CIS 121 — Data Structures and Algorithms
Homework Assignment 8

Assigned: November 6, 2018  Due: November 18, 2018 6 PM E.S.T.

Note: The homework is due electronically on Gradescope and Canvas on Sunday, November 18 by 6:00 pm EDT. You may NOT use any late days on this portion of the assignment; solutions will be distributed at the midterm review session on Sunday to help you study for Midterm 3.

A. Gradescope: You must select the appropriate pages on Gradescope. Gradescope makes this easy for you: before you submit, it asks you to associate pages with the homework questions. Failing to do so will get you 5% off, which cannot be argued against after the fact.

B. Canvas: You must also submit your assignment on Canvas. Forgetting to do so will incur a 10% penalty.

C. \LaTeX: You must use the \texttt{hw121.cls} Latex template provided on the course website, or a 5% penalty will be incurred. Handwritten solutions or solutions not typeset in Latex will not be accepted.

D. Solutions: Please write concise and clear solutions; you will get only a partial credit for correct solutions that are either unnecessarily long or not clear. Please refer to the Written Homework Guidelines for all the requirements. Piazza will also contain a complete sample solution.

E. Algorithms: Whenever you present an algorithm, your answer must include 3 separate sections. Please see Piazza for an example complete solution.

1. A precise description of your algorithm in English. No pseudocode, no code.
2. Proof of correctness of your algorithm
3. Analysis of the running time complexity of your algorithm

F. Collaboration: You are allowed to discuss ideas for solving homework problems in groups of up to 3 people but you must write your solutions independently. Also, you must write on your homework the names of the people with whom you discussed. For more on the collaboration policy, please see the course webpage.

G. Outside Resources: Finally, you are not allowed to use any material outside of the class notes and the textbook. Any violation of this policy may seriously affect your grade in the class. If you’re unsure if something violates our policy, please ask.
1. **[15 pts] Advance Registration** Course registration opened, so you decide to go to Desirae for some advice on which CIS courses to take. Desirae has a list of all CIS courses $C$, and another list $P$ which lists all CIS courses and their associated pre-requisites, if any. She wants to see if you really learned the material in CIS 121, so she wants to quickly test your knowledge. To your surprise, Desirae learned some graph theory from her advisees!

Desirae gives you the graph $G(C, P)$ whose edges $P$ are a mix of directed and undirected edges. To be more precise, she gives you the list $P = P_1 \cup P_2$ where $P_1$ is the list of courses with their associated pre-requisites (i.e., every edge in $P_1$ is an edge from a pre-requisite to a course), and $P_2$ is just a list of pairs of courses (i.e., undirected edges). You can assume that Desirae gives you $P_1$ and $P_2$ as two separate adjacency lists. She promises you that the listings in $P_1$ are correct, and that therefore you can safely assume no cycles exist in $G(C, P_1)$. Your task is to assign a direction to each of the edges in $P_2$ so that $G(C, P)$ remains acyclic. That is, Desirae wants you to tell her for each pair $(c_1, c_2) \in P_2$, which course could be a pre-requisite of the other, so that once everything is put together it all could make sense.

Desirae has no time to waste; she wants a linear time $O(|C| + |P|)$ time algorithm that assigns a direction to each of the edges in $P_2$ such that $G(C, P)$ remains acyclic.

**Reminder from Desirae:** Advance Registration ends November 11th at 11:59pm ☺

2. **[20 pts] Negative Edge Weights** We discussed in lecture and recitation that Dijkstra’s algorithm fails on graphs with negative edge weights. However, with a bit of cleverness, we can use Dijkstra to compute shortest paths in certain graphs with negative weights.

   a. **[5]** Prove or disprove: if there is a single negative weight edge coming out of $s$ (the source) and all other edges have strictly positive weight, then Dijkstra’s algorithm returns the correct answer. You may assume there are no negative weight cycles in this graph.

   b. **[15]** Consider a weighted, directed graph $G = (V, E)$ with two negative edge weights, and all other edges with strictly positive weight. Your goal is to give an $O((m+n) \log n)$ time algorithm to find the shortest distance from $s$ to all other $v \in V$. You are guaranteed that one of the negative edge weights is $s \rightarrow p$ where $p \in V \setminus \{s\}$, but the other negative weight edge can be anywhere. You may assume that introducing these negative weight edges does not cause a negative weight cycle.

   For this problem only, a clear description of your algorithm, an analysis of its time complexity, and a brief justification (∼2-5 sentences) of your approach is sufficient. No detailed proof of correctness is required.

   *Problem 3 is on the next page*
3. [15 pts] MST Weights  Consider a simple undirected graph $G = (V, E)$ with each edge weight equal to 1, and $|V| = n$ and $|E| = m$. For each part below, give the answer and provide a brief justification as to how you arrived to your answer.

   a. [2] Let $T$ be an MST of $G$. What is the weight of $T$? Recall the weight of a tree is just the sum of the weights of all the edges in the tree.

   b. [3] Let’s consider a modification to $G$. Assume we change two edge weights from 1 to now be $\frac{1}{2}$. Now, let $T$ be an MST of the modified $G$. What is the weight of $T$?

   c. [5] Let’s consider another modification to the original $G$. Assume we change three edge weights from 1 to now be $\frac{1}{2}$. Now, let $T$ be an MST of the modified $G$. What are the minimum and maximum possible weights of $T$?

   d. [5] Let’s consider another modification to the original $G$. Assume we change $p$ edge weights from 1 to now be $\frac{1}{2}$, where $p < n$. Now, let $T$ be an MST of the modified $G$. What is the minimum possible weight of $T$?

   5pt Extra Credit:  In the modified graph from part d, what is the maximum possible weight of $T$?

   Note: TAs will not help with this extra credit question.

Feedback:  How long did you spend on this assignment? What did you think of it, and how can we improve written assignments in the future? Let us know anonymously at: https://tinyurl.com/CIS121-18fa-hw-feedback

We really appreciate your feedback!