Binary Search Trees Example Problems—Monday, October 1 / Tuesday, October 2

Problem 1:
Write recursive versions of tree-min and tree-max.

Problem 2
Number of elements smaller than root using preorder traversal of a BST. Given a preorder traversal of a BST. The task is to find the number of elements less than root.

Examples:
- Input: preorder[] = 3, 2, 1, 0, 5, 4, 6. Output: 3
- Input: preorder[] = 5, 4, 3, 2, 1. Output: 4

Problem 3
Let $T$ be a binary search tree whose keys are distinct, let $x$ be a leaf node, and let $y$ be its parent. Show that $y$.key is either the smallest key in $T$ larger than $x$.key or the largest key in $T$ smaller than $x$.key.

Problem 4
Is the operation of deletion commutative in the sense that deleting $x$ and then $y$ from a binary search tree leaves the same tree as deleting $y$ and then $x$? Argue why it is or give a counterexample.

Problem 5
We can sort a given set of $n$ numbers by first building a binary search tree containing these numbers (using insert repeatedly to insert the numbers one by one) and then printing the numbers by an inorder tree walk. What are the worst-case and best-case running times for this sorting algorithm?