
  - appropriate for comparing *mutable* datatypes

What is the result of evaluating the following expression?

```
let p1 : point = { x = 0; y = 0; } in
let p2 : point = p1 in

p1 = p2
```

1. true
2. false
3. runtime error
4. compile-time error

Answer: true

What is the result of evaluating the following expression?

```
let p1 : point = { x = 0; y = 0; } in
let p2 : point = p1 in

p1 == p2
```

1. `true`
2. `false`
3. runtime error
4. compile-time error

Answer: true

What is the result of evaluating the following expression?

```
let p1 : point = { x = 0; y = 0; } in
let p2 : point = { x = 0; y = 0; } in

p1 == p2
```

1. `true`
2. `false`
3. runtime error
4. compile-time error

Answer: false

What is the result of evaluating the following expression?

```
let p1 : point = { x = 0; y = 0; } in
let p2 : point = { x = 0; y = 0; } in
let l1 : point list = [p1] in
let l2 : point list = [p2] in

l1 = l2
```

1. true
2. false
3. runtime error
4. compile-time error

Answer: true

What is the result of evaluating the following expression?

```
let p1 : point = { x = 0; y = 0; } in
let p2 : point = p1 in
let l1 : point list = [p1] in
let l2 : point list = [p2] in

l1 == l2
```

1. `true`
2. `false`
3. runtime error
4. compile-time error

Answer: false

Putting State to Work

Mutable Queues

Announcements

- HW 4: Mutable Queues is available
  - Due: Tuesday, February 16th at 11:59 pm

Have you ever implemented the mutable data structure called a **linked list**, in any language?

1. yes
2. no
3. not sure

A design problem

Suppose you are implementing a website to sell tickets to a very popular music event. To be fair, you would like to allow people to select seats first come, first served. How would you do it?

- Understand the problem
  - Some people may visit the website to buy tickets while others are still selecting their seats
  - Need to remember the order in which people purchase tickets
- Define the interface
  - Need a data structure to store ticket purchasers
  - Need to add purchasers to the *end* of the line
  - Need to allow purchasers at the *beginning* of the line to select seats
  - Both kinds of access must be efficient to handle the volume

(Mutable) Queue Interface

```
module type QUEUE =
sig
  (* abstract type *)
  type 'a queue

  (* Make a new, empty queue *)
  val create : unit -> 'a queue

  (* Determine if the queue is empty *)
  val is_empty : 'a queue -> bool


  (* Add a value to the end of the queue *)
  val enq : 'a -> 'a queue -> unit

  (* Remove the first value (if any) and return it *)
  val deq : 'a queue -> 'a

end
```

We can tell, just looking at this interface, that it is for a MUTABLE data structure. How?

Because queues are mutable, we must allocate a new one every time we need one.

Adding an element to the queue returns unit because it modifies the given queue.

Specify the behavior via test cases

```
let test () : bool =
  let q : int queue = create () in
  enq 1 q;
  enq 2 q;
  1 = deq q
;; run_test "queue test 1" test

let test () : bool =
  let q : int queue = create () in
  enq 1 q;
  enq 2 q;
  let _ = deq q in
  2 = deq q
;; run_test "queue test 2" test
```

What value should replace ??? so that the following test passes?

```
let test () : bool =
  let q : int queue = create () in
  enq 1 q;
  let _ = deq q in
  enq 2 q;
  ??? = deq q

;; run_test "enq after deq" test
```

1. 1
2. 2
3. None
4. failwith “empty queue”

Answer: 2

Implementing Linked Queues

Representing links

Implement the behavior

```
module ListQueue : QUEUE  = struct

  type 'a queue = { mutable contents : ‘a list }

  let create () : 'a queue =
    { contents = [] }

  let is_empty (q:'a queue) : bool =
    q.contents = []

  let enq (x:'a) (q:'a queue) : unit =
    q.contents <- (q.contents @ [x])

  let deq (q:'a queue) : 'a =
    begin match q.contents with
      | [] -> failwith "deq called on empty queue"
      | x::tl -> q.contents <- tl; x
    end
 end
```

Here we are using type abstraction to protect the state. Outside of the module, no one knows that queues are implemented with a mutable structure. So, only these functions can modify this structure.

A Better Implementation

- Implementation is slow because of append:
  - `q.contents @ [x]` copies the entire list each time
  - As the queue gets longer, it takes longer to add data
  - Only has a *single* reference to the beginning of the list

- Let's do it again with TWO references, one to the beginning (head) and one to the end (tail).
  - Dequeue by updating the head reference (as before)
  - Enqueue by updating the tail of the list

- Challenge: The list itself must be mutable
  - because we add to one end and remove from the other

Data Structure for Mutable Queues

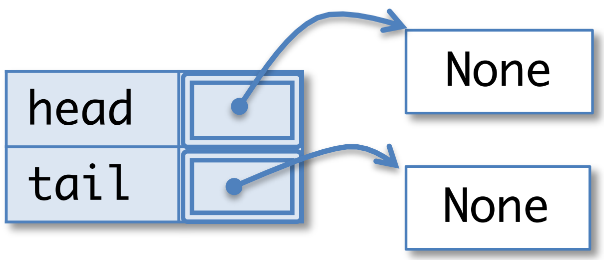
```
type 'a qnode = {
     v: 'a;
     mutable next : 'a qnode option
}

type 'a queue = { mutable head : 'a qnode option;
                  mutable tail : 'a qnode option }
```

There are two parts to a mutable queue:

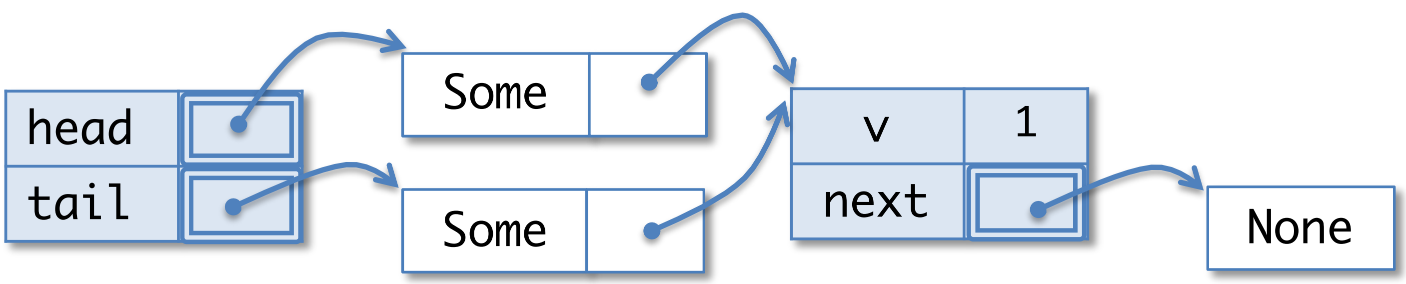
1. the "internal nodes" of the queue, with links from one to the next
2. a record with links to the head and tail nodes

All of the links are *optional* so that the queue can be empty.

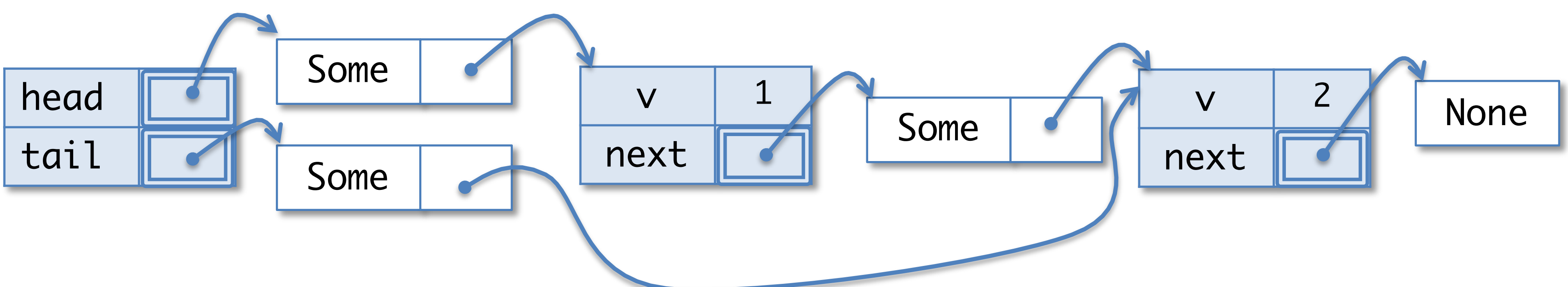
Queues in the Heap



An empty queue

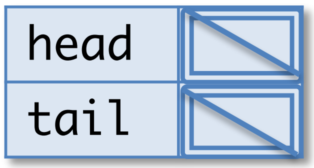


A queue with one element

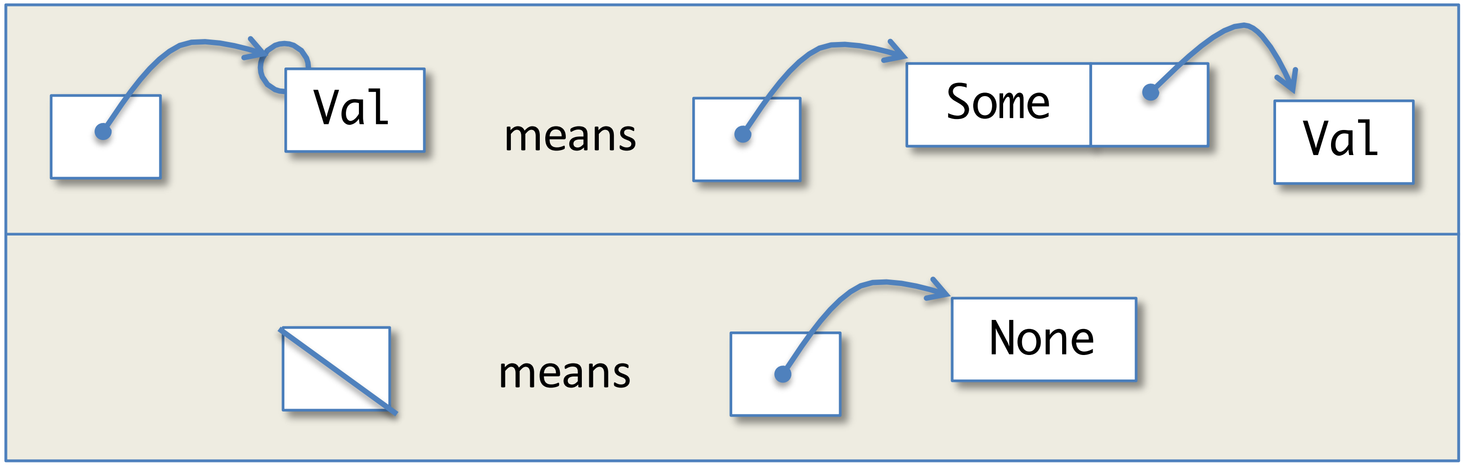


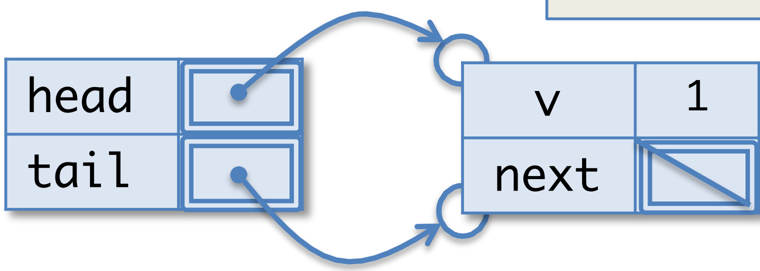
A queue with two elements

Visual Shorthand: Abbreviating Options

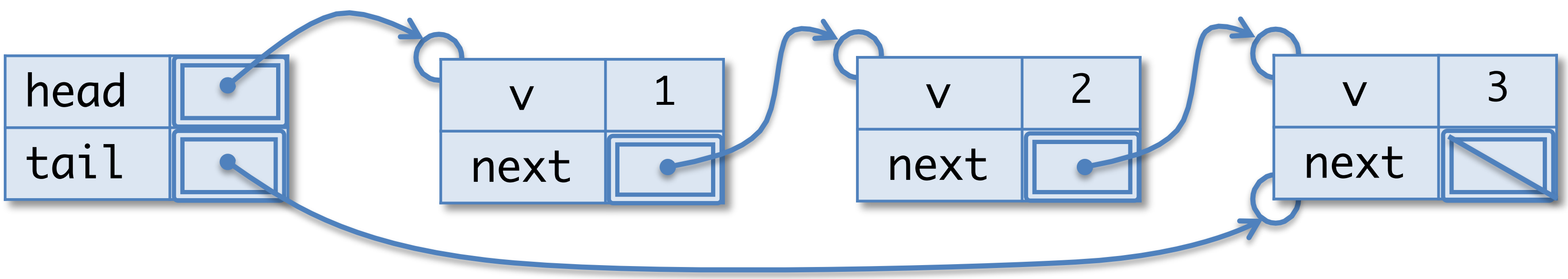


An empty queue



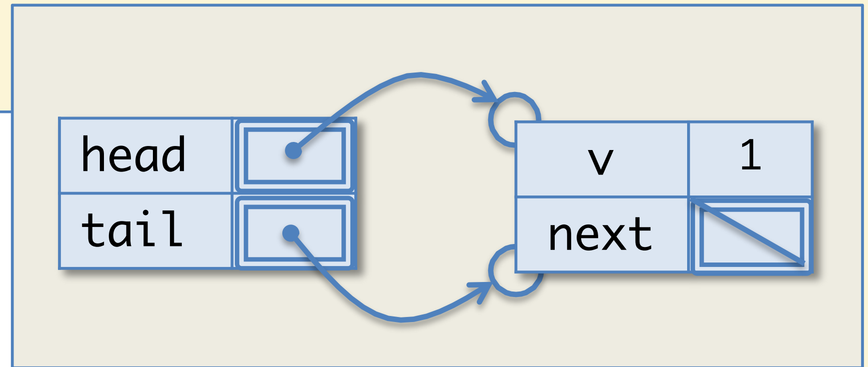


A queue with one element



A queue with three elements

Given the queue datatype shown below, which expression creates a 1-element queue in the heap:




```
type 'a qnode = {
  v: 'a;
  mutable next : 'a qnode option
}

type 'a queue = { mutable head : 'a qnode option;
                  mutable tail : 'a qnode option }
```

1.
```
let q = { head = None; tail = None }
```

2.
```
let q = { head = 1; tail = None }
```

3.
```
let q = let qn = { v= 1; next = None } in
        { head = qn; tail = None }
```

4.
```
let q = let qn = { v= 1; next = None } in
        { head = Some qn; tail = Some qn }
```

Answer: 4