Minimum Spanning Trees—Asynchronous

Readings

- Lecture Notes Chapter 21: Minimum Spanning Trees

Problems

Problem 1
Does Kruskal’s algorithm work on a graph with negative weights? How about Prim’s?

Problem 2
Say we have some MST, $T$, in a positively weighted graph $G$. Construct a graph $G'$ where for any weight $w(e)$ for edge $e$ in $G$, we have weights $(w(e))^2$ in $G'$. Does $T$ still remain an MST in $G'$? Prove your answer. Now if $G$ also had negative weights, would your answer change from the previous part? Prove your answer.

Problem 3
Imagine we have a graph $G$ where all edge weights are equal. Design an algorithm to efficiently find an MST of $G$. Analyze the running time.

Problem 4
Suppose that we have found an MST $T$ of a graph $G$, but soon after, we are told that an edge not in $T$ has a lower weight than we at first thought, and as such our MST is now invalid. Is it guaranteed that we can fix our tree by removing an edge and adding a different one? If so, explain how. If not, provide a counterexample.