

Programming Languages and Techniques (CIS1200)

Lecture 12

Typechecking

Chapter 10

Announcements

- Midterm 1: Friday, September 27th
 - Coverage: up to today (Chapters 1-10)
 - During lecture
Last names: A – Z Meyerson Hall B1
 - 60 minutes; closed book, **closed notes**
 - Review Material
 - old exams on the web site (“schedule” tab)
 - Review Session
 - **Tonight**, 7:00-9:00pm, Towne 100 (will be recorded)
 - Review Videos available
- Dr. Sheth will have extra office hours
 - Thursday from 2-4pm, Levine 264

Module Review

What is a good signature?

Fall 2022 Question 3 #324



Anonymous

15 hours ago in Exams



PIN



STAR



WATCH

29

VIEWS



Hi!

Can someone clarify the difference between ill-typed, unusable, invariant and good interfaces?

Thank you!

[Comment](#) [Edit](#) [Delete](#) [Endorse](#) ...

1 Answer



Luis Sanguedo **STAFF**

12 hours ago



Sure!



Ill-typed: The type signature for one or more functions has an incorrect type such as trying to add an int and a string together to produce an int

Unusable: The provided type declaration would render an implementation unusable. For example, in HW3, not having the definition of an empty set or the `set_of_list` function would render our interface unusable since we'd never be able to create a set type!

Good Signature: Finite Map

```
module type MAP = sig
```

```
  type ('k, 'v) map
```

```
  val empty      : ('k, 'v) map
```

```
  val add        : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
```

```
  val mem        : 'k -> ('k, 'v) map -> bool
```

```
  val get        : 'k -> ('k, 'v) map -> 'v
```

```
  val equals     : ('k, 'v) map -> ('k, 'v) map -> bool
```

```
end
```

Unusable Signature: Finite Map

```
module type MAP = sig
```

```
  type ('k, 'v) map
```

Type is abstract. All we know about it is what is in the signature. All maps must be constructed from operations listed here.

```
  val add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
```

```
  val mem      : 'k -> ('k, 'v) map -> bool
```

```
  val get      : 'k -> ('k, 'v) map -> 'v
```

```
  val equals   : ('k, 'v) map -> ('k, 'v) map -> bool
```

```
end
```

```
let m = ???
```

Clients have no way of constructing a map
Using [] doesn't type check

Good Signature (again): Finite Map

```
module type MAP = sig
```

```
  type ('k, 'v) map
```

```
  val empty      : ('k, 'v) map
```

```
  val add        : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
```

```
  val mem        : 'k -> ('k, 'v) map -> bool
```

```
  val get        : 'k -> ('k, 'v) map -> 'v
```

```
  val equals     : ('k, 'v) map -> ('k, 'v) map -> bool
```

```
end
```

Unsafe Signature: Finite Map

```
module type MAP = sig
```

```
  type ('k, 'v) map = ('k * 'v) list
```

Invariant: keys appear in
order in the list, no duplicate
keys

```
  val empty      : ('k, 'v) map
```

```
  val add        : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
```

```
  val mem        : 'k -> ('k, 'v) map -> bool
```

```
  val get        : 'k -> ('k, 'v) map -> 'v
```

```
  val equals     : ('k, 'v) map -> ('k, 'v) map -> bool
```

```
end
```

```
let m = [ (3, "dos") ; (1, "uno") ; (2, "tres") ] in  
mem m 2 ?
```

Clients can call module code with maps that don't
satisfy the invariant

Good Signature (again): Finite Map

```
module type MAP = sig
```

```
  type ('k, 'v) map
```

```
  val empty      : ('k, 'v) map
```

```
  val add        : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
```

```
  val mem        : 'k -> ('k, 'v) map -> bool
```

```
  val get        : 'k -> ('k, 'v) map -> 'v
```

```
  val equals     : ('k, 'v) map -> ('k, 'v) map -> bool
```

```
end
```

Unsafe Signature: Finite Map

```
module type MAP = sig
```

```
  type ('k, 'v) map
```

Invariant: keys appear in
order in the list, no duplicate
keys

```
  val empty      : ('k, 'v) map
```

```
  val add        : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
```

```
  val mem        : 'k -> ('k, 'v) map -> bool
```

```
  val get        : 'k -> ('k * 'v) list -> 'v
```

```
  val equals     : ('k, 'v) map -> ('k, 'v) map -> bool
```

```
end
```

```
let m = [ (3, "dos") ; (1, "uno") ; (2, "tres") ] in  
get m 2 ?
```

Clients can call module code with maps that don't
satisfy the invariant

Good Signature (again): Finite Map

```
module type MAP = sig
```

```
  type ('k, 'v) map
```

```
  val empty      : ('k, 'v) map
```

```
  val add        : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
```

```
  val mem        : 'k -> ('k, 'v) map -> bool
```

```
  val get        : 'k -> ('k, 'v) map -> 'v
```

```
  val equals     : ('k, 'v) map -> ('k, 'v) map -> bool
```

```
end
```

Unimplementable Signature: Finite Map

```
module type MAP = sig
  type ('k,'v) map = 'k list

  val empty      : map
  val add        : 'k -> 'v -> ('k,'v) map -> ('k,'v) map
  val mem        : 'k -> ('k,'v) map -> bool
  val get        : 'k -> ('k,'v) map -> 'v
  val equals     : ('k1,'v1) map -> ('k2,'v2) map -> bool
end
```

1. Wrong implementation type (get doesn't have enough info)
2. Missing type arguments (empty) --- doesn't compile!
3. Type too generic (equals)

.ml and .mli files

- You've already been using signatures and modules in OCaml.
- A series of type and `val` declarations stored in a file `foo.mli` is considered as defining a signature `F00`
- A series of top-level definitions stored in a file `foo.ml` is considered as defining a module `Foo`

foo.mli

```
type t
val z : t
val f : t -> int
```

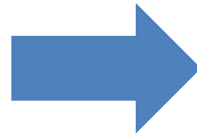
foo.ml

```
type t = int
let z : t = 0
let f (x:t) : int =
  x + 1
```

test.ml

```
;; open Foo
;; print_int
   (Foo.f Foo.z)
```

Files



```
module type F00 = sig
  type t
  val z : t
  val f : t -> int
end
```

```
module Foo : F00 = struct
  type t = int
  let z : t = 0
  let f (x:t) : int =
    x + 1
end
```

```
module Test = struct
  ;; open Foo
  ;; print_int
     (Foo.f Foo.z)
end
```

Typechecking

How does OCaml typecheck your code?

*Historical aside: the algorithm we are about to see is known as the Damas-Hindley-Milner type inference algorithm. Turing Award winner Robin Milner was, among other things, the inventor of "ML" (for "meta language"), from which OCaml gets its "ml".

OCaml Typechecking Errors

```
type ('k,'v) map = ('k * 'v) list
```

```
(* A finite map that contains no entries. *)
```

```
let empty () = []
```

```
let rec mem
```

```
begin ma
```

```
| [] ->
```

```
| (k,v):
```

```
if key
```

```
(key = k) || (mem key rest)
```

```
end
```

```
;; run_test "mem test" (fun () ->
```

```
mem "b" [("a",3); ("b",4)]
```

```
)
```

```
let rec get (key:'k) (m : ('k,'v) map) : 'v =
```

```
begin match m with
```

```
| [] -> failwith "not found"
```

✖ Signature mismatch:

...

Values do not match:

- val empty : unit -> 'a list

is not included in

- val empty : ('k, 'v) map

File "finiteMap.ml", line 13, characters 2-27: Expected declaration

File "finiteMap.ml", line 60, characters 6-11: Actual declaration

Typechecking

How to determine the type of an expression?

1. Recursively determine the types of *all* sub-expressions

– Constants have “obvious” types

3 : int “foo” : string true : bool

– Identifiers may have type annotations

- let and function arguments
- Module signatures/interfaces

2. Expressions that *construct* structured values have compound types built from the types of sub-expressions

(3, “foo”) : int * string

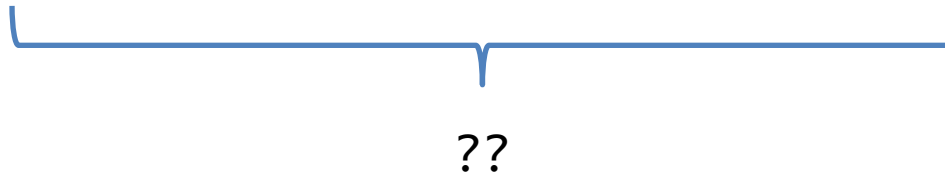
(fun (x:int) -> x + 1) : int -> int

Node(Empty, (3, “foo”), Empty) : (int * string) tree

Typechecking Functions

To typecheck a function:

```
fun (x:int) -> x + x
```



Typechecking Functions

To typecheck a function:

```
fun (x:int) -> x + x
```



$T_{\text{arg}} \rightarrow T_{\text{ans}}$

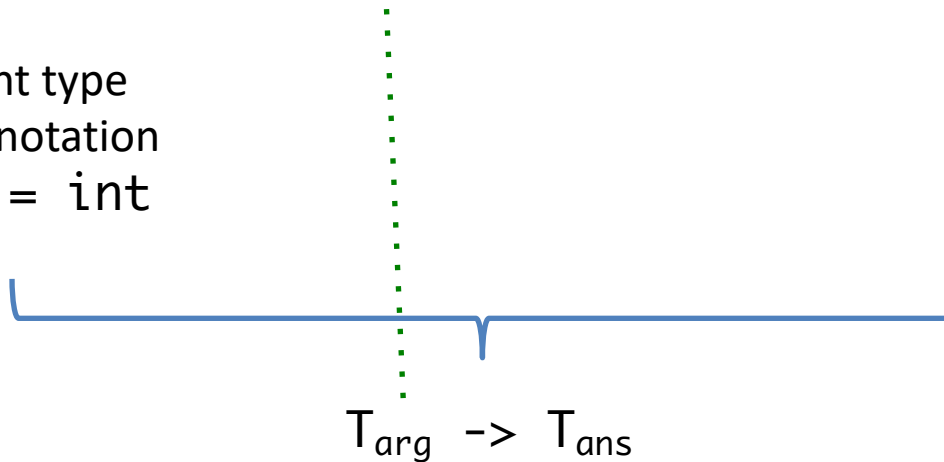
Make up "new names" for the input (argument) and output (answer) types.

Typechecking Functions

To typecheck a function:

fun (x:int) -> x + x

Take the argument type
from the type annotation
(if any*): $T_{arg} = \text{int}$

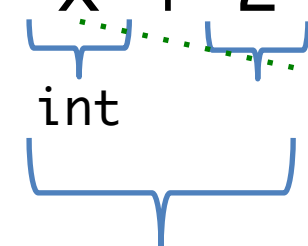


*If there is no annotation, just use the "fresh" name...

Typechecking Functions

To typecheck a function:

`fun (x:int) -> x + 2`



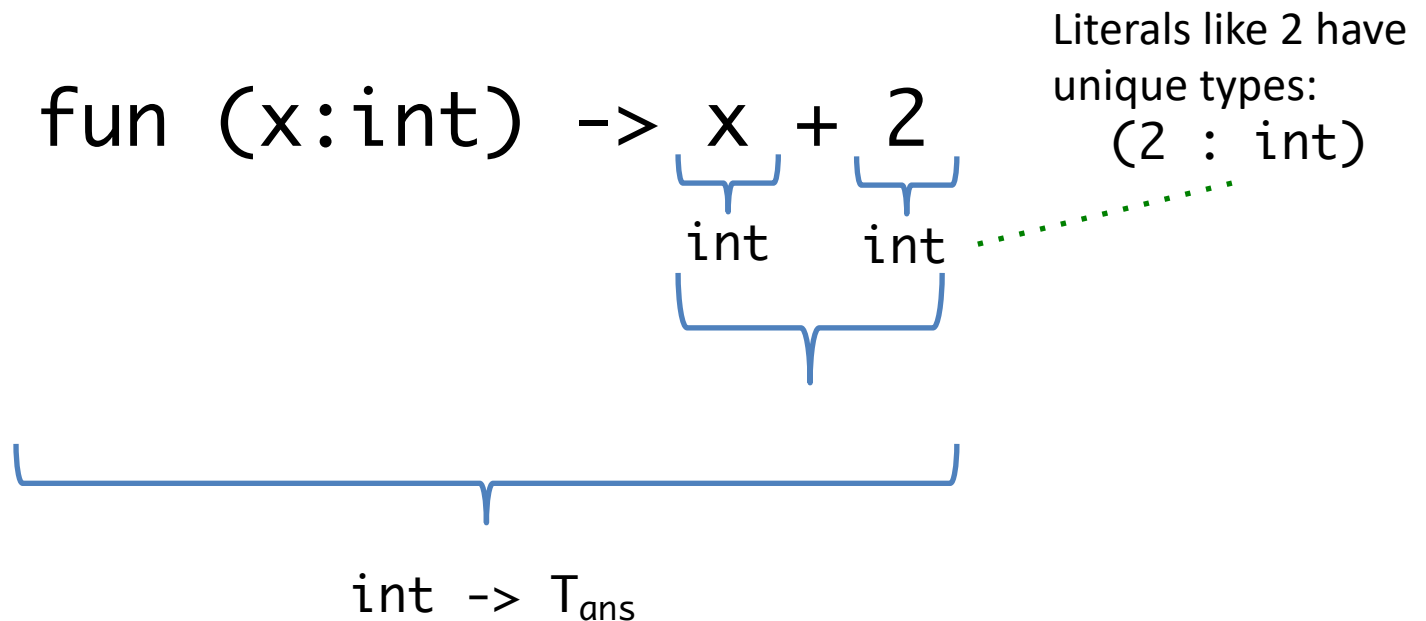
Recursive typecheck the
body of the function in
a "typing context" where
the argument has
the input type:
(x : int)



`int -> Tans`

Typechecking Functions

To typecheck a function:



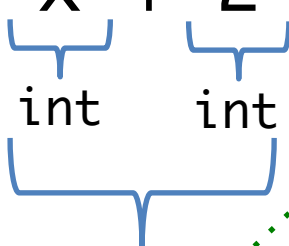
Typechecking Functions

To typecheck a function:

Built-in operations like (+) also have types:

$(+) : \text{int} \rightarrow \text{int} \rightarrow \text{int}$

$\text{fun } (x:\text{int}) \rightarrow x + 2$



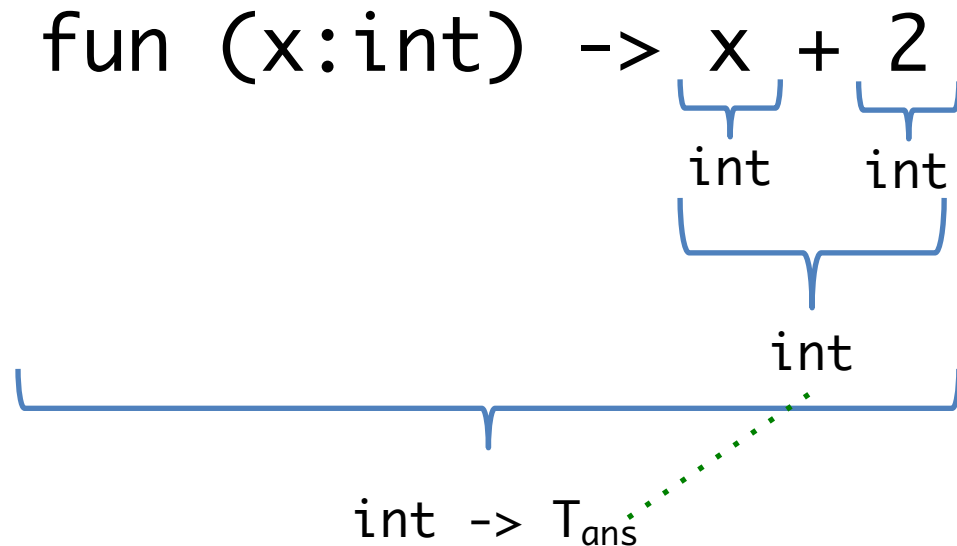
$\text{int} \rightarrow T_{\text{ans}}$



Function application has the result type, assuming the input types are correct.

Typechecking Functions

To typecheck a function:



The "answer" type is the type of the function body.

$$T_{ans} = \text{int}$$

Typechecking Functions

To typecheck a function:

`fun (x:int) -> x + 2`

int -> int

Typechecking II

3. The type of a function-application expression is obtained as the result from the function type:

- Given a function $f : T_{arg} \rightarrow T_{ans}$
- and an argument $e : T_{arg}$ of the input type
- the application $(f\ e) : T_{ans}$ has the answer type

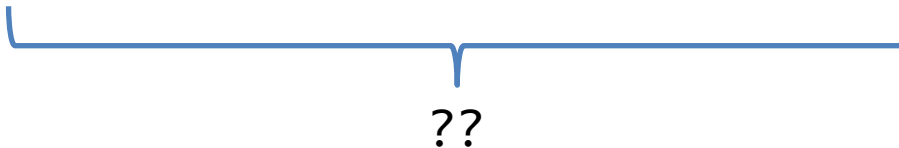
`((fun (x:int) (y:bool) -> y) 3) : ??`

Typechecking II

3. The type of a function-application expression is obtained as the result from the function type:

- Given a function f : $T_{arg} \rightarrow T_{ans}$
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`((fun (x:int) (y:bool) -> y) 3) : ??`



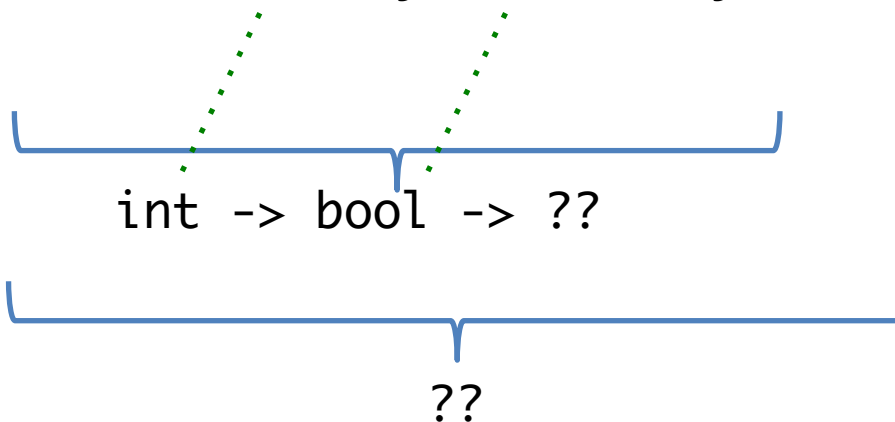
??

Typechecking II

3. The type of a function-application expression is obtained as the result from the function type:

- Given a function f : $T_{arg} \rightarrow T_{ans}$
- and an argument e : T_{arg} of the input type
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`((fun (x:int) (y:bool) -> y) 3) : ??`

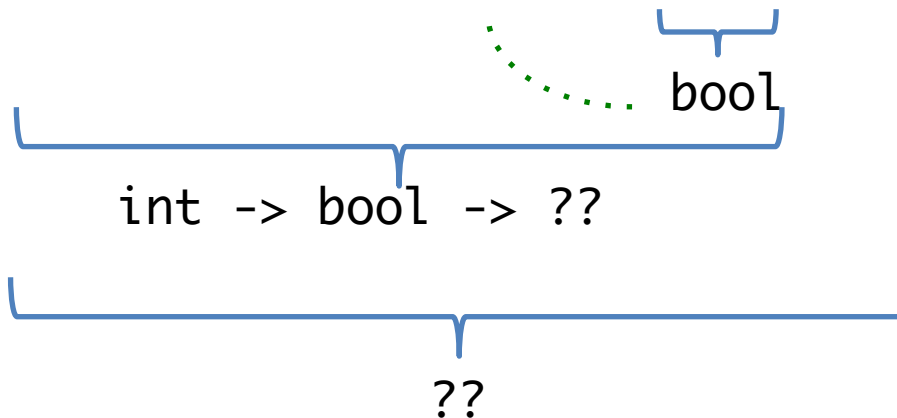


Typechecking II

3. The type of a function-application expression is obtained as the result from the function type:

- Given a function $f : T_{arg} \rightarrow T_{ans}$
- and an argument $e : T_{arg}$ of the input type
- the application $(f\ e) : T_{ans}$ has the answer type

$((\text{fun } (x:\text{int}) (y:\text{bool}) \rightarrow y) \ 3) : ??$

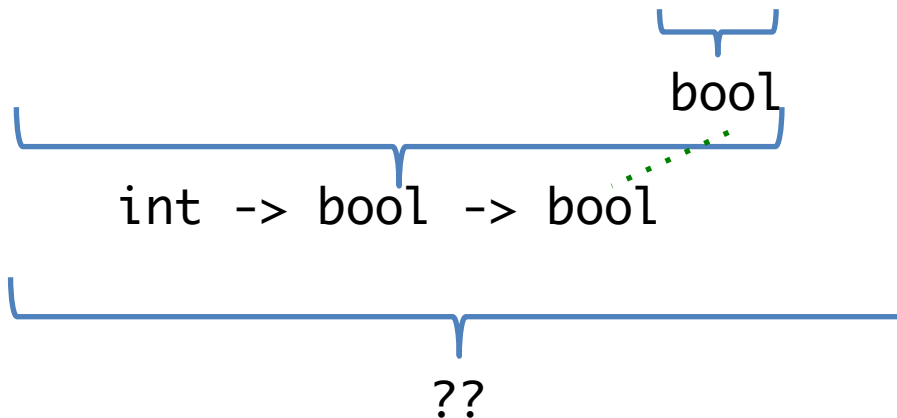


Typechecking II

3. The type of a function-application expression is obtained as the result from the function type:

- Given a function $f : T_{arg} \rightarrow T_{ans}$
- and an argument $e : T_{arg}$ of the input type
- the application $(f\ e) : T_{ans}$ has the answer type

$((\text{fun } (x:\text{int}) (y:\text{bool}) \rightarrow y) \ 3) : ??$

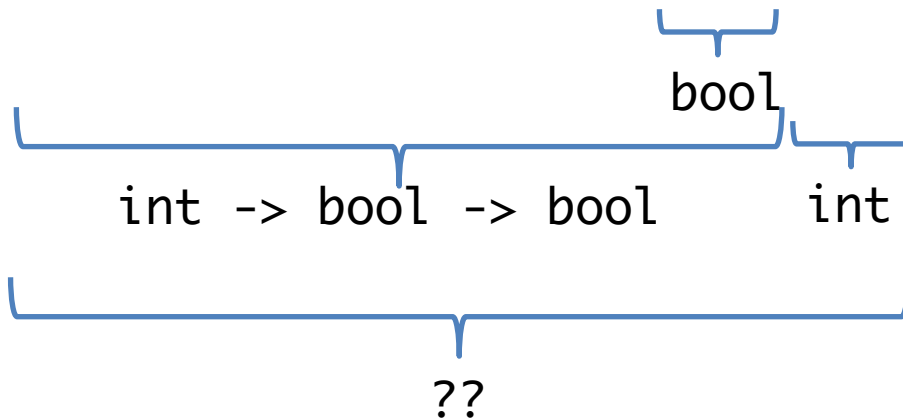


Typechecking II

3. The type of a function-application expression is obtained as the result from the function type:

- Given a function $f : T_{arg} \rightarrow T_{ans}$
- and an argument $e : T_{arg}$ of the input type
- the application $(f\ e) : T_{ans}$ has the answer type

$((\text{fun } (x:\text{int}) (y:\text{bool}) \rightarrow y) \ 3) : ??$

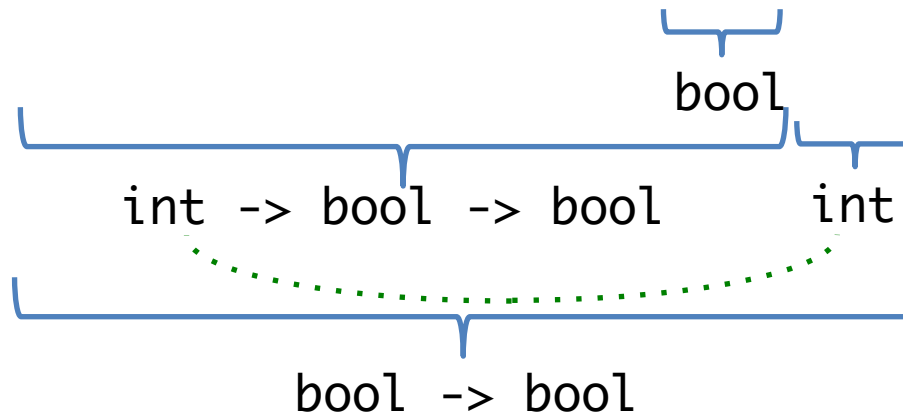


Typechecking II

3. The type of a function-application expression is obtained as the result from the function type:

- Given a function $f : T_{arg} \rightarrow T_{ans}$
- and an argument $e : T_{arg}$ of the input type
- the application $(f\ e) : T_{ans}$ has the answer type

$((\text{fun } (x:\text{int}) (y:\text{bool}) \rightarrow y) \ 3) : ??$



Here:

$T_{arg} = \text{int}$

$T_{ans} = \text{bool} \rightarrow \text{bool}$

Typechecking III

- What about generics? i.e., what if $f : 'a \rightarrow 'a$?
- For generic types we *unify*
 - Given a function $f : T_1 \rightarrow T_2$
 - and an argument $e : U_1$ of the input typeCan “match up” T_1 and U_1 to obtain information about type parameters in T_1 and U_1 based on their usage

- *Unification:*

- Try to match up corresponding parts of the type

(int list) tree $\Leftrightarrow$ 'a tree



- This produces an *instantiation*: e.g. $'a = \text{int list}$
- *Propagate* that instantiation to all occurrences of $'a$
- If unification fails, produce a type checking error

bool tree $\Leftrightarrow$ int tree

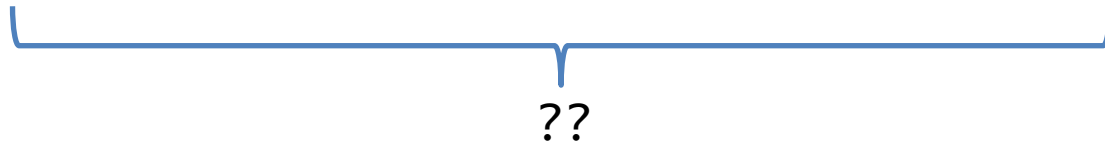


ERROR! $\text{bool} \neq \text{int}$

Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

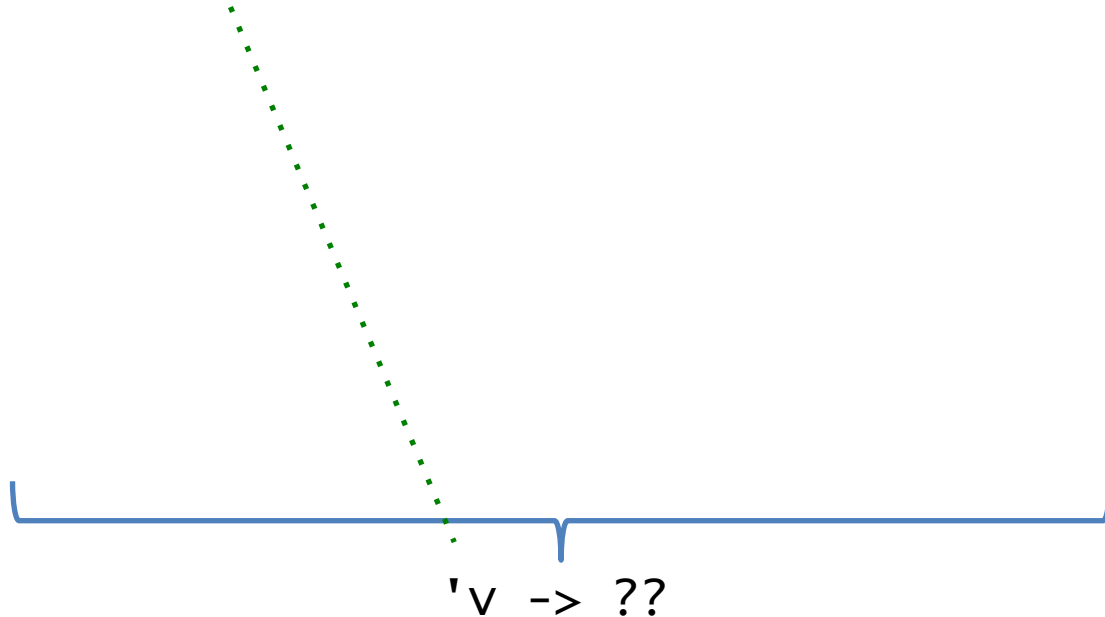
```
fun (x:'v) -> entries (add 3 x empty)
```



Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
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```

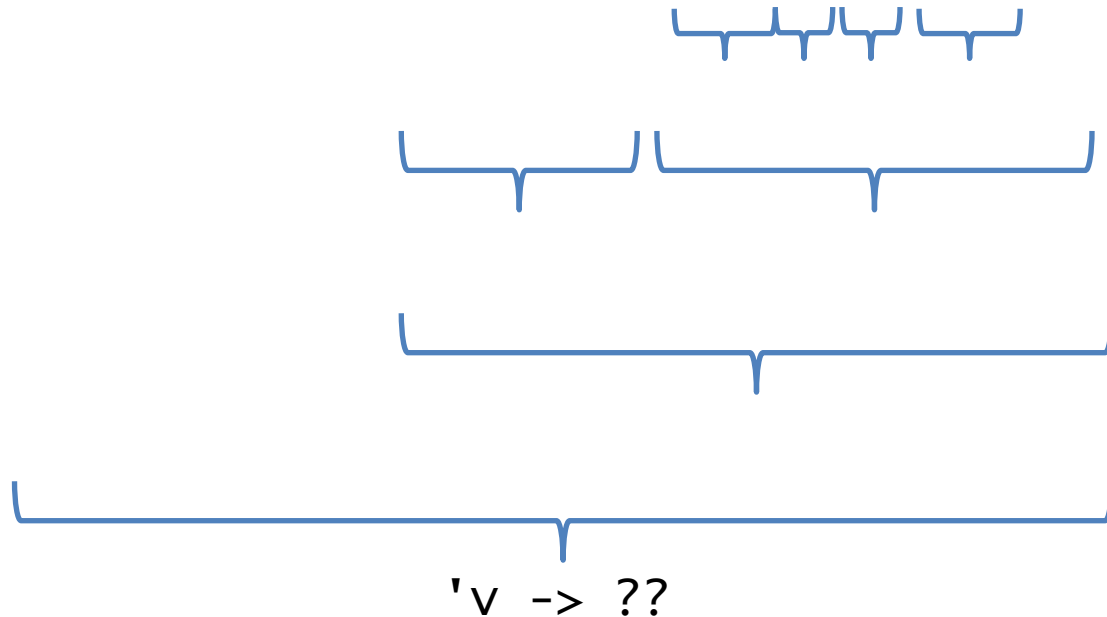
```
fun (x:'v) -> entries (add 3 x empty)
```



Example Typechecking Problem

```
empty    : ('k, 'v) map
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
entries  : ('k, 'v) map -> ('k * 'v) list
```

```
fun (x:'v) -> entries (add 3 x empty)
```



Example Typechecking Problem

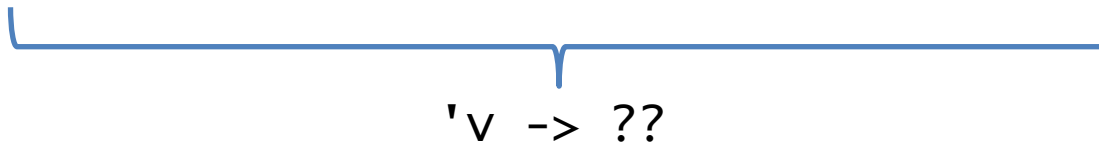
```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

fun (x:'v) -> entries (add 3 x empty)


int



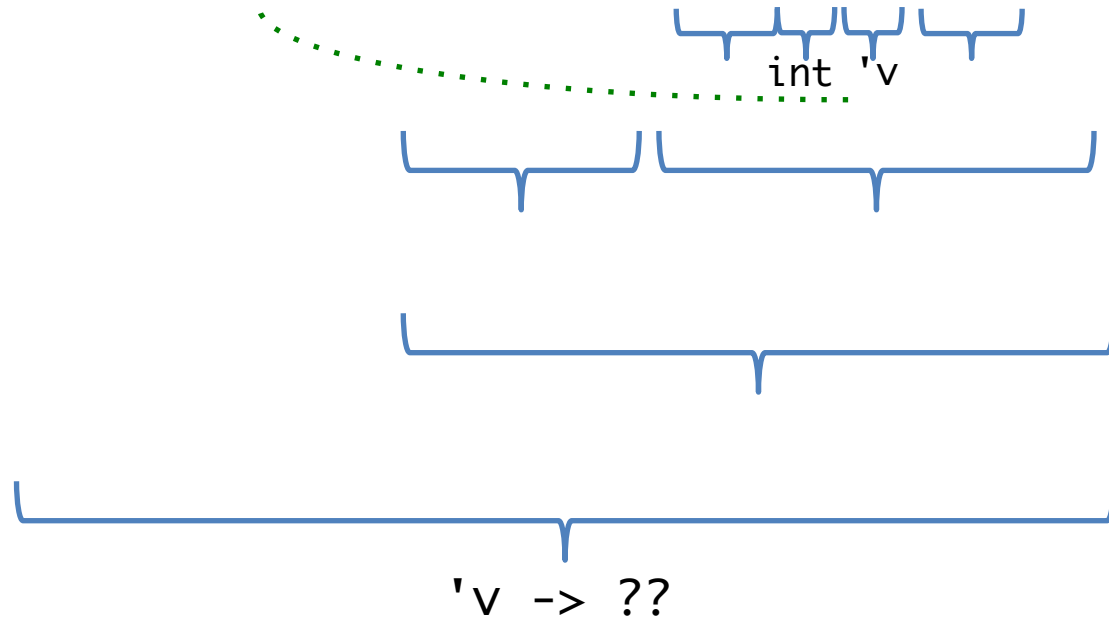



'v -> ??

Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

```
fun (x:'v) -> entries (add 3 x empty)
```



Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

`fun (x:'v) -> entries (add 3 x empty)`

Diagram illustrating the typechecking problem with annotations:

- `add` is annotated with `int 'v ('k, 'v) map` below it.
- `3` is annotated with `int` below it.
- `x` is annotated with `'v` below it.
- `empty` is annotated with `('k, 'v) map` below it.
- The entire expression `add 3 x empty` is annotated with `'v -> ??` below it.

Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

`fun (x:'v) -> entries (add 3 x empty)`

The diagram illustrates the typechecking process for the function `fun (x:'v) -> entries (add 3 x empty)`. It shows nested lambda abstractions represented by blue curly braces. The innermost brace groups the arguments `3`, `x`, and `empty` of the `add` function, with the type `int 'v ('k, 'v) map` written below it. The next brace groups the entire function body `entries (add 3 x empty)`. The outermost brace groups the entire function definition, with the type `'v -> ??` written below it. A green dotted line connects the `'v` in the function signature to the `'v` in the type `int 'v ('k, 'v) map`.

Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

`fun (x:'v) -> entries (add 3 x empty)`

int 'v ('k, 'v) map

??

'v -> ??

Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

fun (x:'v) -> entries (add 3 x empty)

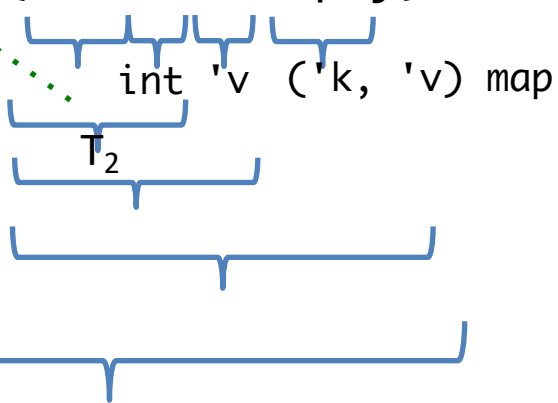
Application:

$T_1 = 'k$

$T_2 = 'v \rightarrow ('k, 'v) \text{ map} \rightarrow ('k, 'v) \text{ map}$

Instantiate: **'k = int**

'v -> ??



Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

fun (x:'v) -> entries (add 3 x empty)

Application:

$T_1 = \text{int}$

$T_2 = 'v \rightarrow (\text{int}, 'v) \text{ map} \rightarrow (\text{int}, 'v) \text{ map}$

Instantiate: $'k = \text{int}$

$'v \rightarrow ??$

Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

fun (x:'v) -> entries (add 3 x empty)

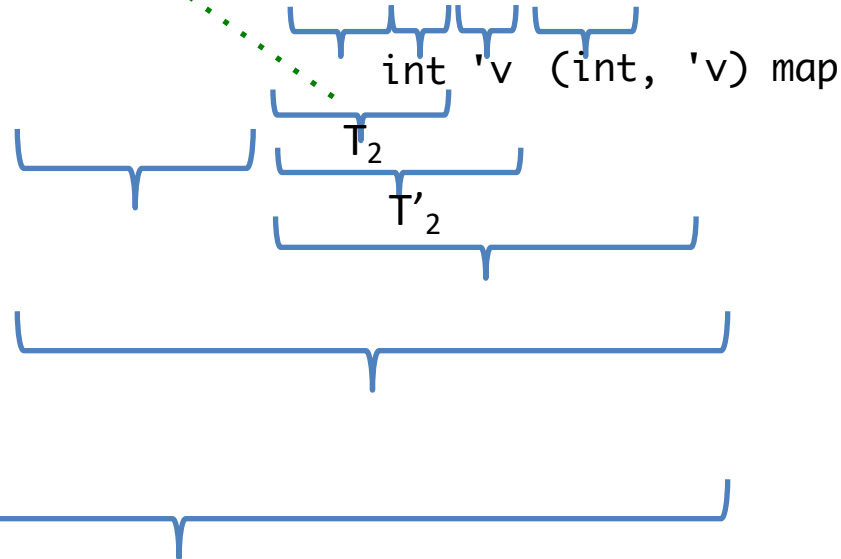
Another Application:

$T'_1 = 'v$

$T'_2 = (int, 'v) map \rightarrow (int, 'v) map$

Instantiate: $'v = 'v$

$'v \rightarrow ??$



Example Typechecking Problem

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

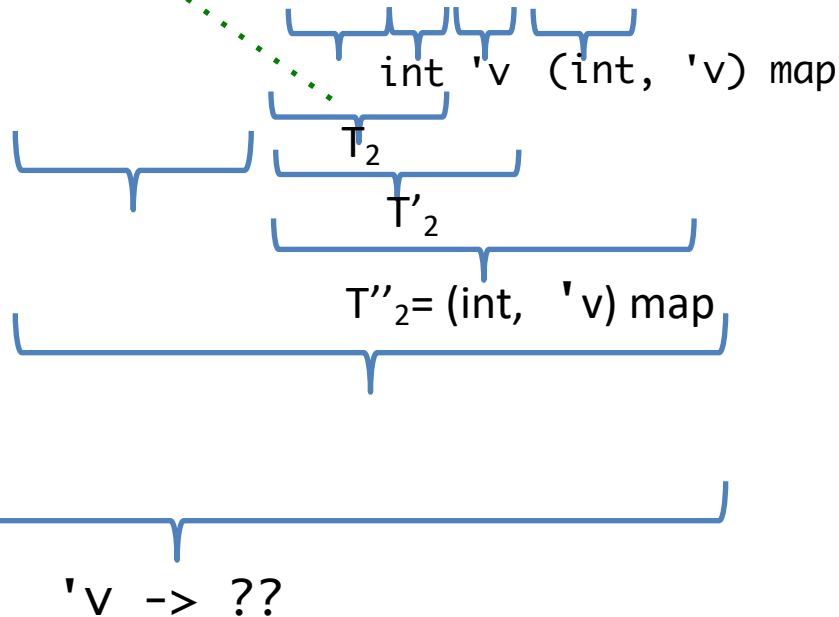
fun (x:'v) -> entries (add 3 x empty)

A third Application:

$T''_1 = (\text{int}, 'v) \text{ map}$

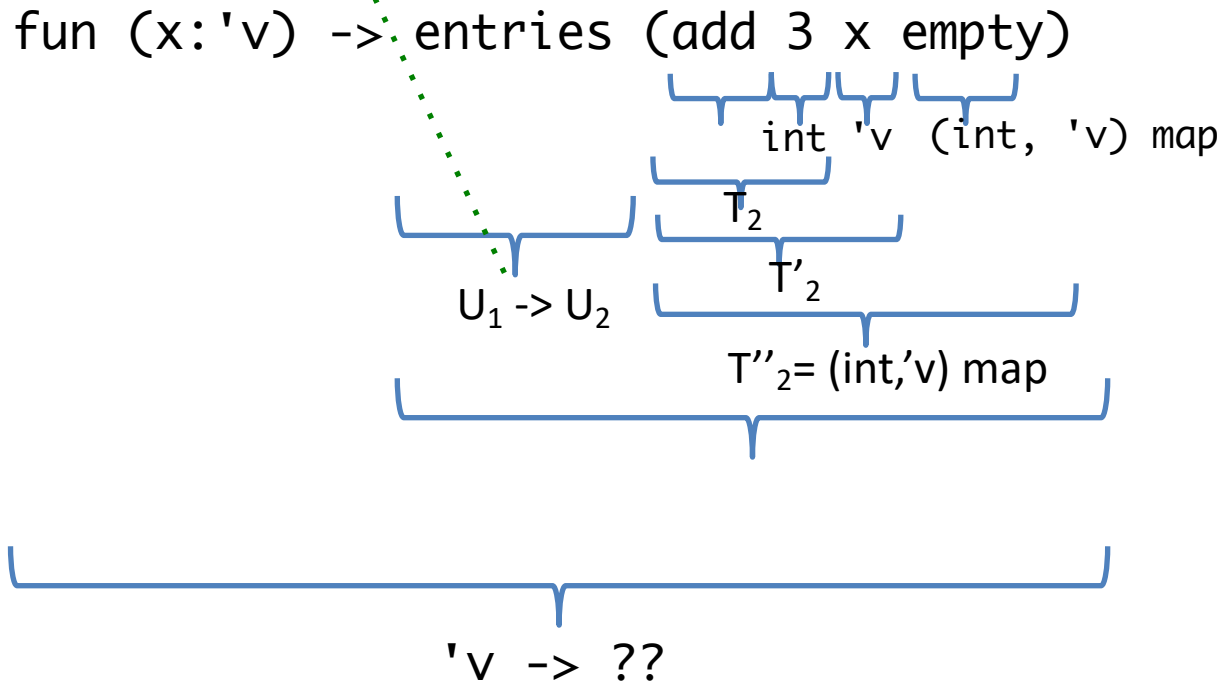
$T''_2 = (\text{int}, 'v) \text{ map}$

Argument and argument
type already agree



Example Typechecking Problem

```
empty    : ('k, 'v) map
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
entries  : ('k, 'v) map -> ('k * 'v) list
```



Example Typechecking Problem

```
empty    : ('k, 'v) map
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
entries  : ('k, 'v) map -> ('k * 'v) list
```

fun (x:'v) -> entries (add 3 x empty)

Another Application:

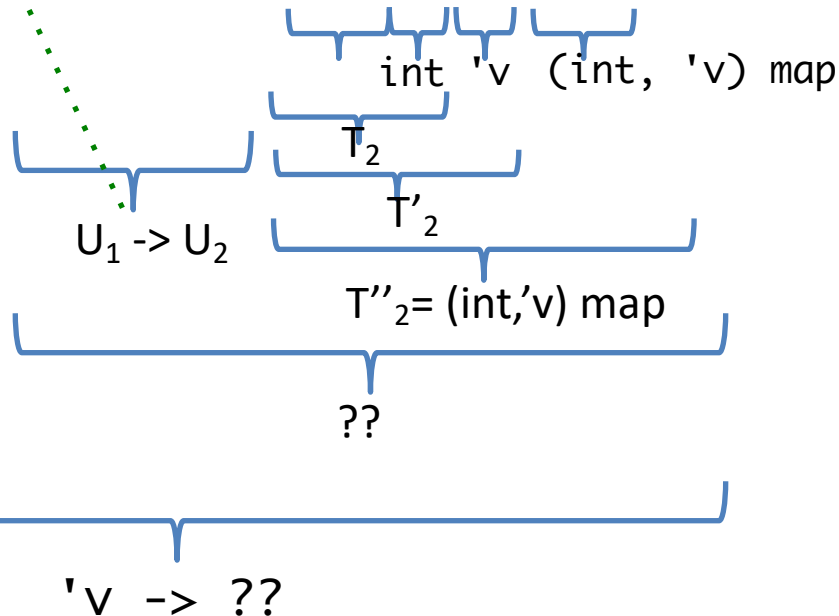
$U_1 = ('k, 'v) \text{ map}$

$U_2 = ('k * 'v) \text{ list}$

Unify U_1 with T''_2

$('k, 'v) \text{ map} \Leftrightarrow (int, 'v) \text{ map}$

Instantiate $'k = int$



Example Typechecking Problem

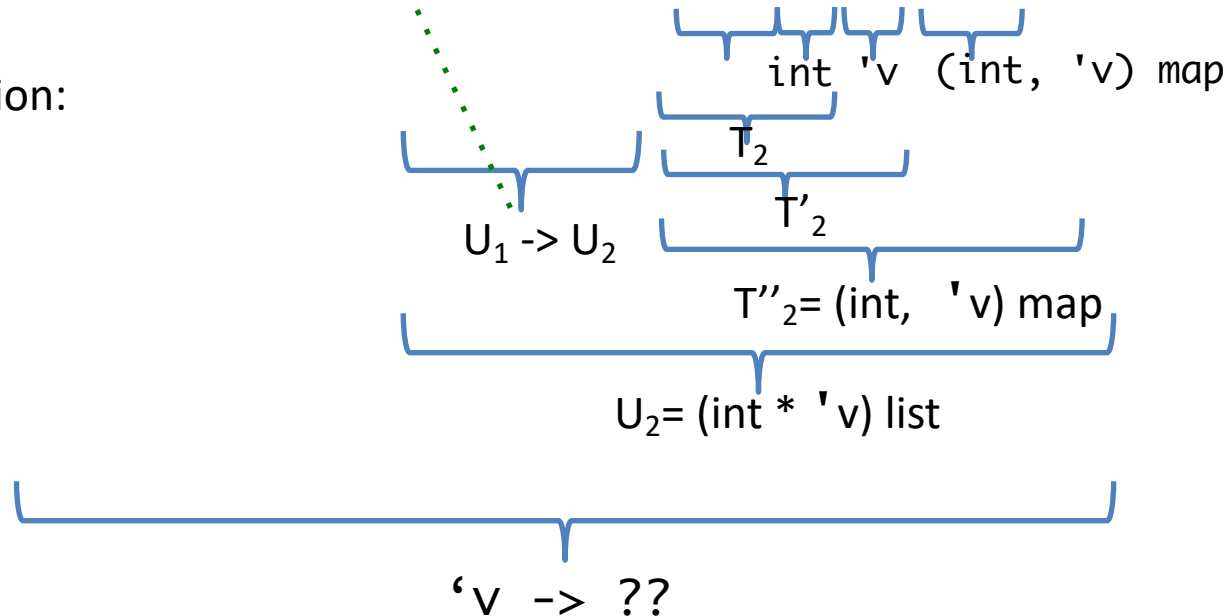
```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

fun (x:'v) -> entries (add 3 x empty)

Another Application:

$U_1 = (\text{int}, 'v) \text{ map}$

$U_2 = (\text{int} * 'v) \text{ list}$



Example Typechecking Problem

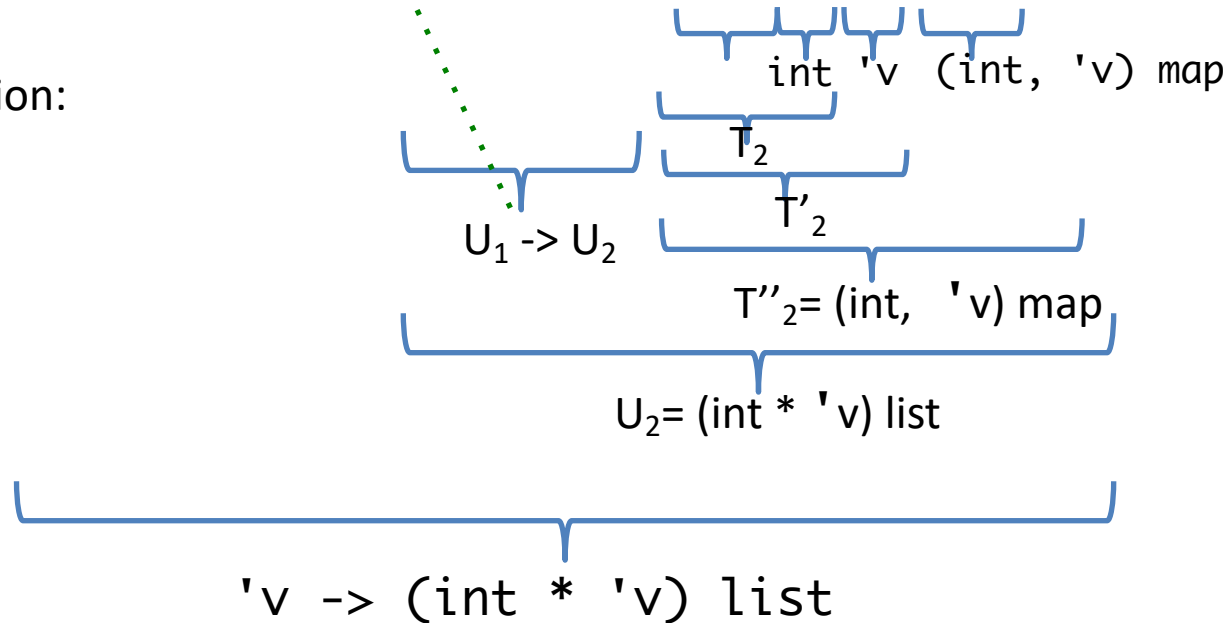
```
empty    : ('k, 'v) map
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
entries  : ('k, 'v) map -> ('k * 'v) list
```

fun (x:'v) -> entries (add 3 x empty)

Another Application:

$U_1 = (\text{int}, 'v) \text{ map}$

$U_2 = (\text{int} * 'v) \text{ list}$



Ill-typed Expressions?

- An expression is ill-typed if, during this type checking process, inconsistent constraints are encountered:

```
empty    : ('k, 'v) map  
add      : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map  
entries  : ('k, 'v) map -> ('k * 'v) list
```

add 3 true (add “foo” false empty)

Error: found `int` but expected `string`

12: What is the type of this expression?

0

```
let e : _____ =  
  transform (fun x y -> x + y)
```

int list -> int list

0%

int list -> int list -> int list

0%

int list -> (int -> int) list

0%

None (it doesn't typecheck)

0%

What is the type of this expression?

```
let e : _____ =  
  transform (fun x y -> x + y)
```

1. int list -> int list
2. int list -> int list -> int list
3. int list -> (int -> int) list
4. None (it doesn't typecheck)

Answer: 3

Dealing with Partiality*

*A function is said to be *partial* if it is not defined for all inputs.

13: Which of these is a function that calculates the maximum value in a (generic) list?

✓ 0

1

0%

2

0%

3

0%

4

0%

Which of these is a function that calculates the maximum value in a (generic) list:

1.

```
let rec list_max (l:'a list) : 'a =  
  begin match l with  
  | [] -> []  
  | h :: t -> max h (list_max t)  
  end
```

2.

```
let rec list_max (l:'a list) : 'a =  
  fold max 0 l
```

3.

```
let rec list_max (l:'a list) : 'a =  
  begin match l with  
  | h :: t -> max h (list_max t)  
  end
```

4. None of the above

Answer: 4

Quiz answer

- list_max isn't defined for the empty list!

```
let rec list_max (l:'a list) : 'a =  
  begin match l with  
    | [] -> failwith "empty list"  
    | [h] -> h  
    | h::t -> max h (list_max t)  
  end
```

Client of list_max

```
(* string_of_max calls list_max *)  
let string_of_max (x:int list) : string =  
  string_of_int (list_max x)
```

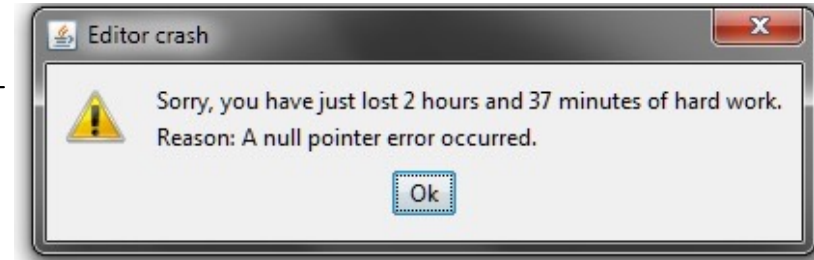
- Oops! `string_of_max` will fail if given `[]`
- Not so easy to debug if `string_of_max` is written by one person and `list_max` is written by another.
- Interface of `list_max` is not very informative
`val list_max : int list -> int`

Solutions to Partiality: Option 1

- *Abort the program:*
 - `failwith "an error message"`
 - Whenever it is called, `failwith` halts the program and reports the error message it is given.
- This solution is appropriate whenever you *know* that a certain case is impossible
 - The compiler isn't smart enough to figure out that the case is impossible...
 - Often happens when there is an invariant on a data structure
- Languages (e.g. OCaml, Java) support *exception handling facilities* to let programs recover from such failures.
 - We'll talk about these when we get to Java

Solutions to Partiality: Option 2

- Return a *default or error value*
 - e.g. define `list_max []` to be `-1`
 - Error codes used often in C programs
 - `null` used often in Java
- But...
 - What if `-1` (or whatever default you choose) really *is* the maximum value?
 - Can lead to many bugs if the default isn't handled properly by the callers
 - *IMPOSSIBLE* to implement generically!
 - No way to generically create a sensible default value for every possible type
 - Sir Tony Hoare, Turing Award winner and inventor of `null` calls it his “billion dollar mistake”!
- *Defaults should be avoided if possible*



Solutions to Partiality: Option 3

Return something that *cannot*
be misinterpreted as a
legitimate non-exceptional
result ...

Optional values

Solutions to Partiality: Option 3

Option Types

- Define a generic datatype of *optional values*

```
type 'a option =  
  | None  
  | Some of 'a
```

Type definition built into
of OCaml. Always available.

- A “partial” function returns an option

```
let list_max (l:list) : int option = ...
```

- Contrast this with “null”, a “legal” return value of any type
 - caller can accidentally forget to check whether null was used; results in NullPointerExceptions or crashes
- Modern language designs (e.g. Apple's Swift, Mozilla's Rust) distinguish between the type String (definitely not null) and String? (optional string)

Example: list_max

- A function that returns the maximum value of a list as an option (None if the list is empty)

```
let list_max (l:'a list) : 'a option =  
  begin match l with  
    | [] -> None  
    | x::tl -> Some (fold max x tl)  
  end
```

Revised client of list_max

```
(* string_of_max calls list_max *)  
let string_of_max (l:int list) : string =  
  begin match (list_max l) with  
  | None -> "no maximum"  
  | Some m -> string_of_int m  
  end
```

- string_of_max will never fail
- The type of list_max makes it explicit that a *client* must check for partiality.

```
val list_max : int list -> int option
```

13: What is the type of this function?



- 'a list -> 'a 0%
- 'a list -> 'a list 0%
- 'a list -> 'b option 0%
- 'a list -> 'a option 0%
- None of the above 0%

What is the type of this function?

```
let head (x: _____) : _____ =  
  begin match x with  
  | [] -> None  
  | h :: t -> Some h  
  end
```

1. 'a list -> 'a
2. 'a list -> 'a list
3. 'a list -> 'b option
4. 'a list -> 'a option
5. None of the above

Answer: 4

13: What is the value of this expression?

0

[1; 0]

0%

1

0%

[Some 1; None]

0%

[None; None]

0%

None of the above

0%

What is the value of this expression?

```
let head (x: 'a list) : 'a option =  
  begin match x with  
    | [] -> None  
    | h :: t -> Some h  
  end in
```

```
[ head [1]; head [] ]
```

1. [1 ; 0]
2. 1
3. [Some 1; None]
4. [None; None]
5. None of the above

Answer: 3

Revising the MAP interface

```
module type MAP = sig

  type ('k, 'v) map

  val empty      : ('k, 'v) map
  val add        : 'k -> 'v -> ('k, 'v) map -> ('k, 'v) map
  val remove     : 'k          -> ('k, 'v) map -> ('k, 'v) map
  val mem        : 'k -> ('k, 'v) map -> bool
  val get        : 'k -> ('k, 'v) map -> 'v option
  val entries    : ('k, 'v) map -> ('k * 'v) list
  val equals     : ('k, 'v) map -> ('k, 'v) map -> bool

end
```

get returns an optional 'v.
Now its type isn't a lie!

Records

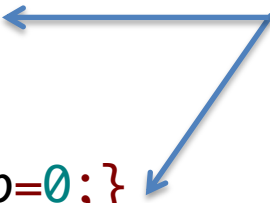
Immutable Records

- Records are like tuples with named fields:

```
(* a type for representing colors *)
```

```
type rgb = {r:int; g:int; b:int;}
```

Curly braces
around record.
Semicolons after
record components.



```
(* some example rgb values *)
```

```
let red    : rgb = {r=255; g=0;  b=0;}
```

```
let blue   : rgb = {r=0;  g=0;  b=255;}
```

```
let green  : rgb = {r=0;  g=255; b=0;}
```

```
let black  : rgb = {r=0;  g=0;  b=0;}
```

```
let white  : rgb = {r=255; g=255; b=255;}
```

- The type `rgb` is a record with three fields: `r`, `g`, and `b`
 - fields can have any types; they don't all have to be the same
- Record values are created using this notation:
`{field1=val1; field2=val2;...}`

Field Projection

- The value in a record field can be obtained by using “dot” notation: `record.field`

```
(* a type for representing colors *)
type rgb = {r:int; g:int; b:int;}

(* using 'dot' notation to project out components *)
(* calculate the average of two colors *)
let average_rgb (c1:rgb) (c2:rgb) : rgb =
  {r = (c1.r + c2.r) / 2;
   g = (c1.g + c2.g) / 2;
   b = (c1.b + c2.b) / 2;}
```

13: What is the type of f in the following program?



unit -> int

0%

unit -> unit

0%

int -> unit

0%

int -> int

0%

f is ill typed

0%

13: What is the type of f in the following program?



unit -> int

0%

unit -> unit

0%

int -> unit

0%

int -> int

0%

f is ill typed

0%

13: What answer does the following function produce when called?

0

17

0%

something else

0%

sometimes 17 and sometimes someth...

0%

f is ill typed

0%

13: What answer does the following function produce when called?

0

17

0%

something else

0%

sometimes 17 and sometimes someth...

0%

f is ill typed

0%