CIS 121-217—Homework 17

David Xu
davix@seas.upenn.edu

Partners: Rajiv Gandhi, Eric Eaton.

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Problem 1

Since f is continuous on the closed interval $[a, b]$, by the Extreme Value Theorem, the function f takes on a maximum value M and a minimum value N on $[a, b]$. Then

$$(b-a)N \leq \int_a^b f(x)\,\mathrm{d}x \leq (b-a)M$$

so

$$N \leq \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x \leq M$$

By the Intermediate Value Theorem, there must exist a value of c with $a \leq c \leq b$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x. \tag{1}$$

Note: I can make a reference to a labeled thing like equation (1) with `\eqref`, or I can do it like with `\autoref` to get Equation 1. Here's a `QED` tombstone to mark the end of my solution. □

Problem 2

I can reference another problem like this: See Problem 3 b or Problem 1.

Problem 3

3 a

The Ackermann function is

$$A(m,n) \coloneqq \begin{cases} n+1 & \text{if } m = 0 \\ A(m-1, 1) & \text{if } m > 0 \text{ and } n = 0 \\ A(m-1, A(m, n-1)) & \text{if } m > 0 \text{ and } n > 0 \end{cases}.$$ □

3 b

This is how you do an aligned equation environment:

$$\begin{aligned} d(f,h) &= \int_a^b |f(x) - h(x)|\,\mathrm{d}x \\ &= \int_a^b |f(x) - g(x) + g(x) - h(x)|\,\mathrm{d}x \\ &\leq \int_a^b (|f(x) - g(x)| + |g(x) - h(x)|)\,\mathrm{d}x \\ &= \int_a^b |f(x) - g(x)|\,\mathrm{d}x + \int_a^b |g(x) - h(x)|\,\mathrm{d}x \\ &= d([f],[g]) + d([g],[h]) \end{aligned}$$ □

or you can do this to suppress numberings for specific lines

$$
\begin{aligned}
d(f,h) &= \int_a^b |f(x) - h(x)| \,\mathrm{d}x \\
&= \int_a^b |f(x) - g(x) + g(x) - h(x)| \,\mathrm{d}x \\
&\leq \int_a^b (|f(x) - g(x)| + |g(x) - h(x)|) \,\mathrm{d}x \\
&= \int_a^b |f(x) - g(x)| \,\mathrm{d}x + \int_a^b |g(x) - h(x)| \,\mathrm{d}x \\
&= d([f],[g]) + d([g],[h]) \qquad (2)
\end{aligned}
$$

so you can have a reference to just (2). □