CIS 121-217—Homework 17

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Problem 1

Since $f$ is continuous on the closed interval $[a, b]$, by the Extreme Value Theorem, the function $f$ takes on a maximum value $M$ and a minimum value $N$ on $[a, b]$. Then

$$(b - a)N \leq \int_a^b f(x) \, dx \leq (b - a)M$$

so

$$N \leq \frac{1}{b - a} \int_a^b f(x) \, dx \leq M$$

By the Intermediate Value Theorem, there must exist a value of $c$ with $a \leq c \leq b$ such that

$$f(c) = \frac{1}{b - a} \int_a^b f(x) \, dx.$$  \hfill (1)$$

Note: I can make a reference to a labeled thing like equation [1] with \autoref, or I can do it like with \autoref to get Equation 1. Here’s a \QED tombstone to mark the end of my solution.

Problem 2

I can reference another problem like this: See Problem 3 b or Problem 1

Problem 3

3 a

The Ackermann function is

$$A(m, n) := \begin{cases} 
  n + 1 & \text{if } m = 0 \\
  A(m - 1, 1) & \text{if } m > 0 \text{ and } n = 0 \\
  A(m - 1, A(m, n - 1)) & \text{if } m > 0 \text{ and } n > 0
\end{cases}.$$  \hfill \Box

3 b

This is how you do an aligned equation environment:

$$d(f, h) = \int_a^b |f(x) - h(x)| \, dx$$

\begin{align*}
  &= \int_a^b |f(x) - g(x) + g(x) - h(x)| \, dx \\
  &\leq \int_a^b (|f(x) - g(x)| + |g(x) - h(x)|) \, dx \\
  &= \int_a^b |f(x) - g(x)| \, dx + \int_a^b |g(x) - h(x)| \, dx \\
  &= d([f], [g]) + d([g], [h])
\end{align*}  \hfill \Box
or you can do this to suppress numberings for specific lines

\[
d(f, h) = \int_a^b |f(x) - h(x)| \, dx \\
= \int_a^b |f(x) - g(x) + g(x) - h(x)| \, dx \\
\leq \int_a^b (|f(x) - g(x)| + |g(x) - h(x)|) \, dx \\
= \int_a^b |f(x) - g(x)| \, dx + \int_a^b |g(x) - h(x)| \, dx \\
= d([f], [g]) + d([g], [h])
\]

so you can have a reference to just \([2]\).