CIS 1200 Midterm I September 30, 2022

Name (printed): _____

PennKey (penn login id): _____

I certify that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

- Please wait to begin the exam until you are told it is time for everyone to start.
- When you begin, please start by writing your username (a.k.a. PennKey, e.g., `swapneel`) clearly at the bottom of *every page*.
- There are 60 total points. The exam length is one hour.
- This exam is *closed book*. Do not collaborate with anyone else when completing this exam.
- For coding problems: aim for accurate syntax, but we will not grade your code style for indentation, spacing, etc.
- There are 13 pages in the exam and an Appendix for your reference. Please do not submit the Appendix.
- Do not spend too much time on any one question. Be sure to recheck all of your answers.
- The last page of the exam can be used as scratch space. By default, we will ignore anything you write on this page. If you write something that you want us to grade, make sure you mark it clearly as an answer to a problem *and* write a clear note on the page with that problem telling us to look at the scratch page.
- Good luck!

1. **List Recursion** (4 points)

   Fill in the blanks in the following function so that it will pass the tests below:

```
let rec upto_helper (n: int) (limit: int) : int list =

  if n > limit then []

  else _____________________________________________________

let upto (n: int) : int list = upto_helper 0 n

;; run_test "upto 0" (fun () -> upto 0 = [0])
;; run_test "upto 5" (fun () -> upto 5 = [0;1;2;3;4;5])
```

2. **Compression** (12 points total)

   Suppose we want a data structure for representing sequences of readings taken at one-second intervals from a temperature sensor, where some of the values may be missing (because the sensor only produces readings every few seconds).

   One way to represent such data—let's call it the *expanded* representation—is as a simple list of values, one for each one-second interval, filling in `0` for intervals when the sensor does not actually produce a reading.

   For example, if the sensor produces `100` at the very first time interval and `101` a few seconds later, we might represent this as:

   ```
   [ 100; 0; 0; 0; 0; 0; 101 ]
   ```

   (We'll assume, for simplicity, that the sensor never produces the value `0` as a valid reading.)

   Alternatively, we could represent the same readings more compactly by giving a list of just the valid readings together with the times at which they occurred:

   ```
   [ (100,0); (101,6) ]
   ```

   (Note that we're calling the first interval "interval `0`," counting from zero like good computer scientists!) We call this the *compressed* representation of our time-series data.

   Your job in this problem will be to write functions that convert back and forth between the expanded and compressed representations.

Continued on next page...

First, let's define a couple of type abbreviations:

```
type expanded = int list

type compressed = (int * int) list
```

(a) (4 points) Now let's write the compression function.

Fill in the blanks in the following helper so that the main function `compress` will pass the tests below:

```
let rec compress_helper (e: expanded) (time: int) : compressed =
  begin match e with
    | [] -> []
    | x::tail ->
        if not (x = 0) then

           ____________________________________________________________

        else

           ____________________________________________________________
  end

let compress (e: expanded) : compressed = compress_helper e 0

;; run_test "compress empty"
      (fun () -> compress [] = [])
;; run_test "compress singleton"
      (fun () -> compress [100] = [(100,0)])
;; run_test "compress example from above"
      (fun () -> compress [ 100; 0; 0; 0; 0; 0; 101 ] = [(100,0); (101,6)])
;; run_test "compress with initial and final zeros"
      (fun () -> compress [0;100;99;0] = [(100,1);(99,2)])
```

(b) (5 points) Next let's write the *un*compression function.

Fill in the blanks in the following helper so that the main function `expand` will pass the tests below:

```
let rec expand_helper (c: compressed) (time: int) : expanded =
  begin match c with

    | [] -> _____________________________________________

    | (x,m) :: rest ->

        if ________________________________  then

             ____________________________________________________________

        else

             ____________________________________________________________
  end

let expand (c: compressed) : expanded =
  expand_helper c 0

;; run_test "expand empty"
       (fun () -> expand [] = [])
;; run_test "expand with initial zeros"
       (fun () -> expand [(100,2)] = [0;0;100])
;; run_test "expand example from earlier"
       (fun () -> expand [(100,0); (101,6)] = [ 100; 0; 0; 0; 0; 0; 101 ])
```

(c) (3 points)

i. "For any `e : expanded`, it is always the case that `expand (compress e) = e`."

True □ False □

If you choose False, give a counterexample:

`e =` _____

ii. "For any `c : compressed`, it is always the case that `compress (expand c) = c`."

True □ False □

If you choose False, give a counterexample:

`c =` _____

3. **ADTs** (8 points total)

Suppose we define the following module to collect together some functions operating on time-series data.

```
module COMPRESSED (* : I *) =
  struct
    type t = compressed

    let import (m: expanded) : t = compress m
    let export (m: t) : expanded = expand m

    (* Check whether any element of a list satisfies a boolean
       predicate (helper function for extend) *)
    let rec exists (test: int*int -> bool) (m: compressed) : bool =
       begin match m with
       | [] -> false
       | hd::tl -> test hd || exists test tl
       end

    (* Add a new reading at the end of a compressed list of readings *)
    let extend (m: t) (newreading: int) (newtime: int) : t =
      if exists (fun (_, oldtime) -> oldtime >= newtime) m then
        failwith "time is less than that of an existing measurement"
      else
        m @ [(newreading, newtime)]
  end
```

The `import` and `extend` functions maintain the invariant that the times of the measurements are sorted in increasing order.

For each of the following possible definitions of the interface `I`, check the box next to the phrase that best describes it:

- *Ill typed* (the interface does not match the module and the code will not compile)
- (Well typed but) *Unusable* (because there is no way to call any of the functions)
- (Well typed and usable but) *does not preserve the invariant* ☹
- *Good* (well typed, usable, and maintains the invariant)

Continued on next page...

(2 points each)

(a)

```
module type I =
  sig
    type t
  end
```

☐ Ill typed ☐ Unusable ☐ Invariant ☹ ☐ Good

(b)

```
module type I =
  sig
    type t
    val import : expanded -> t
    val export : t -> expanded
    val extend : t -> int -> int -> t
  end
```

☐ Ill typed ☐ Unusable ☐ Invariant ☹ ☐ Good

(c)

```
module type I =
  sig
    val import : expanded -> compressed
    val export : compressed -> expanded
    val extend : compressed -> int -> int -> compressed
  end
```

☐ Ill typed ☐ Unusable ☐ Invariant ☹ ☐ Good

(d)

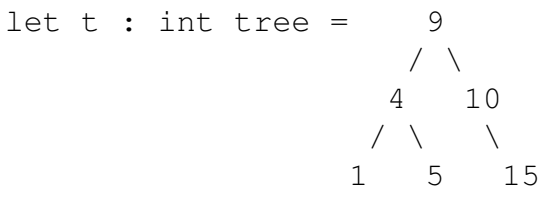
```
module type I =
  sig
    type t
    val export : t -> expanded
    val extend : t -> int -> int -> t
  end
```

☐ Ill typed ☐ Unusable ☐ Invariant ☹ ☐ Good

4. **Binary Search Trees** (12 points total)

   This problem concerns *buggy* implementations of the `lookup` and `insert` functions for binary search trees, the correct versions of which are shown in Appendix B. Note that this problem refers to the `'a tree` type defined there.

   Consider the following tree `t` (as usual, `Empty` constructors are not shown, to avoid clutter).



   (a) (1 point) "Tree `t` satisfies the BST invariants." ☐ True ☐ False

   (b) (5 points) Consider this incorrect definition of `lookup`. (The correct version can be found in Appendix B.)

```
1    let rec bad_lookup (t: int tree) (n: int) : bool =
2      begin match t with
3        | Empty -> n
4        | Node(lt, x, rt) ->
5          if n = x then false
6          else if n <= x then bad_lookup lt n
7          else bad_lookup rt n
8      end
```

   i. "The code above compiles." ☐ True ☐ False

   ii. If you chose `False`, specify which line number the error will be reported for, what error the typechecker will print (don't worry about the exact wording), and one possible fix that makes the program compile.

```
Compile error on line: _____________

Error message        : ___________________________________________

Fix for compile error: ___________________________________________
```

iii. Even after the compile-time error (if any) is fixed, the code will still be buggy: for some inputs it will produce the correct answer, but for others it will not.

Complete each of the test cases below with an `int` value for `x` so that the test passes, demonstrating that this implementation (including your fix for the compilation error, if any) sometimes produces the correct answers and sometimes does not.

Both of your test cases must use the tree `t` shown above.

```
;; run_test "bad_lookup produces correct answer" (fun () ->

      let x = _____________ in
      bad_lookup t x  =  lookup t x)


;; run_test "bad_lookup computes wrong answer" (fun () ->

      let x = _____________ in
      not (bad_lookup t x = lookup t x))
```

(c) (6 points) Consider this incorrect definition of `insert`. (The correct version can be found in Appendix B.)

```
1       let rec bad_insert (t: 'a tree) (n: 'a) : 'a tree =
2        begin match t with
3        | Empty -> Node(Empty, n, Empty)
4        | Node(lt, x, rt) ->
5           if x = n then t
6           else if n < x then Node(lt, x, bad_insert rt n)
7           else Node(bad_insert lt n, x, rt)
8        end
```

i. "The code above compiles." ☐ True ☐ False

ii. If you chose `False`, specify which line number the error will be reported for, what error the typechecker will print (don't worry about the exact wording), and one possible fix that makes the program compile..

```
Compile error on line: _____________

Error message        : ____________________________________

Fix for compile error: ____________________________________
```

iii. Draw two pictures of Binary Search Trees corresponding to the test cases below. For the first, `bad_insert` (after your correction from part (ii), if any) should behave correctly, and for the second it should behave incorrectly, demonstrating that this implementation sometimes produces the correct answers and sometimes does not.

```
;; run_test "bad_insert works correctly" (fun () ->
     bad_insert t1 5 = insert t1 5)

;; run_test "bad_insert works incorrectly" (fun () ->
     not (bad_insert t2 5 = insert t2 5))
```

Works correctly:

```
t1 : int tree = (...draw a picture...)
```

Works incorrectly:

```
t2 : int tree = (...draw a picture...)
```

5. **Generic Types, Higher-Order Functions, Testing** (12 points total)

Recall the `transform` and `fold` functions:

```
let rec transform (f: 'a -> 'b) (xs: 'a list): 'b list =
  begin match xs with
  | [] -> []
  | h::tl -> f h :: transform f tl
  end

let rec fold (combine: 'a -> 'b -> 'b) (base: 'b) (l: 'a list) : 'b =
  begin match l with
  | [] -> base
  | h::tl -> combine h (fold combine base tl)
  end
```

In the first two parts, you are asked to write a test case (involving a list with at least three elements). For example, one possible test case for the `fold` function above could be:

```
;; run_test "fold plus" (fun () -> fold (fun x y -> x+y) 0 [1;2;3] = 6)
```

Your solutions in this problem *must not* use any list library functions. The list constructors `::` and `[]` are fine.

(a) (3 points) Write a test case, involving a list with at least three elements, for the variant of `transform` shown below.

```
let rec squish (f: 'a -> 'a -> 'b) (l: 'a list) : 'b list =
  begin match l with
  | [] -> []
  | [x] -> []
  | x::y::rest -> (f x y) :: (squish f (y::rest))
  end


;; run_test "_____________________________________________________"

   (fun () -> _____________________________________________

              _____________________________________________)
```

(b) (4 points) Write a test case, involving a list with at least three elements, for the variant of `zip` shown below:

```
let rec zip_with (f: 'a -> 'b -> 'c) (l1: 'a list) (l2: 'b list)
                 : 'c list =
  begin match l1, l2 with
  | x::xs, y::ys -> (f x y) :: (zip_with f xs ys)
  | _ -> []
  end


;; run_test "_____________________________________________________"

   (fun () -> _____________________________________________________

               _____________________________________________________)
```

(c) (5 points) Use `transform` or `fold`, along with suitable anonymous function(s), to implement the `chunk` function below.

This function should partition the input list into a tuple of lists `(list1, list2)` based on whether the elements satisfy the given predicate `p`. Elements that satisfy the predicate should be included in `list1` and elements that do not satisfy the predicate should be included in `list2`. For example, the call

```
chunk (fun x -> x mod 2 = 0) [1; 2; 3; 4; 5]
```

should evaluate to

```
([2; 4], [1; 3; 5])
```

since `2` and `4` satisfy the predicate, while `1`, `3`, and `5` do not.

```
let chunk (p: 'a -> bool) (l: 'a list) : ('a list * 'a list) =

  ____________________________________________________________

  ____________________________________________________________

  ____________________________________________________________

  ____________________________________________________________
```

6. **Types** (12 points total)

   For each OCaml value below, fill in the missing type annotations or else write "ill typed" if there is no way to fill in the annotation that does not cause a type error.

   Your answer should be the *most generic* type that OCaml would infer for the value——*i.e.*, if `int list` and `bool list` are both possible types of an expression, you should write `'a list`.

   Some of these expressions refer to the types and functions defined in Appendix A and Appendix B.

   (2 points each)

   (a)
```
let a : ____________________________________________________ =
  [[(1200, "STIT B6"); (1600, "Towne 100")]; [(0100, "MEYH B1")]]
```

   (b)
```
let b : ____________________________________________________ =
  let z : int list list = [[1; 3]; [2]] in
  begin match z with
  | [] -> 0
  | _ -> z
  end
```

   (c)
```
let c (value: (int * int) list) : ___________________________ =
  transform (fun (x, y) -> x) value
```

   (d)
```
let d : ____________________________________________________ =
  fun (l: 'a list) -> fold (fun x acc -> x + acc) 0 l
```

   (e)
```
let e (base: 'a list) : ____________________________________ =
  fold (fun x acc -> [x] @ acc) base
```

   (f)
```
let f : ____________________________________________________ =
  fun f x -> f x
```

Scratch Space

*Use this page for work that you do not want us to grade. If you run out of space elsewhere in the exam and you **do** want to put something here that we should grade, make sure to put a clear note on the page for the problem in question.*