CIS 121

Binary Search Trees
A Binary Search Tree is a Binary Tree in which, at every node \( v \), the values stored in the left subtree of \( v \) are less than the value at \( v \) and the values stored in the right subtree are greater.

The elements in the BST must be comparable.

Duplicates are not allowed in our discussion.

Note that each subtree of a BST is also a BST.
A BST of integers

Describe the values which might appear in the subtrees labeled A, B, C, and D
The SearchTree ADT

- A search tree is a binary search tree which stores homogeneous elements with no duplicates.
- It is dynamic.
- The elements are ordered in the following ways
  - inorder -- as dictated by compareTo()
  - preorder, postorder, levelorder -- as dictated by the structure of the tree
BST Implementation

public class BinarySearchTree<AnyType extends Comparable<? super AnyType>>{
    private static class BinaryNode<AnyType>{
        // Constructors
        BinaryNode( AnyType theElement )
            { this( theElement, null, null ); }

        BinaryNode( AnyType theElement,
            BinaryNode<AnyType> lt, BinaryNode<AnyType> rt )
            { element = theElement; left = lt; right = rt; }

        AnyType element;       // The data in the node
        BinaryNode<AnyType> left;  // Left child reference
        BinaryNode<AnyType> right;  // Right child reference
    }
}
BST Implementation (2)

```java
private BinaryNode<AnyType> root;

public BinarySearchTree() {
    root = null;
}

public void makeEmpty() {
    root = null;
}

public boolean isEmpty() {
    return root == null;
}
```
BST “contains” Method

class BST {
    public boolean contains( AnyType x ) {
        return contains( x, root );
    }

    private boolean contains( AnyType x, BinaryNode<AnyType> t ) {
        if( t == null )
            return false;

        int compareResult = x.compareTo( t.element );

        if( compareResult < 0 )
            return contains( x, t.left );
        else if( compareResult > 0 )
            return contains( x, t.right );
        else
            return true;    // Match
    }
}
Performance of “contains”

- Searching in randomly built BST is $O(lg \, n)$ on average
  - but generally, a BST is not randomly built

- Asymptotic performance is $O$(height) in all cases
Implementation of printTree

```java
public void printTree() {
    printTree(root);
}

private void printTree(BinaryNode<AnyType> t) {
    if (t != null) {
        printTree(t.left);
        System.out.println(t.element);
        printTree(t.right);
    }
}
```
public AnyType findMin() {
    if (isEmpty()) throw new UnderflowException();
    return findMin(root).element;
}

public AnyType findMax() {
    if (isEmpty()) throw new UnderflowException();
    return findMax(root).element;
}

public void insert( AnyType x ) {
    root = insert(x, root);
}

public void remove( AnyType x ) {
    root = remove(x, root);
}
The insert Operation

private BinaryNode<AnyType>
insert( AnyType x, BinaryNode<AnyType> t ) {
    // recursively traverses the tree looking for a null pointer at the point of insertion.

    // If found, constructs a new node and stitches it into the tree.

    // If duplicate found, simply returns with no insertion done.

}
The remove Operation

```java
private BinaryNode<AnyType> remove( AnyType x, BinaryNode<AnyType> t ) {
    if( t == null )
        return t;   // Item not found; do nothing
    int compareResult = x.compareTo( t.element );
    if( compareResult < 0 )
        t.left = remove( x, t.left );
    else if( compareResult > 0 )
        t.right = remove( x, t.right );
    else if( t.left != null && t.right != null ){ // 2 children
        t.element = findMin( t.right ).element;
        t.right = remove( t.element, t.right );
    }
    else  // one child or leaf
        t = ( t.left != null ) ? t.left : t.right;
    return t;
}
```
Implementations of find Max and Min

private BinaryNode<AnyType> findMin( BinaryNode<AnyType> t ) {
    // recursively or iteratively find the min
}

private BinaryNode<AnyType> findMax( BinaryNode<AnyType> t ) {
    // recursively or iteratively find the max
}
Performance of BST methods

What is the asymptotic performance of each of the BST methods?

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Building a BST

- Given an array of elements, what is the performance (best/worst/average) of building a BST from scratch?
Predecessor in BST

- Predecessor of a node $v$ in a BST is the node that holds the data value that immediately precedes the data at $v$ in order.

Finding predecessor

- $v$ has a left subtree
  - then predecessor must be the largest value in the left subtree (the rightmost node in the left subtree)
- $v$ does not have a left subtree
  - predecessor is the first node on path back to root that does not have $v$ in its left subtree
**Successor in BST**

- Successor of a node v in a BST is the node that holds the data value that immediately follows the data at v in order.

**Finding Successor**

- v has right subtree
  - successor is smallest value in right subtree (the leftmost node in the right subtree)

- v does not have right subtree
  - successor is first node on path back to root that does not have v in its right subtree
Tree Iterators

As we know there are several ways to traverse through a BST. For the user to do so, we must supply different kind of iterators. The iterator type defines how the elements are traversed.

- InOrderIterator<T> inOrderIterator();
- PreOrderIterator<T> preOrderIterator();
- PostOrderIterator<T> postOrderIterator();
- LevelOrderIterator<T> levelOrderIterator();
public static void main (String args[]) {
    BinarySearchTree<Integer> tree = new BinarySearchTree<Integer>();

    // store some ints into the tree
    
    InOrderIterator<Integer> itr =
        tree.inOrderIterator();
    while (itr.hasNext()) {
        Object x = itr.next();
        // do something with x
    }
}
The InOrderIterator is a Disguised List Iterator

// An InOrderIterator that uses a list to store
// the complete in-order traversal
import java.util.*;
class InOrderIterator<T> {
    Iterator<T> _listIter;
    List<T> _theList;

    T next()
    {  /*TBD*/
    }

    boolean hasNext()
    {  /*TBD*/
    }

    InOrderIterator(BinarySearchTree.BinaryNode<T> root)
    {  /*TBD*/
    }
}
List-Based InOrderIterator Methods

//constructor
InOrderIterator( BinarySearchTree.BinaryNode<T> root )
{
    fillListInorder( _theList, root );
    _listIter = _theList.iterator( );
}

// constructor helper function
void fillListInorder (List<T> list,  
                      BinarySearchTree.BinaryNode<T> node)
{
    if (node == null) return;
    fillListInorder( list, node.left );
    list.add( node.element );
    fillListInorder( list, node.right );
}
List-based InOrderIterator Methods

Call List Iterator Methods

T next()
{
    return _listIter.next();
}

boolean hasNext()
{
    return _listIter.hasNext();
}
InOrderIterator Class with a Stack

// An InOrderIterator that uses a stack to mimic recursive traversal
class InOrderIterator
{
    Stack<BinarySearchTree.BinaryNode<T>> _theStack;

    //constructor
    InOrderIterator(BinarySearchTree.BinaryNode<T> root){
        _theStack = new Stack();
        fillStack( root );
    }

    // constructor helper function
    void fillStack(BinarySearchTree.BinaryNode<T> node){
        while(node != null){
            _theStack.push(node);
            node = node.left;
        }
    }
}
Stack-Based InOrderIterator

```java
T next(){
    BinarySearchTree.BinaryNode<T> topNode = null;
    try {
        topNode = _theStack.pop();
    }catch (EmptyStackException e)
    {
        return null;
    }
    if(topNode.right != null){
        fillStack(topNode.right);
    }
    return topNode.element;
}

boolean hasNext(){
    return !_theStack.empty();
}
```
More Recursive BST Methods

- boolean isBST (BinaryNode<T> t)
  returns true if the Binary tree is a BST

- int countFullNodes (BinaryNode<T> t)
  returns the number of full nodes (those with 2 children) in a binary tree

- int countLeaves (BinaryNode<T> t)
  counts the number of leaves in a Binary Tree